Computation of positively graded filiform Lie algebras over C

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Summary table (starting on the next page)

Explanation of table

- Column 1 (search) - A character string for text searching purposes
- Column 2 (algebra) - The subclass of positively graded filiform Lie algebra(s)
- Column 3 (Jac) - A check indicates that the class is nonempty
- Column 4 (sol) - Number of Lie algebras in the class

search	algebra	Jac	sol
m1A25	$\mathfrak{m}_{1A}(2,5)$	$\surd$	1
m2A26	$\mathfrak{m}_{2A}(2,6)$	$\surd$	1
m1A36	$\mathfrak{m}_{1A}(3,6)$	$\surd$	1
m1A27	$\mathfrak{m}_{1A}(2,7)$	$\surd$	1
m3A27	$\mathfrak{m}_{3A}(2,7)$	$\surd$	∞
m2A37	$\mathfrak{m}_{2A}(3,7)$	$\surd$	1
m1A47	$\mathfrak{m}_{1A}(4,7)$	$\surd$	1
m2A28	$\mathfrak{m}_{2A}(2,8)$	$\surd$	1
m4A28	$\mathfrak{m}_{4A}(2,8)$	$\surd$	∞
m1A38	$\mathfrak{m}_{1A}(3,8)$	$\surd$	1
m3A38	$\mathfrak{m}_{3A}(3,8)$	$\surd$	∞
m2A48	$\mathfrak{m}_{2A}(4,8)$	$\surd$	1
m1A58	$\mathfrak{m}_{1A}(5,8)$	$\surd$	1
m1A29	$\mathfrak{m}_{1A}(2,9)$	$\surd$	1
m3A29	$\mathfrak{m}_{3A}(2,9)$	$\surd$	1
m5A29	$\mathfrak{m}_{5A}(2,9)$	$\surd$	∞
m2A39	$\mathfrak{m}_{2A}(3,9)$	$\surd$	1
m4A39	$\mathfrak{m}_{4A}(3,9)$	$\surd$	∞
m1A49	$\mathfrak{m}_{1A}(4,9)$	$\surd$	1
m3A49	$\mathfrak{m}_{3A}(4,9)$	$\surd$	∞
m2A59	$\mathfrak{m}_{2A}(5,9)$	$\surd$	1
m1A69	$\mathfrak{m}_{1A}(6,9)$	$\surd$	1
m2A210	$\mathfrak{m}_{2A}(2,10)$	$\surd$	1
m4A210	$\mathfrak{m}_{4A}(2,10)$	$\surd$	1
m6A210	$\mathfrak{m}_{6A}(2,10)$	$\surd$	∞
m1A310	$\mathfrak{m}_{1A}(3,10)$	$\surd$	1
m3A310	$\mathfrak{m}_{3A}(3,10)$	$\surd$	∞
m5A310	$\mathfrak{m}_{5A}(3,10)$	$\surd$	∞
m2A410	$\mathfrak{m}_{2A}(4,10)$	$\surd$	1
m4A410	$\mathfrak{m}_{4A}(4,10)$	$\surd$	∞
m1A510	$\mathfrak{m}_{1A}(5,10)$	$\surd$	1
m3A510	$\mathfrak{m}_{3A}(5,10)$	$\surd$	∞
m2A610	$\mathfrak{m}_{2A}(6,10)$	$\surd$	1
m1A710	$\mathfrak{m}_{1A}(7,10)$	$\surd$	1
m1A211	$\mathfrak{m}_{1A}(2,11)$	$\surd$	1
m3A211	$\mathfrak{m}_{3A}(2,11)$	$\surd$	1
m5A211	$\mathfrak{m}_{5A}(2,11)$	$\surd$	1
m7A211	$\mathfrak{m}_{7A}(2,11)$	$\surd$	∞
m2A311	$\mathfrak{m}_{2A}(3,11)$	$\surd$	1
m4A311	$\mathfrak{m}_{4A}(3,11)$	$\surd$	1
m6A311	$\mathfrak{m}_{6A}(3,11)$	$\surd$	∞
m1A411	$\mathfrak{m}_{1A}(4,11)$	$\surd$	1

search	algebra	Jac	sol
m3A411	$\mathfrak{m}_{3A}(4,11)$	$\surd$	∞
m5A411	$\mathfrak{m}_{5A}(4,11)$	$\surd$	∞
m2A511	$\mathfrak{m}_{2A}(5,11)$	$\surd$	1
m4A511	$\mathfrak{m}_{4A}(5,11)$	$\surd$	∞
m1A611	$\mathfrak{m}_{1A}(6,11)$	$\surd$	1
m3A611	$\mathfrak{m}_{3A}(6,11)$	$\surd$	∞
m2A711	$\mathfrak{m}_{2A}(7,11)$	$\surd$	1
m1A811	$\mathfrak{m}_{1A}(8,11)$	$\surd$	1
m2A212	$\mathfrak{m}_{2A}(2,12)$	$\surd$	1
m4A212	$\mathfrak{m}_{4A}(2,12)$		0
m6A212	$\mathfrak{m}_{6A}(2,12)$		0
m8A212	$\mathfrak{m}_{8A}(2,12)$	$\surd$	2
m1A312	$\mathfrak{m}_{1A}(3,12)$	$\surd$	1
m3A312	$\mathfrak{m}_{3A}(3,12)$	$\surd$	∞
m5A312	$\mathfrak{m}_{5A}(3,12)$	$\surd$	∞
m7A312	$\mathfrak{m}_{7A}(3,12)$	$\surd$	∞
m2A412	$\mathfrak{m}_{2A}(4,12)$	$\surd$	1
m4A412	$\mathfrak{m}_{4A}(4,12)$	$\surd$	∞
m6A412	$\mathfrak{m}_{6A}(4,12)$	$\surd$	∞
m1A512	$\mathfrak{m}_{1A}(5,12)$	$\surd$	1
m3A512	$\mathfrak{m}_{3A}(5,12)$	$\surd$	∞
m5A512	$\mathfrak{m}_{5A}(5,12)$	$\surd$	∞
m2A612	$\mathfrak{m}_{2A}(6,12)$	$\surd$	1
m4A612	$\mathfrak{m}_{4A}(6,12)$	$\surd$	∞
m1A712	$\mathfrak{m}_{1A}(7,12)$	$\surd$	1
m3A712	$\mathfrak{m}_{3A}(7,12)$	$\surd$	∞
m2A812	$\mathfrak{m}_{2A}(8,12)$	$\surd$	1
m1A912	$\mathfrak{m}_{1A}(9,12)$	$\surd$	1
m1A213	$\mathfrak{m}_{1A}(2,13)$	$\surd$	1
m3A213	$\mathfrak{m}_{3A}(2,13)$	$\surd$	1
m9A213	$\mathfrak{m}_{9A}(2,13)$	$\surd$	2
m2A313	$\mathfrak{m}_{2A}(3,13)$	$\surd$	1
m4A313	$\mathfrak{m}_{4A}(3,13)$	$\surd$	1
m6A313	$\mathfrak{m}_{6A}(3,13)$	$\surd$	1
m8A313	$\mathfrak{m}_{8A}(3,13)$	$\surd$	∞
m1A413	$\mathfrak{m}_{1A}(4,13)$	$\surd$	1
m3A413	$\mathfrak{m}_{3A}(4,13)$	$\surd$	∞
m5A413	$\mathfrak{m}_{5A}(4,13)$	$\surd$	∞
m7A413	$\mathfrak{m}_{7A}(4,13)$	$\surd$	∞
m2A513	$\mathfrak{m}_{2A}(5,13)$	$\surd$	1
m4A513	$\mathfrak{m}_{4A}(5,13)$	$\surd$	∞
m6A513	$\mathfrak{m}_{6A}(5,13)$	$\surd$	∞

search	algebra	Jac	sol
m1A613	$\mathfrak{m}_{1A}(6,13)$	$\surd$	1
m3A613	$\mathfrak{m}_{3A}(6,13)$	$\surd$	∞
m5A613	$\mathfrak{m}_{5A}(6,13)$	$\surd$	∞
m2A713	$\mathfrak{m}_{2A}(7,13)$	$\surd$	1
m4A713	$\mathfrak{m}_{4A}(7,13)$	$\surd$	∞
m1A813	$\mathfrak{m}_{1A}(8,13)$	$\surd$	1
m3A813	$\mathfrak{m}_{3A}(8,13)$	$\surd$	∞
m2A913	$\mathfrak{m}_{2A}(9,13)$	$\surd$	1
m1A1013	$\mathfrak{m}_{1A}(10,13)$	$\surd$	1
m2A214	$\mathfrak{m}_{2A}(2,14)$	$\surd$	1
m4A214	$\mathfrak{m}_{4A}(2,14)$		0
m10A214	$\mathfrak{m}_{10A}(2,14)$	$\surd$	2
m1A314	$\mathfrak{m}_{1A}(3,14)$	$\surd$	1
m3A314	$\mathfrak{m}_{3A}(3,14)$	$\surd$	∞
m5A314	$\mathfrak{m}_{5A}(3,14)$	$\surd$	1
m7A314	$\mathfrak{m}_{7A}(3,14)$	$\surd$	1
m9A314	$\mathfrak{m}_{9A}(3,14)$	$\surd$	∞
m2A414	$\mathfrak{m}_{2A}(4,14)$	$\surd$	1
m4A414	$\mathfrak{m}_{4A}(4,14)$	$\surd$	∞
m6A414	$\mathfrak{m}_{6A}(4,14)$	$\surd$	∞
m8A414	$\mathfrak{m}_{8A}(4,14)$	$\surd$	∞
m1A514	$\mathfrak{m}_{1A}(5,14)$	$\surd$	1
m3A514	$\mathfrak{m}_{3A}(5,14)$	$\surd$	∞
m5A514	$\mathfrak{m}_{5A}(5,14)$	$\surd$	∞
m7A514	$\mathfrak{m}_{7A}(5,14)$	$\surd$	∞
m2A614	$\mathfrak{m}_{2A}(6,14)$	$\surd$	1
m4A614	$\mathfrak{m}_{4A}(6,14)$	$\surd$	∞
m6A614	$\mathfrak{m}_{6A}(6,14)$	$\surd$	∞
m1A714	$\mathfrak{m}_{1A}(7,14)$	$\surd$	1
m3A714	$\mathfrak{m}_{3A}(7,14)$	$\surd$	∞
m5A714	$\mathfrak{m}_{5A}(7,14)$	$\surd$	∞
m2A814	$\mathfrak{m}_{2A}(8,14)$	$\surd$	1
m4A814	$\mathfrak{m}_{4A}(8,14)$	$\surd$	∞
m1A914	$\mathfrak{m}_{1A}(9,14)$	$\surd$	1
m3A914	$\mathfrak{m}_{3A}(9,14)$	$\surd$	∞
m2A1014	$\mathfrak{m}_{2A}(10,14)$	$\surd$	1
m1A1114	$\mathfrak{m}_{1A}(11,14)$	$\surd$	1
m1A215	$\mathfrak{m}_{1A}(2,15)$	$\surd$	1
m3A215	$\mathfrak{m}_{3A}(2,15)$	$\surd$	1
m11A215	$\mathfrak{m}_{11A}(2,15)$	$\surd$	2
m2A315	$\mathfrak{m}_{2A}(3,15)$	$\surd$	1
m4A315	$\mathfrak{m}_{4A}(3,15)$	$\surd$	1

search	algebra	Jac	sol
m6A315	$\mathfrak{m}_{6A}(3,15)$		0
m8A315	$\mathfrak{m}_{8A}(3,15)$		0
m10A315	$\mathfrak{m}_{10A}(3,15)$	$\surd$	5
m1A415	$\mathfrak{m}_{1A}(4,15)$	$\surd$	1
m3A415	$\mathfrak{m}_{3A}(4,15)$	$\surd$	∞
m5A415	$\mathfrak{m}_{5A}(4,15)$	$\surd$	∞
m7A415	$\mathfrak{m}_{7A}(4,15)$	$\surd$	∞
m9A415	$\mathfrak{m}_{9A}(4,15)$	$\surd$	∞
m2A515	$\mathfrak{m}_{2A}(5,15)$	$\surd$	1
m4A515	$\mathfrak{m}_{4A}(5,15)$	$\surd$	∞
m6A515	$\mathfrak{m}_{6A}(5,15)$	$\surd$	∞
m8A515	$\mathfrak{m}_{8A}(5,15)$	$\surd$	∞
m1A615	$\mathfrak{m}_{1A}(6,15)$	$\surd$	1
m3A615	$\mathfrak{m}_{3A}(6,15)$	$\surd$	∞
m5A615	$\mathfrak{m}_{5A}(6,15)$	$\surd$	∞
m7A615	$\mathfrak{m}_{7A}(6,15)$	$\surd$	∞
m2A715	$\mathfrak{m}_{2A}(7,15)$	$\surd$	1
m4A715	$\mathfrak{m}_{4A}(7,15)$	$\surd$	∞
m6A715	$\mathfrak{m}_{6A}(7,15)$	$\surd$	∞
m1A815	$\mathfrak{m}_{1A}(8,15)$	$\surd$	1
m3A815	$\mathfrak{m}_{3A}(8,15)$	$\surd$	∞
m5A815	$\mathfrak{m}_{5A}(8,15)$	$\surd$	∞
m2A915	$\mathfrak{m}_{2A}(9,15)$	$\surd$	1
m4A915	$\mathfrak{m}_{4A}(9,15)$	$\surd$	∞
m1A1015	$\mathfrak{m}_{1A}(10,15)$	$\surd$	1
m3A1015	$\mathfrak{m}_{3A}(10,15)$	$\surd$	∞
m2A1115	$\mathfrak{m}_{2A}(11,15)$	$\surd$	1
m1A1215	$\mathfrak{m}_{1A}(12,15)$	$\surd$	1
m2B26	$\mathfrak{m}_{2B}(2,6)$	$\surd$	1
m2B28	$\mathfrak{m}_{2B}(2,8)$		0
m4B28	$\mathfrak{m}_{4B}(2,8)$	$\surd$	1
m3B38	$\mathfrak{m}_{3B}(3,8)$	$\surd$	1
m2B48	$\mathfrak{m}_{2B}(4,8)$	$\surd$	1
m2B210	$\mathfrak{m}_{2B}(2,10)$		0
m4B210	$\mathfrak{m}_{4B}(2,10)$		0
m6B210	$\mathfrak{m}_{6B}(2,10)$	$\surd$	2
m3B310	$\mathfrak{m}_{3B}(3,10)$	$\surd$	1
m5B310	$\mathfrak{m}_{5B}(3,10)$	$\surd$	∞
m2B410	$\mathfrak{m}_{2B}(4,10)$		0
m4B410	$\mathfrak{m}_{4B}(4,10)$	$\surd$	1
m3B510	$\mathfrak{m}_{3B}(5,10)$	$\surd$	1
m2B610	$\mathfrak{m}_{2B}(6,10)$	$\surd$	1

search	algebra	Jac	sol
m2B212	$\mathfrak{m}_{2B}(2,12)$		0
m4B212	$\mathfrak{m}_{4B}(2,12)$		0
m6B212	$\mathfrak{m}_{6B}(2,12)$		0
m8B212	$\mathfrak{m}_{8B}(2,12)$	$\surd$	4
m3B312	$\mathfrak{m}_{3B}(3,12)$		0
m5B312	$\mathfrak{m}_{5B}(3,12)$		0
m7B312	$\mathfrak{m}_{7B}(3,12)$	$\surd$	2
m2B412	$\mathfrak{m}_{2B}(4,12)$		0
m4B412	$\mathfrak{m}_{4B}(4,12)$	$\surd$	1
m6B412	$\mathfrak{m}_{6B}(4,12)$	$\surd$	∞
m3B512	$\mathfrak{m}_{3B}(5,12)$	$\surd$	1
m5B512	$\mathfrak{m}_{5B}(5,12)$	$\surd$	∞
m2B612	$\mathfrak{m}_{2B}(6,12)$		0
m4B612	$\mathfrak{m}_{4B}(6,12)$	$\surd$	1
m3B712	$\mathfrak{m}_{3B}(7,12)$	$\surd$	1
m2B812	$\mathfrak{m}_{2B}(8,12)$	$\surd$	1
m2B214	$\mathfrak{m}_{2B}(2,14)$		0
m4B214	$\mathfrak{m}_{4B}(2,14)$		0
m10B214	$\mathfrak{m}_{10B}(2,14)$		0
m3B314	$\mathfrak{m}_{3B}(3,14)$		0
m5B314	$\mathfrak{m}_{5B}(3,14)$		0
m7B314	$\mathfrak{m}_{7B}(3,14)$		0
m9B314	$\mathfrak{m}_{9B}(3,14)$	$\surd$	4
m2B414	$\mathfrak{m}_{2B}(4,14)$		0
m4B414	$\mathfrak{m}_{4B}(4,14)$	$\surd$	1
m6B414	$\mathfrak{m}_{6B}(4,14)$	$\surd$	1
m8B414	$\mathfrak{m}_{8B}(4,14)$	$\surd$	∞
m3B514	$\mathfrak{m}_{3B}(5,14)$		0
m5B514	$\mathfrak{m}_{5B}(5,14)$	$\surd$	1
m7B514	$\mathfrak{m}_{7B}(5,14)$	$\surd$	∞
m2B614	$\mathfrak{m}_{2B}(6,14)$		0
m4B614	$\mathfrak{m}_{4B}(6,14)$	$\surd$	1
m6B614	$\mathfrak{m}_{6B}(6,14)$	$\surd$	∞
m3B714	$\mathfrak{m}_{3B}(7,14)$	$\surd$	1
m5B714	$\mathfrak{m}_{5B}(7,14)$	$\surd$	∞
m2B814	$\mathfrak{m}_{2B}(8,14)$		0
m4B814	$\mathfrak{m}_{4B}(8,14)$	$\surd$	1
m3B914	$\mathfrak{m}_{3B}(9,14)$	$\surd$	1
m2B1014	$\mathfrak{m}_{2B}(10,14)$	$\surd$	1

Algebra details

$\mathfrak{m}_{1A}(2,5)$

m1A25 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_2,e_3]=e_5$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,6)$

m2A26 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_2,e_3]=e_5 \qquad [e_2,e_4]=e_6$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(3,6)$

m1A36 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_2,e_3]=e_6$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2,7)$

m1A27 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_2,e_5]=e_7$$
$$[e_3,e_4]=-e_7$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2,7)$

m3A27 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned} [e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\ [e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\ [e_1,e_6] &= e_7 & [e_2,e_3] &= e_5 \\ [e_2,e_4] &= e_6 & [e_2,e_5] &= \alpha_{2,5}^{7} e_7 \\ [e_3,e_4] &= \alpha_{3,4}^{7} e_7 \end{aligned}$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 \qquad = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^{7} \to x_1$$
$$\alpha_{2,5}^{7} \to x_2$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(3,7)$

m2A37 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned} [e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\ [e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\ [e_1,e_6] &= e_7 & [e_2,e_3] &= e_6 \\ [e_2,e_4] &= e_7 \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(4,7)$

m1A47 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_7
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,8)$

m2A28 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_2, e_5] &= e_7 & [e_2, e_6] &= 2e_8 \\
[e_3, e_4] &= -e_7 & [e_3, e_5] &= -e_8
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2,8)$

m4A28 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_2, e_3] &= e_5 & [e_2, e_4] &= e_6 \\
[e_2, e_5] &= \alpha_{2,5}^{7} e_7 & [e_2, e_6] &= \alpha_{2,6}^{8} e_8 \\
[e_3, e_4] &= \alpha_{3,4}^{7} e_7 & [e_3, e_5] &= \alpha_{3,5}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha^7_{2,5} - \alpha^7_{3,4} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha^7_{2,5} - \alpha^8_{2,6} - \alpha^8_{3,5} &= 0\\
(e_1, e_3, e_4): &\quad \alpha^7_{3,4} - \alpha^8_{3,5} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^7_{3,4} \to x_1$$
$$\alpha^8_{3,5} \to x_2$$
$$\alpha^8_{2,6} \to x_3$$
$$\alpha^7_{2,5} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_4 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_2 - x_3 + x_4 &= 0\\
(e_1, e_3, e_4): &\quad x_1 - x_2 &= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 1 = 0$$
$$x_2 + x_4 - 1 = 0$$
$$x_3 - 2x_4 + 1 = 0$$

$\mathfrak{m}_{1A}(3, 8)$

m1A38 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_2, e_5] &= e_8 & [e_3, e_4] &= -e_8
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3,8)$

m3A38 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_2,e_3] &= e_6 & [e_2,e_4] &= e_7 \\
[e_2,e_5] &= \alpha_{2,5}^{8} e_8 & [e_3,e_4] &= \alpha_{3,4}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 \qquad = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,5}^{8} &\to x_1 \\
\alpha_{3,4}^{8} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(4,8)$

m2A48 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_2,e_3] &= e_7 & [e_2,e_4] &= e_8
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(5,8)$

m1A58 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_2,e_3]=e_8$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2,9)$

m1A29 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_2,e_7]=e_9$$
$$[e_3,e_6]=-e_9 \qquad [e_4,e_5]=e_9$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2,9)$

m3A29 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_2,e_5]=e_7$$
$$[e_2,e_6]=2e_8 \qquad [e_2,e_7]=0$$
$$[e_3,e_4]=-e_7 \qquad [e_3,e_5]=-e_8$$
$$[e_3,e_6]=2e_9 \qquad [e_4,e_5]=-3e_9$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_7 \\
&[e_2, e_6] = 2e_8 && [e_2, e_7] = \alpha_{2,7}^9 e_9 \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^9 e_9 && [e_4, e_5] = \alpha_{4,5}^9 e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^9 - \alpha_{3,6}^9 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^9 - \alpha_{4,5}^9 - 1 &&= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^9 &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,6}^9 &= 2 \\
\alpha_{4,5}^9 &= -3 \\
\alpha_{2,7}^9 &= 0
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,6}^9 &\to x_1 \\
\alpha_{4,5}^9 &\to x_2 \\
\alpha_{2,7}^9 &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_3 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_1 - x_2 - 1 &&= 0 \\
(e_2, e_3, e_4): &\quad -x_3 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 - 2 = 0$$
$$x_2 + 3 = 0$$
$$x_3 = 0$$

Solution 1:

$$x_1 = 2$$
$$x_2 = -3$$
$$x_3 = 0$$

$\mathfrak{m}_{5A}(2,9)$

m5A29 (this line included for string searching purposes)

Original brackets:

$$\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_4, e_5] = \alpha_{4,5}^{9} e_9
\end{array}$$

Non-trivial Jacobi Tests:

$$\begin{array}{lll}
(e_1, e_2, e_4): & -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 & = 0 \\
(e_1, e_2, e_5): & \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} & = 0 \\
(e_1, e_3, e_4): & \alpha_{3,4}^{7} - \alpha_{3,5}^{8} & = 0 \\
(e_1, e_2, e_6): & \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} & = 0 \\
(e_1, e_3, e_5): & \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} & = 0 \\
(e_2, e_3, e_4): & \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} & = 0
\end{array}$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,7}^{9} \to x_1$$

$$\alpha_{2,6}^{8} \to x_2$$
$$\alpha_{2,5}^{7} \to x_3$$
$$\alpha_{4,5}^{9} \to x_4$$
$$\alpha_{3,6}^{9} \to x_5$$
$$\alpha_{3,4}^{7} \to x_6$$
$$\alpha_{3,5}^{8} \to x_7$$

Jacobi Tests

$$\begin{array}{lll}
(e_1, e_2, e_4): & -x_3 - x_6 + 1 & = 0 \\
(e_1, e_2, e_5): & -x_2 + x_3 - x_7 & = 0 \\
(e_1, e_3, e_4): & x_6 - x_7 & = 0 \\
(e_1, e_2, e_6): & -x_1 + x_2 - x_5 & = 0 \\
(e_1, e_3, e_5): & -x_4 - x_5 + x_7 & = 0 \\
(e_2, e_3, e_4): & x_1 x_6 + x_4 - x_5 & = 0
\end{array}$$

Groebner basis (7 variables, 5 linear, 1 nonlinear)

$$x_1 + x_5 + 2x_7 - 1 = 0$$
$$x_2 + 2x_7 - 1 = 0$$
$$x_3 + x_7 - 1 = 0$$
$$x_4 + x_5 - x_7 = 0$$
$$x_5 x_7 + 2x_5 + 2x_7^2 - 2x_7 = 0$$
$$x_6 - x_7 = 0$$

$\mathfrak{m}_{2A}(3, 9)$

m2A39 (this line included for string searching purposes)

Original brackets:

$$\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_2, e_5] = e_8 \\
[e_2, e_6] = 2e_9 & [e_3, e_4] = -e_8 \\
[e_3, e_5] = -e_9 &
\end{array}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3,9)$

m4A39 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] &= \alpha_{2,6}^{9} e_9 & [e_3, e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] &= \alpha_{3,5}^{9} e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0 \\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,5}^{8} &\to x_1 \\
\alpha_{2,6}^{9} &\to x_2 \\
\alpha_{3,4}^{8} &\to x_3 \\
\alpha_{3,5}^{9} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -x_1 - x_3 + 1 &= 0 \\
(e_1, e_2, e_5):&\quad x_1 - x_2 - x_4 &= 0 \\
(e_1, e_3, e_4):&\quad x_3 - x_4 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 + x_4 - 1 &= 0 \\
x_2 + 2x_4 - 1 &= 0 \\
x_3 - x_4 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(4,9)$

m1A49 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_5] &= e_9 \\
[e_3, e_4] &= -e_9
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4,9)$

m3A49 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_7 \\
[e_2, e_4] &= e_8 & [e_2, e_5] &= \alpha_{2,5}^{9} e_9 \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 \qquad = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^{9} &\to x_1 \\
\alpha_{2,5}^{9} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(5,9)$

m2A59 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_8 \\
[e_2,e_4] &= e_9
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(6,9)$

m1A69 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_9
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,10)$

m2A210 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_2,e_7] &= e_9 & [e_2,e_8] &= 3e_{10} \\
[e_3,e_6] &= -e_9 & [e_3,e_7] &= -2e_{10} \\
[e_4,e_5] &= e_9 & [e_4,e_6] &= e_{10}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2, 10)$

m4A210 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_2, e_5] &= e_7 & [e_2, e_6] &= 2e_8 \\
[e_2, e_7] &= 0 & [e_2, e_8] &= -5e_{10} \\
[e_3, e_4] &= -e_7 & [e_3, e_5] &= -e_8 \\
[e_3, e_6] &= 2e_9 & [e_3, e_7] &= 5e_{10} \\
[e_4, e_5] &= -3e_9 & [e_4, e_6] &= -3e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_2, e_5] &= e_7 & [e_2, e_6] &= 2e_8 \\
[e_2, e_7] &= \alpha_{2,7}^{9} e_9 & [e_2, e_8] &= \alpha_{2,8}^{10} e_{10} \\
[e_3, e_4] &= -e_7 & [e_3, e_5] &= -e_8 \\
[e_3, e_6] &= \alpha_{3,6}^{9} e_9 & [e_3, e_7] &= \alpha_{3,7}^{10} e_{10} \\
[e_4, e_5] &= \alpha_{4,5}^{9} e_9 & [e_4, e_6] &= \alpha_{4,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6):&\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0 \\
(e_1, e_3, e_5):&\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0 \\
(e_2, e_3, e_4):&\quad -\alpha_{2,7}^{9} &&= 0 \\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0 \\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0 \\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0 \\
(e_2, e_3, e_5):&\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0
\end{aligned}
$$

Solution 1:

$$\begin{aligned}
\alpha_{3,7}^{10} &= 5\\
\alpha_{2,7}^{9} &= 0\\
\alpha_{2,8}^{10} &= -5\\
\alpha_{4,5}^{9} &= -3\\
\alpha_{3,6}^{9} &= 2\\
\alpha_{4,6}^{10} &= -3
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,7}^{10} \to x_1$$

$$\alpha_{2,7}^{9} \to x_2$$

$$\alpha_{2,8}^{10} \to x_3$$

$$\alpha_{4,5}^{9} \to x_4$$

$$\alpha_{3,6}^{9} \to x_5$$

$$\alpha_{4,6}^{10} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): \quad & -x_2 - x_5 + 2 && = 0\\
(e_1, e_3, e_5): \quad & -x_4 - x_5 - 1 && = 0\\
(e_2, e_3, e_4): \quad & -x_2 && = 0\\
(e_1, e_2, e_7): \quad & -x_1 + x_2 - x_3 && = 0\\
(e_1, e_3, e_6): \quad & -x_1 + x_5 - x_6 && = 0\\
(e_1, e_4, e_5): \quad & x_4 - x_6 && = 0\\
(e_2, e_3, e_5): \quad & -x_1 - x_3 && = 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 - 5 = 0$$

$$x_2 = 0$$

$$x_3 + 5 = 0$$

$$x_4 + 3 = 0$$

$$x_5 - 2 = 0$$
$$x_6 + 3 = 0$$

Solution 1:

$$x_1 = 5$$
$$x_2 = 0$$
$$x_3 = -5$$
$$x_4 = -3$$
$$x_5 = 2$$
$$x_6 = -3$$

$\mathfrak{m}_{6A}(2, 10)$

m6A210 (this line included for string searching purposes)

Original brackets:

$$\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_2, e_3] = e_5 & [e_2, e_4] = e_6 \\
[e_2, e_5] = \alpha_{2,5}^{7} e_7 & [e_2, e_6] = \alpha_{2,6}^{8} e_8 \\
[e_2, e_7] = \alpha_{2,7}^{9} e_9 & [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_4, e_5] = \alpha_{4,5}^{9} e_9 & [e_4, e_6] = \alpha_{4,6}^{10} e_{10}
\end{array}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha^{7}_{2,5} - \alpha^{7}_{3,4} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha^{7}_{2,5} - \alpha^{8}_{2,6} - \alpha^{8}_{3,5} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha^{7}_{3,4} - \alpha^{8}_{3,5} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha^{8}_{2,6} - \alpha^{9}_{2,7} - \alpha^{9}_{3,6} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha^{8}_{3,5} - \alpha^{9}_{3,6} - \alpha^{9}_{4,5} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha^{9}_{2,7}\alpha^{7}_{3,4} - \alpha^{9}_{3,6} + \alpha^{9}_{4,5} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha^{9}_{2,7} - \alpha^{10}_{2,8} - \alpha^{10}_{3,7} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha^{9}_{3,6} - \alpha^{10}_{3,7} - \alpha^{10}_{4,6} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha^{9}_{4,5} - \alpha^{10}_{4,6} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha^{7}_{2,5}\alpha^{10}_{3,7} + \alpha^{10}_{2,8}\alpha^{8}_{3,5} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{10}_{3,7} \to x_1$$

$$\alpha^{9}_{2,7} \to x_2$$

$$\alpha^{8}_{2,6} \to x_3$$

$$\alpha^{7}_{2,5} \to x_4$$

$$\alpha^{10}_{2,8} \to x_5$$

$$\alpha^{9}_{4,5} \to x_6$$

$$\alpha^{9}_{3,6} \to x_7$$

$$\alpha^{7}_{3,4} \to x_8$$

$$\alpha^{8}_{3,5} \to x_9$$

$$\alpha^{10}_{4,6} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_4 - x_8 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_3 + x_4 - x_9 &&= 0\\
(e_1, e_3, e_4): &\quad x_8 - x_9 &&= 0\\
(e_1, e_2, e_6): &\quad -x_2 + x_3 - x_7 &&= 0\\
(e_1, e_3, e_5): &\quad -x_6 - x_7 + x_9 &&= 0\\
(e_2, e_3, e_4): &\quad x_2x_8 + x_6 - x_7 &&= 0\\
(e_1, e_2, e_7): &\quad -x_1 + x_2 - x_5 &&= 0\\
(e_1, e_3, e_6): &\quad -x_1 - x_{10} + x_7 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{10} + x_6 &&= 0\\
(e_2, e_3, e_5): &\quad -x_1x_4 + x_5x_9 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 8 linear, 1 nonlinear)

$$x_1 + 2x_{10} - x_9 = 0$$
$$-x_{10} + x_2 + 3x_9 - 1 = 0$$
$$x_3 + 2x_9 - 1 = 0$$
$$x_4 + x_9 - 1 = 0$$
$$-3x_{10} + x_5 + 4x_9 - 1 = 0$$
$$-x_{10} + x_6 = 0$$
$$x_{10} + x_7 - x_9 = 0$$
$$x_8 - x_9 = 0$$
$$-x_{10}x_9 - 2x_{10} + 3x_9^2 = 0$$

$\mathfrak{m}_{1A}(3, 10)$

m1A310 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_2, e_7] &= e_{10} & [e_3, e_6] &= -e_{10}\\
[e_4, e_5] &= e_{10}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3, 10)$

m3A310 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_5] = e_8 && [e_2, e_6] = 2e_9 \\
&[e_2, e_7] = \alpha_{2,7}^{10} e_{10} && [e_3, e_4] = -e_8 \\
&[e_3, e_5] = -e_9 && [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
&[e_4, e_5] = \alpha_{4,5}^{10} e_{10} &&
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,7}^{10} \to x_1$$

$$\alpha_{4,5}^{10} \to x_2$$

$$\alpha_{3,6}^{10} \to x_3$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_3 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_2 - x_3 - 1 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 - 2 = 0$$

$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(3, 10)$

m5A310 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_3] = e_6 && [e_2, e_4] = e_7 \\
&[e_2, e_5] = \alpha_{2,5}^{8} e_8 && [e_2, e_6] = \alpha_{2,6}^{9} e_9 \\
&[e_2, e_7] = \alpha_{2,7}^{10} e_{10} && [e_3, e_4] = \alpha_{3,4}^{8} e_8 \\
&[e_3, e_5] = \alpha_{3,5}^{9} e_9 && [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
&[e_4, e_5] = \alpha_{4,5}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1, e_2, e_4): && -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 && = 0 \\
&(e_1, e_2, e_5): && \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} && = 0 \\
&(e_1, e_3, e_4): && \alpha_{3,4}^{8} - \alpha_{3,5}^{9} && = 0 \\
&(e_1, e_2, e_6): && \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} && = 0 \\
&(e_1, e_3, e_5): && \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} && = 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{3,6}^{10} &\to x_1 \\
\alpha_{2,6}^{9} &\to x_2 \\
\alpha_{4,5}^{10} &\to x_3 \\
\alpha_{3,4}^{8} &\to x_4 \\
\alpha_{2,7}^{10} &\to x_5 \\
\alpha_{2,5}^{8} &\to x_6 \\
\alpha_{3,5}^{9} &\to x_7
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_4 - x_6 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_2 + x_6 - x_7 &= 0\\
(e_1, e_3, e_4): &\quad x_4 - x_7 &= 0\\
(e_1, e_2, e_6): &\quad -x_1 + x_2 - x_5 &= 0\\
(e_1, e_3, e_5): &\quad -x_1 - x_3 + x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 + x_5 + 2x_7 - 1 = 0$$
$$x_2 + 2x_7 - 1 = 0$$
$$x_3 - x_5 - 3x_7 + 1 = 0$$
$$x_4 - x_7 = 0$$
$$x_6 + x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(4, 10)$

m2A410 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_2, e_5] &= e_9 & [e_2, e_6] &= 2e_{10}\\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(4, 10)$

m4A410 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_3] = e_7 && [e_2, e_4] = e_8 \\
&[e_2, e_5] = \alpha_{2,5}^{9} e_9 && [e_2, e_6] = \alpha_{2,6}^{10} e_{10} \\
&[e_3, e_4] = \alpha_{3,4}^{9} e_9 && [e_3, e_5] = \alpha_{3,5}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1, e_2, e_4): && -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 && = 0 \\
&(e_1, e_2, e_5): && \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} && = 0 \\
&(e_1, e_3, e_4): && \alpha_{3,4}^{9} - \alpha_{3,5}^{10} && = 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{10} \to x_1$$

$$\alpha_{3,4}^{9} \to x_2$$

$$\alpha_{3,5}^{10} \to x_3$$

$$\alpha_{2,5}^{9} \to x_4$$

Jacobi Tests

$$
\begin{aligned}
&(e_1, e_2, e_4): && -x_2 - x_4 + 1 && = 0 \\
&(e_1, e_2, e_5): && -x_1 - x_3 + x_4 && = 0 \\
&(e_1, e_3, e_4): && x_2 - x_3 && = 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - 2x_4 + 1 = 0$$

$$x_2 + x_4 - 1 = 0$$

$$x_3 + x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(5, 10)$

m1A510 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_2, e_5] = e_{10} \qquad [e_3, e_4] = -e_{10}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(5, 10)$

m3A510 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_2, e_3] = e_8 \qquad [e_2, e_4] = e_9$$
$$[e_2, e_5] = \alpha_{2,5}^{10} e_{10} \qquad [e_3, e_4] = \alpha_{3,4}^{10} e_{10}$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4) : \quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^{10} \to x_1$$
$$\alpha_{2,5}^{10} \to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4) : \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(6, 10)$

m2A610 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_2, e_3] &= e_9 & [e_2, e_4] &= e_{10} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(7, 10)$

m1A710 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_2, e_3] &= e_{10} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2, 11)$

m1A211 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_2, e_9] &= e_{11} \\ [e_3, e_8] &= -e_{11} & [e_4, e_7] &= e_{11} \\ [e_5, e_6] &= -e_{11} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2, 11)$

m3A211 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_9 \\
[e_2, e_8] &= 3e_{10} & [e_2, e_9] &= 0 \\
[e_3, e_6] &= -e_9 & [e_3, e_7] &= -2e_{10} \\
[e_3, e_8] &= 3e_{11} & [e_4, e_5] &= e_9 \\
[e_4, e_6] &= e_{10} & [e_4, e_7] &= -5e_{11} \\
[e_5, e_6] &= 6e_{11}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_9 \\
[e_2, e_8] &= 3e_{10} & [e_2, e_9] &= \alpha_{2,9}^{11} e_{11} \\
[e_3, e_6] &= -e_9 & [e_3, e_7] &= -2e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_4, e_5] &= e_9 \\
[e_4, e_6] &= e_{10} & [e_4, e_7] &= \alpha_{4,7}^{11} e_{11} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8) &: \quad -\alpha_{2,9}^{11} - \alpha_{3,8}^{11} + 3 &&= 0 \\
(e_1, e_3, e_7) &: \quad -\alpha_{3,8}^{11} - \alpha_{4,7}^{11} - 2 &&= 0 \\
(e_1, e_4, e_6) &: \quad -\alpha_{4,7}^{11} - \alpha_{5,6}^{11} + 1 &&= 0 \\
(e_2, e_3, e_6) &: \quad -\alpha_{2,9}^{11} &&= 0 \\
(e_2, e_4, e_5) &: \quad \alpha_{2,9}^{11} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha_{5,6}^{11} = 6$$
$$\alpha_{2,9}^{11} = 0$$
$$\alpha_{4,7}^{11} = -5$$
$$\alpha_{3,8}^{11} = 3$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{5,6}^{11} \to x_1$$
$$\alpha_{2,9}^{11} \to x_2$$
$$\alpha_{4,7}^{11} \to x_3$$
$$\alpha_{3,8}^{11} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8):&\quad -x_2 - x_4 + 3 && = 0\\
(e_1, e_3, e_7):&\quad -x_3 - x_4 - 2 && = 0\\
(e_1, e_4, e_6):&\quad -x_1 - x_3 + 1 && = 0\\
(e_2, e_3, e_6):&\quad -x_2 && = 0\\
(e_2, e_4, e_5):&\quad x_2 && = 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 - 6 = 0$$
$$x_2 = 0$$
$$x_3 + 5 = 0$$
$$x_4 - 3 = 0$$

Solution 1:

$$x_1 = 6$$
$$x_2 = 0$$
$$x_3 = -5$$
$$x_4 = 3$$

$\mathfrak{m}_{5A}(2,11)$

m5A211 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_2,e_5]=e_7$$
$$[e_2,e_6]=2e_8 \qquad [e_2,e_7]=0$$
$$[e_2,e_8]=-5e_{10} \qquad [e_2,e_9]=-\frac{5e_{11}}{2}$$
$$[e_3,e_4]=-e_7 \qquad [e_3,e_5]=-e_8$$
$$[e_3,e_6]=2e_9 \qquad [e_3,e_7]=5e_{10}$$
$$[e_3,e_8]=-\frac{5e_{11}}{2} \qquad [e_4,e_5]=-3e_9$$
$$[e_4,e_6]=-3e_{10} \qquad [e_4,e_7]=\frac{15e_{11}}{2}$$
$$[e_5,e_6]=-\frac{21e_{11}}{2}$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_2,e_5]=e_7$$
$$[e_2,e_6]=2e_8 \qquad [e_2,e_7]=\alpha_{2,7}^{9}e_9$$
$$[e_2,e_8]=\alpha_{2,8}^{10}e_{10} \qquad [e_2,e_9]=\alpha_{2,9}^{11}e_{11}$$
$$[e_3,e_4]=-e_7 \qquad [e_3,e_5]=-e_8$$
$$[e_3,e_6]=\alpha_{3,6}^{9}e_9 \qquad [e_3,e_7]=\alpha_{3,7}^{10}e_{10}$$
$$[e_3,e_8]=\alpha_{3,8}^{11}e_{11} \qquad [e_4,e_5]=\alpha_{4,5}^{9}e_9$$
$$[e_4,e_6]=\alpha_{4,6}^{10}e_{10} \qquad [e_4,e_7]=\alpha_{4,7}^{11}e_{11}$$
$$[e_5,e_6]=\alpha_{5,6}^{11}e_{11}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha^9_{2,7} - \alpha^9_{3,6} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha^9_{3,6} - \alpha^9_{4,5} - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha^9_{2,7} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha^9_{2,7} - \alpha^{10}_{2,8} - \alpha^{10}_{3,7} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha^9_{3,6} - \alpha^{10}_{3,7} - \alpha^{10}_{4,6} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha^9_{4,5} - \alpha^{10}_{4,6} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha^{10}_{2,8} - \alpha^{10}_{3,7} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha^{10}_{2,8} - \alpha^{11}_{2,9} - \alpha^{11}_{3,8} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha^{10}_{3,7} - \alpha^{11}_{3,8} - \alpha^{11}_{4,7} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha^{10}_{4,6} - \alpha^{11}_{4,7} - \alpha^{11}_{5,6} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha^{11}_{2,9}\alpha^9_{3,6} - 2\alpha^{11}_{3,8} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha^{11}_{2,9}\alpha^9_{4,5} - \alpha^{11}_{4,7} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha^{11}_{2,9} &= -5/2\\
\alpha^{10}_{3,7} &= 5\\
\alpha^9_{2,7} &= 0\\
\alpha^{10}_{2,8} &= -5\\
\alpha^9_{4,5} &= -3\\
\alpha^{11}_{4,7} &= 15/2\\
\alpha^9_{3,6} &= 2\\
\alpha^{11}_{3,8} &= -5/2\\
\alpha^{11}_{5,6} &= -21/2\\
\alpha^{10}_{4,6} &= -3
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha^{11}_{2,9} &\to x_1\\
\alpha^{10}_{3,7} &\to x_2\\
\alpha^9_{2,7} &\to x_3
\end{aligned}
$$

$$\alpha_{2,8}^{10} \to x_4$$

$$\alpha_{4,5}^{9} \to x_5$$

$$\alpha_{4,7}^{11} \to x_6$$

$$\alpha_{3,6}^{9} \to x_7$$

$$\alpha_{3,8}^{11} \to x_8$$

$$\alpha_{5,6}^{11} \to x_9$$

$$\alpha_{4,6}^{10} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_3 - x_7 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_5 - x_7 - 1 &&= 0 \\
(e_2, e_3, e_4): &\quad -x_3 &&= 0 \\
(e_1, e_2, e_7): &\quad -x_2 + x_3 - x_4 &&= 0 \\
(e_1, e_3, e_6): &\quad -x_{10} - x_2 + x_7 &&= 0 \\
(e_1, e_4, e_5): &\quad -x_{10} + x_5 &&= 0 \\
(e_2, e_3, e_5): &\quad -x_2 - x_4 &&= 0 \\
(e_1, e_2, e_8): &\quad -x_1 + x_4 - x_8 &&= 0 \\
(e_1, e_3, e_7): &\quad x_2 - x_6 - x_8 &&= 0 \\
(e_1, e_4, e_6): &\quad x_{10} - x_6 - x_9 &&= 0 \\
(e_2, e_3, e_6): &\quad x_1 x_7 - 2x_8 &&= 0 \\
(e_2, e_4, e_5): &\quad x_1 x_5 - x_6 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 10 linear, 0 nonlinear)

$$2x_1 + 5 = 0$$

$$x_2 - 5 = 0$$

$$x_3 = 0$$

$$x_4 + 5 = 0$$

$$x_5 + 3 = 0$$

$$2x_6 - 15 = 0$$

$$x_7 - 2 = 0$$

$$2x_8 + 5 = 0$$

$$2x_9 + 21 = 0$$

$$x_{10} + 3 = 0$$

Solution 1:

$$x_1 = -5/2$$
$$x_2 = 5$$
$$x_3 = 0$$
$$x_4 = -5$$
$$x_5 = -3$$
$$x_6 = 15/2$$
$$x_7 = 2$$
$$x_8 = -5/2$$
$$x_9 = -21/2$$
$$x_10 = -3$$

$\mathfrak{m}_{7A}(2, 11)$

m7A211 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] = \alpha_{3,8}^{11} e_{11} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_5, e_6] = \alpha_{5,6}^{11} e_{11} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4):\quad & -\alpha_{2,5}^{7}-\alpha_{3,4}^{7}+1 &&=0\\
(e_1,e_2,e_5):\quad & \alpha_{2,5}^{7}-\alpha_{2,6}^{8}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_3,e_4):\quad & \alpha_{3,4}^{7}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_2,e_6):\quad & \alpha_{2,6}^{8}-\alpha_{2,7}^{9}-\alpha_{3,6}^{9} &&=0\\
(e_1,e_3,e_5):\quad & \alpha_{3,5}^{8}-\alpha_{3,6}^{9}-\alpha_{4,5}^{9} &&=0\\
(e_2,e_3,e_4):\quad & \alpha_{2,7}^{9}\alpha_{3,4}^{7}-\alpha_{3,6}^{9}+\alpha_{4,5}^{9} &&=0\\
(e_1,e_2,e_7):\quad & \alpha_{2,7}^{9}-\alpha_{2,8}^{10}-\alpha_{3,7}^{10} &&=0\\
(e_1,e_3,e_6):\quad & \alpha_{3,6}^{9}-\alpha_{3,7}^{10}-\alpha_{4,6}^{10} &&=0\\
(e_1,e_4,e_5):\quad & \alpha_{4,5}^{9}-\alpha_{4,6}^{10} &&=0\\
(e_2,e_3,e_5):\quad & -\alpha_{2,5}^{7}\alpha_{3,7}^{10}+\alpha_{2,8}^{10}\alpha_{3,5}^{8} &&=0\\
(e_1,e_2,e_8):\quad & \alpha_{2,8}^{10}-\alpha_{2,9}^{11}-\alpha_{3,8}^{11} &&=0\\
(e_1,e_3,e_7):\quad & \alpha_{3,7}^{10}-\alpha_{3,8}^{11}-\alpha_{4,7}^{11} &&=0\\
(e_1,e_4,e_6):\quad & \alpha_{4,6}^{10}-\alpha_{4,7}^{11}-\alpha_{5,6}^{11} &&=0\\
(e_2,e_3,e_6):\quad & -\alpha_{2,6}^{8}\alpha_{3,8}^{11}+\alpha_{2,9}^{11}\alpha_{3,6}^{9}-\alpha_{5,6}^{11} &&=0\\
(e_2,e_4,e_5):\quad & -\alpha_{2,5}^{7}\alpha_{4,7}^{11}+\alpha_{2,9}^{11}\alpha_{4,5}^{9}+\alpha_{5,6}^{11} &&=0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{11}\to x_1$$
$$\alpha_{3,7}^{10}\to x_2$$
$$\alpha_{2,7}^{9}\to x_3$$
$$\alpha_{2,6}^{8}\to x_4$$
$$\alpha_{2,5}^{7}\to x_5$$
$$\alpha_{2,8}^{10}\to x_6$$
$$\alpha_{4,5}^{9}\to x_7$$
$$\alpha_{4,7}^{11}\to x_8$$
$$\alpha_{3,6}^{9}\to x_9$$
$$\alpha_{3,8}^{11}\to x_{10}$$
$$\alpha_{5,6}^{11}\to x_{11}$$

$$\alpha_{3,4}^{7} \to x_{12}$$
$$\alpha_{3,5}^{8} \to x_{13}$$
$$\alpha_{4,6}^{10} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4) &: \quad -x_{12} - x_5 + 1 && = 0 \\
(e_1, e_2, e_5) &: \quad -x_{13} - x_4 + x_5 && = 0 \\
(e_1, e_3, e_4) &: \quad x_{12} - x_{13} && = 0 \\
(e_1, e_2, e_6) &: \quad -x_3 + x_4 - x_9 && = 0 \\
(e_1, e_3, e_5) &: \quad x_{13} - x_7 - x_9 && = 0 \\
(e_2, e_3, e_4) &: \quad x_{12}x_3 + x_7 - x_9 && = 0 \\
(e_1, e_2, e_7) &: \quad -x_2 + x_3 - x_6 && = 0 \\
(e_1, e_3, e_6) &: \quad -x_{14} - x_2 + x_9 && = 0 \\
(e_1, e_4, e_5) &: \quad -x_{14} + x_7 && = 0 \\
(e_2, e_3, e_5) &: \quad x_{13}x_6 - x_2x_5 && = 0 \\
(e_1, e_2, e_8) &: \quad -x_1 - x_{10} + x_6 && = 0 \\
(e_1, e_3, e_7) &: \quad -x_{10} + x_2 - x_8 && = 0 \\
(e_1, e_4, e_6) &: \quad -x_{11} + x_{14} - x_8 && = 0 \\
(e_2, e_3, e_6) &: \quad x_1x_9 - x_{10}x_4 - x_{11} && = 0 \\
(e_2, e_4, e_5) &: \quad x_1x_7 + x_{11} - x_5x_8 && = 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 3 nonlinear)

$$x_1 + x_{11} + 5x_{13} - 6x_{14} - 1 = 0$$
$$-x_{13} + 2x_{14} + x_2 = 0$$
$$3x_{13} - x_{14} + x_3 - 1 = 0$$
$$2x_{13} + x_4 - 1 = 0$$
$$x_{13} + x_5 - 1 = 0$$
$$4x_{13} - 3x_{14} + x_6 - 1 = 0$$
$$-x_{14} + x_7 = 0$$
$$x_{11} - x_{14} + x_8 = 0$$
$$-x_{13} + x_{14} + x_9 = 0$$
$$x_{10} - x_{11} - x_{13} + 3x_{14} = 0$$
$$x_{11}x_{13} + x_{11}x_{14} - 2x_{11} + 4x_{13}x_{14} - 6x_{14}^2 = 0$$
$$2x_{11}x_{14}^2 - 8x_{11}x_{14} + 6x_{11} + 15x_{13}x_{14}^2 - 12x_{13}x_{14} - 12x_{14}^3 + 14x_{14}^2 = 0$$
$$x_{12} - x_{13} = 0$$
$$3x_{13}^2 - x_{13}x_{14} - 2x_{14} = 0$$

$\mathfrak{m}_{2A}(3, 11)$

m2A311 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_7] = e_{10}$$
$$[e_2, e_8] = 3e_{11} \qquad [e_3, e_6] = -e_{10}$$
$$[e_3, e_7] = -2e_{11} \qquad [e_4, e_5] = e_{10}$$
$$[e_4, e_6] = e_{11}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3, 11)$

m4A311 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_5] = e_8$$
$$[e_2, e_6] = 2e_9 \qquad [e_2, e_7] = \frac{5e_{10}}{3}$$
$$[e_2, e_8] = 0 \qquad [e_3, e_4] = -e_8$$
$$[e_3, e_5] = -e_9 \qquad [e_3, e_6] = \frac{e_{10}}{3}$$
$$[e_3, e_7] = \frac{5e_{11}}{3} \qquad [e_4, e_5] = -\frac{4e_{10}}{3}$$
$$[e_4, e_6] = -\frac{4e_{11}}{3}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= 2e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_3, e_4] &= -e_8 \\
[e_3, e_5] &= -e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6) &: \quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0 \\
(e_1, e_3, e_5) &: \quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0 \\
(e_1, e_2, e_7) &: \quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0 \\
(e_1, e_3, e_6) &: \quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0 \\
(e_1, e_4, e_5) &: \quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0 \\
(e_2, e_3, e_4) &: \quad -\alpha_{2,8}^{11} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,8}^{11} &= 0 \\
\alpha_{3,6}^{10} &= 1/3 \\
\alpha_{4,6}^{11} &= -4/3 \\
\alpha_{3,7}^{11} &= 5/3 \\
\alpha_{4,5}^{10} &= -4/3 \\
\alpha_{2,7}^{10} &= 5/3
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,8}^{11} \to x_1$$

$$\alpha_{3,6}^{10} \to x_2$$

$$\alpha_{4,6}^{11} \to x_3$$

$$\alpha_{3,7}^{11} \to x_4$$

$$\alpha_{4,5}^{10} \to x_5$$

$$\alpha_{2,7}^{10} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):& \quad -x_2 - x_6 + 2 &= 0\\
(e_1, e_3, e_5):& \quad -x_2 - x_5 - 1 &= 0\\
(e_1, e_2, e_7):& \quad -x_1 - x_4 + x_6 &= 0\\
(e_1, e_3, e_6):& \quad x_2 - x_3 - x_4 &= 0\\
(e_1, e_4, e_5):& \quad -x_3 + x_5 &= 0\\
(e_2, e_3, e_4):& \quad -x_1 &= 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 = 0$$

$$3x_2 - 1 = 0$$

$$3x_3 + 4 = 0$$

$$3x_4 - 5 = 0$$

$$3x_5 + 4 = 0$$

$$3x_6 - 5 = 0$$

Solution 1:

$$x_1 = 0$$

$$x_2 = 1/3$$

$$x_3 = -4/3$$

$$x_4 = 5/3$$

$$x_5 = -4/3$$

$$x_6 = 5/3$$

$\mathfrak{m}_{6A}(3, 11)$

m6A311 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] &= \alpha_{2,6}^{9} e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_3, e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] &= \alpha_{3,5}^{9} e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0 \\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &= 0 \\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &= 0 \\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0 \\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0 \\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0 \\
(e_2, e_3, e_4):&\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,8}^{11} &\to x_1 \\
\alpha_{3,6}^{10} &\to x_2 \\
\alpha_{4,6}^{11} &\to x_3 \\
\alpha_{3,7}^{11} &\to x_4
\end{aligned}
$$

$$\alpha^{9}_{2,6} \to x_5$$
$$\alpha^{10}_{4,5} \to x_6$$
$$\alpha^{8}_{3,4} \to x_7$$
$$\alpha^{10}_{2,7} \to x_8$$
$$\alpha^{8}_{2,5} \to x_9$$
$$\alpha^{9}_{3,5} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_7 - x_9 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_{10} - x_5 + x_9 &&= 0\\
(e_1, e_3, e_4):&\quad -x_{10} + x_7 &&= 0\\
(e_1, e_2, e_6):&\quad -x_2 + x_5 - x_8 &&= 0\\
(e_1, e_3, e_5):&\quad x_{10} - x_2 - x_6 &&= 0\\
(e_1, e_2, e_7):&\quad -x_1 - x_4 + x_8 &&= 0\\
(e_1, e_3, e_6):&\quad x_2 - x_3 - x_4 &&= 0\\
(e_1, e_4, e_5):&\quad -x_3 + x_6 &&= 0\\
(e_2, e_3, e_4):&\quad x_1 x_7 + x_3 - x_4 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 1 nonlinear)

$$x_1 - 5x_{10} - 3x_8 + 2 = 0$$
$$2x_{10} + x_2 + x_8 - 1 = 0$$
$$-3x_{10} + x_3 - x_8 + 1 = 0$$
$$5x_{10} + x_4 + 2x_8 - 2 = 0$$
$$2x_{10} + x_5 - 1 = 0$$
$$-3x_{10} + x_6 - x_8 + 1 = 0$$
$$-x_{10} + x_7 = 0$$
$$5x_{10}^2 + 3x_{10}x_8 + 6x_{10} + 3x_8 - 3 = 0$$
$$x_{10} + x_9 - 1 = 0$$

$\mathfrak{m}_{1A}(4, 11)$

m1A411 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_{11} \\
[e_3, e_6] &= -e_{11} & [e_4, e_5] &= e_{11}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4, 11)$

m3A411 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_9 \\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6):& \quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &&= 0 \\
(e_1, e_3, e_5):& \quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\alpha_{3,6}^{11} \to x_1
$$

$$
\alpha_{4,5}^{11} \to x_2
$$

$$\alpha_{2,7}^{11} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):& \quad -x_1 - x_3 + 2 &= 0\\
(e_1, e_3, e_5):& \quad -x_1 - x_2 - 1 &= 0
\end{aligned}$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 - 2 = 0$$

$$x_2 - x_3 + 3 = 0$$

$\mathfrak{m}_{5A}(4, 11)$

m5A411 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_7\\
[e_2, e_4] &= e_8 & [e_2, e_5] &= \alpha_{2,5}^{9} e_9\\
[e_2, e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11}\\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10}\\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4):& \quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0\\
(e_1, e_2, e_5):& \quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_3, e_4):& \quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_2, e_6):& \quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0\\
(e_1, e_3, e_5):& \quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0
\end{aligned}$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,6}^{10} \to x_1$$
$$\alpha_{3,5}^{10} \to x_2$$
$$\alpha_{3,6}^{11} \to x_3$$
$$\alpha_{2,5}^{9} \to x_4$$
$$\alpha_{3,4}^{9} \to x_5$$
$$\alpha_{4,5}^{11} \to x_6$$
$$\alpha_{2,7}^{11} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_4 - x_5 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_2 + x_4 &&= 0\\
(e_1, e_3, e_4): &\quad -x_2 + x_5 &&= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_3 - x_7 &&= 0\\
(e_1, e_3, e_5): &\quad x_2 - x_3 - x_6 &&= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$3x_1 + 2x_6 - 2x_7 - 1 = 0$$
$$3x_2 - x_6 + x_7 - 1 = 0$$
$$3x_3 + 2x_6 + x_7 - 1 = 0$$
$$3x_4 + x_6 - x_7 - 2 = 0$$
$$3x_5 - x_6 + x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(5, 11)$

m2A511 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{10}\\
[e_2, e_6] &= 2e_{11} & [e_3, e_4] &= -e_{10}\\
[e_3, e_5] &= -e_{11}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(5, 11)$

m4A511 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9 & [e_2, e_5] &= \alpha_{2,5}^{10} e_{10} \\
[e_2, e_6] &= \alpha_{2,6}^{11} e_{11} & [e_3, e_4] &= \alpha_{3,4}^{10} e_{10} \\
[e_3, e_5] &= \alpha_{3,5}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{3,5}^{11} &\to x_1 \\
\alpha_{3,4}^{10} &\to x_2 \\
\alpha_{2,5}^{10} &\to x_3 \\
\alpha_{2,6}^{11} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_3 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad -x_1 + x_3 - x_4 &= 0 \\
(e_1, e_3, e_4): &\quad -x_1 + x_2 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
2x_1 + x_4 - 1 &= 0 \\
2x_2 + x_4 - 1 &= 0 \\
2x_3 - x_4 - 1 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(6,11)$

m1A611 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_{11} \\
[e_3,e_4] &= -e_{11}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(6,11)$

m3A611 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_9 \\
[e_2,e_4] &= e_{10} & [e_2,e_5] &= \alpha_{2,5}^{11}e_{11} \\
[e_3,e_4] &= \alpha_{3,4}^{11}e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,5}^{11} &\to x_1 \\
\alpha_{3,4}^{11} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(7, 11)$

m2A711 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(8, 11)$

m1A811 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{11}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2, 12)$

m2A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_9] &= e_{11} & [e_2, e_{10}] &= 4e_{12} \\
[e_3, e_8] &= -e_{11} & [e_3, e_9] &= -3e_{12} \\
[e_4, e_7] &= e_{11} & [e_4, e_8] &= 2e_{12} \\
[e_5, e_6] &= -e_{11} & [e_5, e_7] &= -e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2, 12)$

m4A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_7] &= e_9 & [e_2, e_8] &= 3e_{10} \\
[e_2, e_9] &= \alpha_{2,9}^{11} e_{11} & [e_2, e_{10}] &= \alpha_{2,10}^{12} e_{12} \\
[e_3, e_6] &= -e_9 & [e_3, e_7] &= -2e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_3, e_9] &= \alpha_{3,9}^{12} e_{12} \\
[e_4, e_5] &= e_9 & [e_4, e_6] &= e_{10} \\
[e_4, e_7] &= \alpha_{4,7}^{11} e_{11} & [e_4, e_8] &= \alpha_{4,8}^{12} e_{12} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11} & [e_5, e_7] &= \alpha_{5,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{11} - \alpha_{3,8}^{11} + 3 &&= 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{11} - \alpha_{4,7}^{11} - 2 &&= 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{11} - \alpha_{5,6}^{11} + 1 &&= 0 \\
(e_2, e_3, e_6): &\quad -\alpha_{2,9}^{11} &&= 0 \\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11} &&= 0 \\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0 \\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0 \\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0 \\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0 \\
(e_2, e_3, e_7): &\quad -2\alpha_{2,10}^{12} - \alpha_{3,9}^{12} &&= 0 \\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12} &&= 0 \\
(e_3, e_4, e_5): &\quad \alpha_{3,9}^{12} &&= 0
\end{aligned}
$$

No solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,9}^{11} \to x_1$$

$$\alpha_{3,9}^{12} \to x_2$$

$$\alpha_{2,10}^{12} \to x_3$$

$$\alpha_{4,8}^{12} \to x_4$$

$$\alpha_{4,7}^{11} \to x_5$$

$$\alpha_{5,6}^{11} \to x_6$$

$$\alpha_{3,8}^{11} \to x_7$$

$$\alpha_{5,7}^{12} \to x_8$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_7 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_5 - x_7 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_5 - x_6 + 1 &&= 0\\
(e_2, e_3, e_6): &\quad -x_1 &&= 0\\
(e_2, e_4, e_5): &\quad x_1 &&= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_2 - x_3 &&= 0\\
(e_1, e_3, e_8): &\quad -x_2 - x_4 + x_7 &&= 0\\
(e_1, e_4, e_7): &\quad -x_4 + x_5 - x_8 &&= 0\\
(e_1, e_5, e_6): &\quad x_6 - x_8 &&= 0\\
(e_2, e_3, e_7): &\quad -x_2 - 2x_3 &&= 0\\
(e_2, e_4, e_6): &\quad x_3 &&= 0\\
(e_3, e_4, e_5): &\quad x_2 &&= 0
\end{aligned}
$$

Groebner basis (8 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6A}(2, 12)$

m6A212 (this line included for string searching purposes)

Original brackets:

$[e_1, e_2] = e_3$	$[e_1, e_3] = e_4$
$[e_1, e_4] = e_5$	$[e_1, e_5] = e_6$
$[e_1, e_6] = e_7$	$[e_1, e_7] = e_8$
$[e_1, e_8] = e_9$	$[e_1, e_9] = e_{10}$
$[e_1, e_{10}] = e_{11}$	$[e_1, e_{11}] = e_{12}$
$[e_2, e_5] = e_7$	$[e_2, e_6] = 2e_8$
$[e_2, e_7] = \alpha_{2,7}^{9} e_9$	$[e_2, e_8] = \alpha_{2,8}^{10} e_{10}$
$[e_2, e_9] = \alpha_{2,9}^{11} e_{11}$	$[e_2, e_{10}] = \alpha_{2,10}^{12} e_{12}$
$[e_3, e_4] = -e_7$	$[e_3, e_5] = -e_8$
$[e_3, e_6] = \alpha_{3,6}^{9} e_9$	$[e_3, e_7] = \alpha_{3,7}^{10} e_{10}$
$[e_3, e_8] = \alpha_{3,8}^{11} e_{11}$	$[e_3, e_9] = \alpha_{3,9}^{12} e_{12}$
$[e_4, e_5] = \alpha_{4,5}^{9} e_9$	$[e_4, e_6] = \alpha_{4,6}^{10} e_{10}$
$[e_4, e_7] = \alpha_{4,7}^{11} e_{11}$	$[e_4, e_8] = \alpha_{4,8}^{12} e_{12}$
$[e_5, e_6] = \alpha_{5,6}^{11} e_{11}$	$[e_5, e_7] = \alpha_{5,7}^{12} e_{12}$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10} - \alpha_{2,7}^{9}\alpha_{3,9}^{12} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10} - 2\alpha_{4,8}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,9}^{12}\alpha_{4,5}^{9} + \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{11} \to x_1$$

$$\alpha_{3,9}^{12} \to x_2$$

$$\alpha_{3,7}^{10} \to x_3$$

$$\alpha_{2,7}^{9} \to x_4$$

$$\alpha_{2,10}^{12} \to x_5$$

$$\alpha_{2,8}^{10} \to x_6$$

$$\alpha_{4,5}^{9} \to x_7$$

$$\alpha_{4,8}^{12} \to x_8$$

$$\alpha_{4,7}^{11} \to x_9$$
$$\alpha_{3,6}^{9} \to x_{10}$$
$$\alpha_{3,8}^{11} \to x_{11}$$
$$\alpha_{5,6}^{11} \to x_{12}$$
$$\alpha_{5,7}^{12} \to x_{13}$$
$$\alpha_{4,6}^{10} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_6):&\quad -x_{10}-x_4+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_{10}-x_7-1 &&=0\\
(e_2,e_3,e_4):&\quad -x_4 &&=0\\
(e_1,e_2,e_7):&\quad -x_3+x_4-x_6 &&=0\\
(e_1,e_3,e_6):&\quad x_{10}-x_{14}-x_3 &&=0\\
(e_1,e_4,e_5):&\quad -x_{14}+x_7 &&=0\\
(e_2,e_3,e_5):&\quad -x_3-x_6 &&=0\\
(e_1,e_2,e_8):&\quad -x_1-x_{11}+x_6 &&=0\\
(e_1,e_3,e_7):&\quad -x_{11}+x_3-x_9 &&=0\\
(e_1,e_4,e_6):&\quad -x_{12}+x_{14}-x_9 &&=0\\
(e_2,e_3,e_6):&\quad x_1x_{10}-2x_{11} &&=0\\
(e_2,e_4,e_5):&\quad x_1x_7-x_9 &&=0\\
(e_1,e_2,e_9):&\quad x_1-x_2-x_5 &&=0\\
(e_1,e_3,e_8):&\quad x_{11}-x_2-x_8 &&=0\\
(e_1,e_4,e_7):&\quad -x_{13}-x_8+x_9 &&=0\\
(e_1,e_5,e_6):&\quad x_{12}-x_{13} &&=0\\
(e_2,e_3,e_7):&\quad -x_2x_4+x_3x_5 &&=0\\
(e_2,e_4,e_6):&\quad x_{14}x_5-2x_8 &&=0\\
(e_3,e_4,e_5):&\quad -x_{13}+x_2x_7+x_8 &&=0
\end{aligned}$$

Groebner basis (14 variables, 1 linear, 0 nonlinear)

$$1=0$$

$\mathfrak{m}_{8A}(2, 12)$

m8A212 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_3] = e_5 \qquad [e_2, e_4] = e_6$$
$$[e_2, e_5] = \frac{9e_7}{10} \qquad [e_2, e_6] = \frac{4e_8}{5}$$
$$[e_2, e_7] = \frac{5e_9}{7} \qquad [e_2, e_8] = \frac{9e_{10}}{14}$$
$$[e_2, e_9] = \frac{7e_{11}}{12} \qquad [e_2, e_{10}] = \frac{8e_{12}}{15}$$
$$[e_3, e_4] = \frac{e_7}{10} \qquad [e_3, e_5] = \frac{e_8}{10}$$
$$[e_3, e_6] = \frac{3e_9}{35} \qquad [e_3, e_7] = \frac{e_{10}}{14}$$
$$[e_3, e_8] = \frac{5e_{11}}{84} \qquad [e_3, e_9] = \frac{e_{12}}{20}$$
$$[e_4, e_5] = \frac{e_9}{70} \qquad [e_4, e_6] = \frac{e_{10}}{70}$$
$$[e_4, e_7] = \frac{e_{11}}{84} \qquad [e_4, e_8] = \frac{e_{12}}{105}$$
$$[e_5, e_6] = \frac{e_{11}}{420} \qquad [e_5, e_7] = \frac{e_{12}}{420}$$

Solution 2

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_2, e_3] = e_5 && [e_2, e_4] = e_6 \\
&[e_2, e_5] = e_7 && [e_2, e_6] = e_8 \\
&[e_2, e_7] = e_9 && [e_2, e_8] = e_{10} \\
&[e_2, e_9] = e_{11} && [e_2, e_{10}] = e_{12} \\
&[e_3, e_4] = 0 && [e_3, e_5] = 0 \\
&[e_3, e_6] = 0 && [e_3, e_7] = 0 \\
&[e_3, e_8] = 0 && [e_3, e_9] = 0 \\
&[e_4, e_5] = 0 && [e_4, e_6] = 0 \\
&[e_4, e_7] = 0 && [e_4, e_8] = 0 \\
&[e_5, e_6] = 0 && [e_5, e_7] = 0
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_2, e_3] = e_5 && [e_2, e_4] = e_6 \\
&[e_2, e_5] = \alpha_{2,5}^{7} e_7 && [e_2, e_6] = \alpha_{2,6}^{8} e_8 \\
&[e_2, e_7] = \alpha_{2,7}^{9} e_9 && [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
&[e_2, e_9] = \alpha_{2,9}^{11} e_{11} && [e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} \\
&[e_3, e_4] = \alpha_{3,4}^{7} e_7 && [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
&[e_3, e_6] = \alpha_{3,6}^{9} e_9 && [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
&[e_3, e_8] = \alpha_{3,8}^{11} e_{11} && [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
&[e_4, e_5] = \alpha_{4,5}^{9} e_9 && [e_4, e_6] = \alpha_{4,6}^{10} e_{10} \\
&[e_4, e_7] = \alpha_{4,7}^{11} e_{11} && [e_4, e_8] = \alpha_{4,8}^{12} e_{12} \\
&[e_5, e_6] = \alpha_{5,6}^{11} e_{11} && [e_5, e_7] = \alpha_{5,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10} + \alpha_{2,8}^{10}\alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11} + \alpha_{2,9}^{11}\alpha_{3,6}^{9} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11} + \alpha_{2,9}^{11}\alpha_{4,5}^{9} + \alpha_{5,6}^{11} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10} - \alpha_{2,7}^{9}\alpha_{3,9}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10} - \alpha_{2,6}^{8}\alpha_{4,8}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,4}^{7}\alpha_{5,7}^{12} - \alpha_{3,5}^{8}\alpha_{4,8}^{12} + \alpha_{3,9}^{12}\alpha_{4,5}^{9} &&= 0
\end{aligned}$$

Solution 1:

$$\alpha_{2,9}^{11} = 7/12$$
$$\alpha_{3,9}^{12} = 1/20$$
$$\alpha_{3,7}^{10} = 1/14$$
$$\alpha_{2,7}^{9} = 5/7$$
$$\alpha_{2,6}^{8} = 4/5$$
$$\alpha_{2,5}^{7} = 9/10$$
$$\alpha_{2,10}^{12} = 8/15$$
$$\alpha_{2,8}^{10} = 9/14$$
$$\alpha_{4,5}^{9} = 1/70$$
$$\alpha_{4,8}^{12} = 1/105$$
$$\alpha_{4,7}^{11} = 1/84$$
$$\alpha_{3,6}^{9} = 3/35$$
$$\alpha_{3,8}^{11} = 5/84$$
$$\alpha_{5,6}^{11} = 1/420$$
$$\alpha_{5,7}^{12} = 1/420$$
$$\alpha_{3,4}^{7} = 1/10$$
$$\alpha_{3,5}^{8} = 1/10$$
$$\alpha_{4,6}^{10} = 1/70$$

Solution 2:

$$\alpha_{2,9}^{11} = 1$$
$$\alpha_{3,9}^{12} = 0$$
$$\alpha_{3,7}^{10} = 0$$
$$\alpha_{2,7}^{9} = 1$$
$$\alpha_{2,6}^{8} = 1$$
$$\alpha_{2,5}^{7} = 1$$
$$\alpha_{2,10}^{12} = 1$$
$$\alpha_{2,8}^{10} = 1$$
$$\alpha_{4,5}^{9} = 0$$
$$\alpha_{4,8}^{12} = 0$$
$$\alpha_{4,7}^{11} = 0$$
$$\alpha_{3,6}^{9} = 0$$
$$\alpha_{3,8}^{11} = 0$$
$$\alpha_{5,6}^{11} = 0$$
$$\alpha_{5,7}^{12} = 0$$
$$\alpha_{3,4}^{7} = 0$$
$$\alpha_{3,5}^{8} = 0$$
$$\alpha_{4,6}^{10} = 0$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{11} \to x_1$$
$$\alpha_{3,9}^{12} \to x_2$$
$$\alpha_{3,7}^{10} \to x_3$$
$$\alpha_{2,7}^{9} \to x_4$$
$$\alpha_{2,6}^{8} \to x_5$$
$$\alpha_{2,5}^{7} \to x_6$$
$$\alpha_{2,10}^{12} \to x_7$$
$$\alpha_{2,8}^{10} \to x_8$$
$$\alpha_{4,5}^{9} \to x_9$$

$$\alpha^{12}_{4,8} \to x_{10}$$
$$\alpha^{11}_{4,7} \to x_{11}$$
$$\alpha^{9}_{3,6} \to x_{12}$$
$$\alpha^{11}_{3,8} \to x_{13}$$
$$\alpha^{11}_{5,6} \to x_{14}$$
$$\alpha^{12}_{5,7} \to x_{15}$$
$$\alpha^{7}_{3,4} \to x_{16}$$
$$\alpha^{8}_{3,5} \to x_{17}$$
$$\alpha^{10}_{4,6} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_{16} - x_6 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_{17} - x_5 + x_6 &&= 0\\
(e_1, e_3, e_4):&\quad x_{16} - x_{17} &&= 0\\
(e_1, e_2, e_6):&\quad -x_{12} - x_4 + x_5 &&= 0\\
(e_1, e_3, e_5):&\quad -x_{12} + x_{17} - x_9 &&= 0\\
(e_2, e_3, e_4):&\quad -x_{12} + x_{16}x_4 + x_9 &&= 0\\
(e_1, e_2, e_7):&\quad -x_3 + x_4 - x_8 &&= 0\\
(e_1, e_3, e_6):&\quad x_{12} - x_{18} - x_3 &&= 0\\
(e_1, e_4, e_5):&\quad -x_{18} + x_9 &&= 0\\
(e_2, e_3, e_5):&\quad x_{17}x_8 - x_3x_6 &&= 0\\
(e_1, e_2, e_8):&\quad -x_1 - x_{13} + x_8 &&= 0\\
(e_1, e_3, e_7):&\quad -x_{11} - x_{13} + x_3 &&= 0\\
(e_1, e_4, e_6):&\quad -x_{11} - x_{14} + x_{18} &&= 0\\
(e_2, e_3, e_6):&\quad x_1x_{12} - x_{13}x_5 - x_{14} &&= 0\\
(e_2, e_4, e_5):&\quad x_1x_9 - x_{11}x_6 + x_{14} &&= 0\\
(e_1, e_2, e_9):&\quad x_1 - x_2 - x_7 &&= 0\\
(e_1, e_3, e_8):&\quad -x_{10} + x_{13} - x_2 &&= 0\\
(e_1, e_4, e_7):&\quad -x_{10} + x_{11} - x_{15} &&= 0\\
(e_1, e_5, e_6):&\quad x_{14} - x_{15} &&= 0\\
(e_2, e_3, e_7):&\quad -x_{15} - x_2x_4 + x_3x_7 &&= 0\\
(e_2, e_4, e_6):&\quad -x_{10}x_5 + x_{18}x_7 &&= 0\\
(e_3, e_4, e_5):&\quad -x_{10}x_{17} + x_{15}x_{16} + x_2x_9 &&= 0
\end{aligned}$$

Groebner basis (18 variables, 8 linear, 11 nonlinear)

$$3x_1 + 6x_{17}x_{18} + 15x_{17} - 7x_{18}^2 - 18x_{18} - 3 = 0$$
$$-6x_{17}x_{18} - x_{17} + 7x_{18}^2 + 4x_{18} + x_2 = 0$$
$$-x_{17} + 2x_{18} + x_3 = 0$$
$$3x_{17} - x_{18} + x_4 - 1 = 0$$
$$2x_{17} + x_5 - 1 = 0$$
$$x_{17} + x_6 - 1 = 0$$
$$24x_{17}x_{18} + 18x_{17} - 28x_{18}^2 - 30x_{18} + 3x_7 - 3 = 0$$
$$4x_{17} - 3x_{18} + x_8 - 1 = 0$$
$$-x_{18} + x_9 = 0$$
$$3x_{10} + 12x_{17}x_{18} - 14x_{18}^2 - 3x_{18} = 0$$
$$3x_{11} + 6x_{17}x_{18} - 7x_{18}^2 - 3x_{18} = 0$$
$$x_{12} - x_{17} + x_{18} = 0$$
$$3x_{13} - 6x_{17}x_{18} - 3x_{17} + 7x_{18}^2 + 9x_{18} = 0$$
$$3x_{14} - 6x_{17}x_{18} + 7x_{18}^2 = 0$$
$$3x_{15} - 6x_{17}x_{18} + 7x_{18}^2 = 0$$
$$x_{16} - x_{17} = 0$$
$$3x_{17}^2 - x_{17}x_{18} - 2x_{18} = 0$$
$$x_{17}x_{18}^2 - 7x_{18}^3 = 0$$
$$70x_{18}^4 - x_{18}^3 = 0$$

Solution 1:

$$x_1 = 7/12$$
$$x_2 = 1/20$$
$$x_3 = 1/14$$
$$x_4 = 5/7$$
$$x_5 = 4/5$$
$$x_6 = 9/10$$
$$x_7 = 8/15$$
$$x_8 = 9/14$$
$$x_9 = 1/70$$
$$x_10 = 1/105$$

$$x_1 1 = 1/84$$
$$x_1 2 = 3/35$$
$$x_1 3 = 5/84$$
$$x_1 4 = 1/420$$
$$x_1 5 = 1/420$$
$$x_1 6 = 1/10$$
$$x_1 7 = 1/10$$
$$x_1 8 = 1/70$$

Solution 2:

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 0$$
$$x_4 = 1$$
$$x_5 = 1$$
$$x_6 = 1$$
$$x_7 = 1$$
$$x_8 = 1$$
$$x_9 = 0$$
$$x_1 0 = 0$$
$$x_1 1 = 0$$
$$x_1 2 = 0$$
$$x_1 3 = 0$$
$$x_1 4 = 0$$
$$x_1 5 = 0$$
$$x_1 6 = 0$$
$$x_1 7 = 0$$
$$x_1 8 = 0$$

$\mathfrak{m}_{1A}(3,12)$

m1A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_1,e_{11}]=e_{12}\\
&[e_2,e_9]=e_{12} && [e_3,e_8]=-e_{12}\\
&[e_4,e_7]=e_{12} && [e_5,e_6]=-e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3,12)$

m3A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_1,e_{11}]=e_{12}\\
&[e_2,e_7]=e_{10} && [e_2,e_8]=3e_{11}\\
&[e_2,e_9]=\alpha_{2,9}^{12}e_{12} && [e_3,e_6]=-e_{10}\\
&[e_3,e_7]=-2e_{11} && [e_3,e_8]=\alpha_{3,8}^{12}e_{12}\\
&[e_4,e_5]=e_{10} && [e_4,e_6]=e_{11}\\
&[e_4,e_7]=\alpha_{4,7}^{12}e_{12} && [e_5,e_6]=\alpha_{5,6}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -\alpha_{2,9}^{12}-\alpha_{3,8}^{12}+3 &&=0\\
(e_1,e_3,e_7):&\quad -\alpha_{3,8}^{12}-\alpha_{4,7}^{12}-2 &&=0\\
(e_1,e_4,e_6):&\quad -\alpha_{4,7}^{12}-\alpha_{5,6}^{12}+1 &&=0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,9}^{12} \to x_1$$
$$\alpha_{4,7}^{12} \to x_2$$
$$\alpha_{5,6}^{12} \to x_3$$
$$\alpha_{3,8}^{12} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_4 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_4 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_2 - x_3 + 1 &&= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 3 = 0$$
$$x_2 + x_4 + 2 = 0$$
$$x_3 - x_4 - 3 = 0$$

$\mathfrak{m}_{5A}(3, 12)$

m5A312 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_2, e_5] &= e_8 & [e_2, e_6] &= 2e_9\\
[e_2, e_7] &= \alpha_{2,7}^{10} e_{10} & [e_2, e_8] &= \alpha_{2,8}^{11} e_{11}\\
[e_2, e_9] &= \alpha_{2,9}^{12} e_{12} & [e_3, e_4] &= -e_8\\
[e_3, e_5] &= -e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10}\\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12}\\
[e_4, e_5] &= \alpha_{4,5}^{10} e_{10} & [e_4, e_6] &= \alpha_{4,6}^{11} e_{11}\\
[e_4, e_7] &= \alpha_{4,7}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,8}^{11} \to x_1$$
$$\alpha_{3,6}^{10} \to x_2$$
$$\alpha_{4,6}^{11} \to x_3$$
$$\alpha_{3,8}^{12} \to x_4$$
$$\alpha_{3,7}^{11} \to x_5$$
$$\alpha_{2,9}^{12} \to x_6$$
$$\alpha_{4,5}^{10} \to x_7$$
$$\alpha_{2,7}^{10} \to x_8$$
$$\alpha_{4,7}^{12} \to x_9$$
$$\alpha_{5,6}^{12} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_8 + 2 && = 0\\
(e_1, e_3, e_5): &\quad -x_2 - x_7 - 1 && = 0\\
(e_1, e_2, e_7): &\quad -x_1 - x_5 + x_8 && = 0\\
(e_1, e_3, e_6): &\quad x_2 - x_3 - x_5 && = 0\\
(e_1, e_4, e_5): &\quad -x_3 + x_7 && = 0\\
(e_2, e_3, e_4): &\quad -x_1 && = 0\\
(e_1, e_2, e_8): &\quad x_1 - x_4 - x_6 && = 0\\
(e_1, e_3, e_7): &\quad -x_4 + x_5 - x_9 && = 0\\
(e_1, e_4, e_6): &\quad -x_{10} + x_3 - x_9 && = 0\\
(e_2, e_3, e_5): &\quad -x_4 - x_6 && = 0
\end{aligned}
$$

Groebner basis (10 variables, 9 linear, 0 nonlinear)

$$x_1 = 0$$

$$3x_2 - 1 = 0$$

$$3x_3 + 4 = 0$$

$$-x_{10} + x_4 - 3 = 0$$

$$3x_5 - 5 = 0$$

$$x_{10} + x_6 + 3 = 0$$

$$3x_7 + 4 = 0$$

$$3x_8 - 5 = 0$$

$$3x_{10} + 3x_9 + 4 = 0$$

$\mathfrak{m}_{7A}(3, 12)$

m7A312 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_6 & [e_2, e_4] &= e_7 \\
[e_2, e_5] &= \alpha_{2,5}^{8} e_8 & [e_2, e_6] &= \alpha_{2,6}^{9} e_9 \\
[e_2, e_7] &= \alpha_{2,7}^{10} e_{10} & [e_2, e_8] &= \alpha_{2,8}^{11} e_{11} \\
[e_2, e_9] &= \alpha_{2,9}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] &= \alpha_{3,5}^{9} e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{10} e_{10} & [e_4, e_6] &= \alpha_{4,6}^{11} e_{11} \\
[e_4, e_7] &= \alpha_{4,7}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &&= 0 \\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &&= 0 \\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &&= 0 \\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &&= 0 \\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &&= 0 \\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0 \\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0 \\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0 \\
(e_2, e_3, e_4):&\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &&= 0 \\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0 \\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0 \\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0 \\
(e_2, e_3, e_5):&\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{12} + \alpha_{2,9}^{12}\alpha_{3,5}^{9} + \alpha_{5,6}^{12} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,8}^{11} \to x_1$$

$$\alpha^{12}_{5,6} \to x_2$$
$$\alpha^{10}_{3,6} \to x_3$$
$$\alpha^{11}_{4,6} \to x_4$$
$$\alpha^{12}_{3,8} \to x_5$$
$$\alpha^{11}_{3,7} \to x_6$$
$$\alpha^{12}_{2,9} \to x_7$$
$$\alpha^{9}_{2,6} \to x_8$$
$$\alpha^{10}_{4,5} \to x_9$$
$$\alpha^{8}_{3,4} \to x_{10}$$
$$\alpha^{10}_{2,7} \to x_{11}$$
$$\alpha^{12}_{4,7} \to x_{12}$$
$$\alpha^{8}_{2,5} \to x_{13}$$
$$\alpha^{9}_{3,5} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{10}-x_{13}+1 &&=0\\
(e_1,e_2,e_5):&\quad x_{13}-x_{14}-x_8 &&=0\\
(e_1,e_3,e_4):&\quad x_{10}-x_{14} &&=0\\
(e_1,e_2,e_6):&\quad -x_{11}-x_3+x_8 &&=0\\
(e_1,e_3,e_5):&\quad x_{14}-x_3-x_9 &&=0\\
(e_1,e_2,e_7):&\quad -x_1+x_{11}-x_6 &&=0\\
(e_1,e_3,e_6):&\quad x_3-x_4-x_6 &&=0\\
(e_1,e_4,e_5):&\quad -x_4+x_9 &&=0\\
(e_2,e_3,e_4):&\quad x_1x_{10}+x_4-x_6 &&=0\\
(e_1,e_2,e_8):&\quad x_1-x_5-x_7 &&=0\\
(e_1,e_3,e_7):&\quad -x_{12}-x_5+x_6 &&=0\\
(e_1,e_4,e_6):&\quad -x_{12}-x_2+x_4 &&=0\\
(e_2,e_3,e_5):&\quad -x_{13}x_5+x_{14}x_7+x_2 &&=0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 1 nonlinear)

$$x_1-3x_{11}-5x_{14}+2=0$$
$$-x_{11}+x_{12}-3x_{14}+x_2+1=0$$

$$x_{11} + 2x_{14} + x_3 - 1 = 0$$
$$-x_{11} - 3x_{14} + x_4 + 1 = 0$$
$$2x_{11} + x_{12} + 5x_{14} + x_5 - 2 = 0$$
$$2x_{11} + 5x_{14} + x_6 - 2 = 0$$
$$-5x_{11} - x_{12} - 10x_{14} + x_7 + 4 = 0$$
$$2x_{14} + x_8 - 1 = 0$$
$$-x_{11} - 3x_{14} + x_9 + 1 = 0$$
$$x_{10} - x_{14} = 0$$
$$3x_{11}x_{14} + 3x_{11} + 5x_{14}^2 + 6x_{14} - 3 = 0$$
$$x_{13} + x_{14} - 1 = 0$$

$\mathfrak{m}_{2A}(4, 12)$

m2A412 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_7] &= e_{11} & [e_2, e_8] &= 3e_{12} \\
[e_3, e_6] &= -e_{11} & [e_3, e_7] &= -2e_{12} \\
[e_4, e_5] &= e_{11} & [e_4, e_6] &= e_{12}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(4, 12)$

m4A412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_1,e_{11}]=e_{12}\\
&[e_2,e_5]=e_9 && [e_2,e_6]=2e_{10}\\
&[e_2,e_7]=\alpha_{2,7}^{11}e_{11} && [e_2,e_8]=\alpha_{2,8}^{12}e_{12}\\
&[e_3,e_4]=-e_9 && [e_3,e_5]=-e_{10}\\
&[e_3,e_6]=\alpha_{3,6}^{11}e_{11} && [e_3,e_7]=\alpha_{3,7}^{12}e_{12}\\
&[e_4,e_5]=\alpha_{4,5}^{11}e_{11} && [e_4,e_6]=\alpha_{4,6}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1,e_2,e_6): && -\alpha_{2,7}^{11}-\alpha_{3,6}^{11}+2 &&=0\\
&(e_1,e_3,e_5): && -\alpha_{3,6}^{11}-\alpha_{4,5}^{11}-1 &&=0\\
&(e_1,e_2,e_7): && \alpha_{2,7}^{11}-\alpha_{2,8}^{12}-\alpha_{3,7}^{12} &&=0\\
&(e_1,e_3,e_6): && \alpha_{3,6}^{11}-\alpha_{3,7}^{12}-\alpha_{4,6}^{12} &&=0\\
&(e_1,e_4,e_5): && \alpha_{4,5}^{11}-\alpha_{4,6}^{12} &&=0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,7}^{12}&\to x_1\\
\alpha_{2,8}^{12}&\to x_2\\
\alpha_{4,6}^{12}&\to x_3\\
\alpha_{3,6}^{11}&\to x_4\\
\alpha_{4,5}^{11}&\to x_5\\
\alpha_{2,7}^{11}&\to x_6
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -x_4-x_6+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_4-x_5-1 &&=0\\
(e_1,e_2,e_7):&\quad -x_1-x_2+x_6 &&=0\\
(e_1,e_3,e_6):&\quad -x_1-x_3+x_4 &&=0\\
(e_1,e_4,e_5):&\quad -x_3+x_5 &&=0
\end{aligned}
$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$x_1+2x_6-5=0$$

$$x_2-3x_6+5=0$$

$$x_3-x_6+3=0$$

$$x_4+x_6-2=0$$

$$x_5-x_6+3=0$$

$\mathfrak{m}_{6A}(4,12)$

m6A412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_2,e_3]&=e_7 & [e_2,e_4]&=e_8\\
[e_2,e_5]&=\alpha_{2,5}^{9}e_9 & [e_2,e_6]&=\alpha_{2,6}^{10}e_{10}\\
[e_2,e_7]&=\alpha_{2,7}^{11}e_{11} & [e_2,e_8]&=\alpha_{2,8}^{12}e_{12}\\
[e_3,e_4]&=\alpha_{3,4}^{9}e_9 & [e_3,e_5]&=\alpha_{3,5}^{10}e_{10}\\
[e_3,e_6]&=\alpha_{3,6}^{11}e_{11} & [e_3,e_7]&=\alpha_{3,7}^{12}e_{12}\\
[e_4,e_5]&=\alpha_{4,5}^{11}e_{11} & [e_4,e_6]&=\alpha_{4,6}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{9}_{2,5}-\alpha^{9}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{9}_{2,5}-\alpha^{10}_{2,6}-\alpha^{10}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{9}_{3,4}-\alpha^{10}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{10}_{2,6}-\alpha^{11}_{2,7}-\alpha^{11}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{10}_{3,5}-\alpha^{11}_{3,6}-\alpha^{11}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{11}_{2,7}-\alpha^{12}_{2,8}-\alpha^{12}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{11}_{3,6}-\alpha^{12}_{3,7}-\alpha^{12}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{11}_{4,5}-\alpha^{12}_{4,6} &&=0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\begin{gathered}
\alpha^{12}_{3,7}\to x_1\\
\alpha^{10}_{2,6}\to x_2\\
\alpha^{12}_{2,8}\to x_3\\
\alpha^{12}_{4,6}\to x_4\\
\alpha^{10}_{3,5}\to x_5\\
\alpha^{11}_{3,6}\to x_6\\
\alpha^{9}_{2,5}\to x_7\\
\alpha^{9}_{3,4}\to x_8\\
\alpha^{11}_{4,5}\to x_9\\
\alpha^{11}_{2,7}\to x_{10}
\end{gathered}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_7-x_8+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_2-x_5+x_7 &&=0\\
(e_1,e_3,e_4):&\quad -x_5+x_8 &&=0\\
(e_1,e_2,e_6):&\quad -x_{10}+x_2-x_6 &&=0\\
(e_1,e_3,e_5):&\quad x_5-x_6-x_9 &&=0\\
(e_1,e_2,e_7):&\quad -x_1+x_{10}-x_3 &&=0\\
(e_1,e_3,e_6):&\quad -x_1-x_4+x_6 &&=0\\
(e_1,e_4,e_5):&\quad -x_4+x_9 &&=0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$3x_1 + x_{10} + 5x_9 - 1 = 0$$
$$-2x_{10} + 3x_2 + 2x_9 - 1 = 0$$
$$-4x_{10} + 3x_3 - 5x_9 + 1 = 0$$
$$x_4 - x_9 = 0$$
$$x_{10} + 3x_5 - x_9 - 1 = 0$$
$$x_{10} + 3x_6 + 2x_9 - 1 = 0$$
$$-x_{10} + 3x_7 + x_9 - 2 = 0$$
$$x_{10} + 3x_8 - x_9 - 1 = 0$$

$\mathfrak{m}_{1A}(5, 12)$

m1A512 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_7] = e_{12} \qquad [e_3, e_6] = -e_{12}$$
$$[e_4, e_5] = e_{12}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(5, 12)$

m3A512 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_5] = e_{10} \qquad [e_2, e_6] = 2e_{11}$$
$$[e_2, e_7] = \alpha_{2,7}^{12} e_{12} \qquad [e_3, e_4] = -e_{10}$$
$$[e_3, e_5] = -e_{11} \qquad [e_3, e_6] = \alpha_{3,6}^{12} e_{12}$$
$$[e_4, e_5] = \alpha_{4,5}^{12} e_{12}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): \quad & -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 && = 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{12} \to x_1$$

$$\alpha_{4,5}^{12} \to x_2$$

$$\alpha_{3,6}^{12} \to x_3$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): \quad & -x_1 - x_3 + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -x_2 - x_3 - 1 && = 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 - 2 = 0$$

$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(5, 12)$

m5A512 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_8 & [e_2, e_4] &= e_9 \\
[e_2, e_5] &= \alpha_{2,5}^{10} e_{10} & [e_2, e_6] &= \alpha_{2,6}^{11} e_{11} \\
[e_2, e_7] &= \alpha_{2,7}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{10} e_{10} \\
[e_3, e_5] &= \alpha_{3,5}^{11} e_{11} & [e_3, e_6] &= \alpha_{3,6}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4):& \quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &&= 0\\
(e_1, e_2, e_5):& \quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_3, e_4):& \quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_2, e_6):& \quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &&= 0\\
(e_1, e_3, e_5):& \quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{11} \to x_1$$

$$\alpha_{2,7}^{12} \to x_2$$

$$\alpha_{3,4}^{10} \to x_3$$

$$\alpha_{2,5}^{10} \to x_4$$

$$\alpha_{3,6}^{12} \to x_5$$

$$\alpha_{3,5}^{11} \to x_6$$

$$\alpha_{4,5}^{12} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):& \quad -x_3 - x_4 + 1 &&= 0\\
(e_1, e_2, e_5):& \quad -x_1 + x_4 - x_6 &&= 0\\
(e_1, e_3, e_4):& \quad x_3 - x_6 &&= 0\\
(e_1, e_2, e_6):& \quad x_1 - x_2 - x_5 &&= 0\\
(e_1, e_3, e_5):& \quad -x_5 + x_6 - x_7 &&= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 + 2x_6 - 1 = 0$$

$$x_2 + 3x_6 - x_7 - 1 = 0$$

$$x_3 - x_6 = 0$$

$$x_4 + x_6 - 1 = 0$$

$$x_5 - x_6 + x_7 = 0$$

$\mathfrak{m}_{2A}(6,12)$

m2A612 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_2,e_5]=e_{11} \qquad [e_2,e_6]=2e_{12}$$
$$[e_3,e_4]=-e_{11} \qquad [e_3,e_5]=-e_{12}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(6,12)$

m4A612 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_2,e_3]=e_9 \qquad [e_2,e_4]=e_{10}$$
$$[e_2,e_5]=\alpha_{2,5}^{11}e_{11} \qquad [e_2,e_6]=\alpha_{2,6}^{12}e_{12}$$
$$[e_3,e_4]=\alpha_{3,4}^{11}e_{11} \qquad [e_3,e_5]=\alpha_{3,5}^{12}e_{12}$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{11}-\alpha_{3,4}^{11}+1 = 0$$
$$(e_1,e_2,e_5): \quad \alpha_{2,5}^{11}-\alpha_{2,6}^{12}-\alpha_{3,5}^{12} = 0$$
$$(e_1,e_3,e_4): \quad \alpha_{3,4}^{11}-\alpha_{3,5}^{12} = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,5}^{11}\to x_1$$

$$\alpha_{2,6}^{12} \to x_2$$

$$\alpha_{3,4}^{11} \to x_3$$

$$\alpha_{3,5}^{12} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_1 - x_3 + 1 &= 0\\
(e_1, e_2, e_5):&\quad x_1 - x_2 - x_4 &= 0\\
(e_1, e_3, e_4):&\quad x_3 - x_4 &= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 1 = 0$$

$$x_2 + 2x_4 - 1 = 0$$

$$x_3 - x_4 = 0$$

$\mathfrak{m}_{1A}(7, 12)$

m1A712 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_2, e_5] &= e_{12} & [e_3, e_4] &= -e_{12}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(7, 12)$

m3A712 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{10} & [e_2, e_4] &= e_{11} \\
[e_2, e_5] &= \alpha_{2,5}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^{12} &\to x_1 \\
\alpha_{2,5}^{12} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(8, 12)$

m2A812 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{11} & [e_2, e_4] &= e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(9, 12)$

m1A912 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2, 13)$

m1A213 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_{11}] &= e_{13} \\
[e_3, e_{10}] &= -e_{13} & [e_4, e_9] &= e_{13} \\
[e_5, e_8] &= -e_{13} & [e_6, e_7] &= e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2, 13)$

m3A213 (this line included for string searching purposes)

Solution 1

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_9] = e_{11} \\
[e_2, e_{10}] = 4e_{12} & [e_2, e_{11}] = 0 \\
[e_3, e_8] = -e_{11} & [e_3, e_9] = -3e_{12} \\
[e_3, e_{10}] = 4e_{13} & [e_4, e_7] = e_{11} \\
[e_4, e_8] = 2e_{12} & [e_4, e_9] = -7e_{13} \\
[e_5, e_6] = -e_{11} & [e_5, e_7] = -e_{12} \\
[e_5, e_8] = 9e_{13} & [e_6, e_7] = -10e_{13}
\end{array}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_9] = e_{11} \\
[e_2, e_{10}] = 4e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_3, e_8] = -e_{11} & [e_3, e_9] = -3e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_4, e_7] = e_{11} \\
[e_4, e_8] = 2e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_5, e_6] = -e_{11} & [e_5, e_7] = -e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{2,11}^{13} - \alpha_{3,10}^{13} + 4 &&= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{13} - \alpha_{4,9}^{13} - 3 &&= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{13} - \alpha_{5,8}^{13} + 2 &&= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{13} - \alpha_{6,7}^{13} - 1 &&= 0 \\
(e_2, e_3, e_8): &\quad -\alpha_{2,11}^{13} &&= 0 \\
(e_2, e_4, e_7): &\quad \alpha_{2,11}^{13} &&= 0 \\
(e_2, e_5, e_6): &\quad -\alpha_{2,11}^{13} &&= 0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha_{5,8}^{13} &= 9 \\
\alpha_{3,10}^{13} &= 4 \\
\alpha_{4,9}^{13} &= -7 \\
\alpha_{2,11}^{13} &= 0 \\
\alpha_{6,7}^{13} &= -10
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha_{5,8}^{13} &\to x_1 \\
\alpha_{3,10}^{13} &\to x_2 \\
\alpha_{4,9}^{13} &\to x_3 \\
\alpha_{2,11}^{13} &\to x_4 \\
\alpha_{6,7}^{13} &\to x_5
\end{aligned}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_2 - x_4 + 4 &&= 0 \\
(e_1, e_3, e_9): &\quad -x_2 - x_3 - 3 &&= 0 \\
(e_1, e_4, e_8): &\quad -x_1 - x_3 + 2 &&= 0 \\
(e_1, e_5, e_7): &\quad -x_1 - x_5 - 1 &&= 0 \\
(e_2, e_3, e_8): &\quad -x_4 &&= 0 \\
(e_2, e_4, e_7): &\quad x_4 &&= 0 \\
(e_2, e_5, e_6): &\quad -x_4 &&= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 - 9 = 0$$
$$x_2 - 4 = 0$$
$$x_3 + 7 = 0$$
$$x_4 = 0$$
$$x_5 + 10 = 0$$

Solution 1:

$$x_1 = 9$$
$$x_2 = 4$$
$$x_3 = -7$$
$$x_4 = 0$$
$$x_5 = -10$$

$\mathfrak{m}_{9A}(2, 13)$

m9A213 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7$$
$$[e_2, e_6] = e_8 \qquad [e_2, e_7] = e_9$$
$$[e_2, e_8] = e_{10} \qquad [e_2, e_9] = e_{11}$$
$$[e_2, e_{10}] = e_{12} \qquad [e_2, e_{11}] = e_{13}$$
$$[e_3, e_4] = 0 \qquad [e_3, e_5] = 0$$
$$[e_3, e_6] = 0 \qquad [e_3, e_7] = 0$$
$$[e_3, e_8] = 0 \qquad [e_3, e_9] = 0$$
$$[e_3, e_{10}] = 0 \qquad [e_4, e_5] = 0$$
$$[e_4, e_6] = 0 \qquad [e_4, e_7] = 0$$
$$[e_4, e_8] = 0 \qquad [e_4, e_9] = 0$$
$$[e_5, e_6] = 0 \qquad [e_5, e_7] = 0$$
$$[e_5, e_8] = 0 \qquad [e_6, e_7] = 0$$

Solution 2

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = \frac{9e_7}{10} \\
&[e_2, e_6] = \frac{4e_8}{5} && [e_2, e_7] = \frac{5e_9}{7} \\
&[e_2, e_8] = \frac{9e_{10}}{14} && [e_2, e_9] = \frac{7e_{11}}{12} \\
&[e_2, e_{10}] = \frac{8e_{12}}{15} && [e_2, e_{11}] = \frac{27e_{13}}{55} \\
&[e_3, e_4] = \frac{e_7}{10} && [e_3, e_5] = \frac{e_8}{10} \\
&[e_3, e_6] = \frac{3e_9}{35} && [e_3, e_7] = \frac{e_{10}}{14} \\
&[e_3, e_8] = \frac{5e_{11}}{84} && [e_3, e_9] = \frac{e_{12}}{20} \\
&[e_3, e_{10}] = \frac{7e_{13}}{165} && [e_4, e_5] = \frac{e_9}{70} \\
&[e_4, e_6] = \frac{e_{10}}{70} && [e_4, e_7] = \frac{e_{11}}{84} \\
&[e_4, e_8] = \frac{e_{12}}{105} && [e_4, e_9] = \frac{e_{13}}{132} \\
&[e_5, e_6] = \frac{e_{11}}{420} && [e_5, e_7] = \frac{e_{12}}{420} \\
&[e_5, e_8] = \frac{3e_{13}}{1540} && [e_6, e_7] = \frac{e_{13}}{2310}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_5 \\
[e_2, e_4] &= e_6 & [e_2, e_5] &= \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] &= \alpha_{2,6}^{8} e_8 & [e_2, e_7] &= \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] &= \alpha_{2,8}^{10} e_{10} & [e_2, e_9] &= \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{10}] &= \alpha_{2,10}^{12} e_{12} & [e_2, e_{11}] &= \alpha_{2,11}^{13} e_{13} \\
[e_3, e_4] &= \alpha_{3,4}^{7} e_7 & [e_3, e_5] &= \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] &= \alpha_{3,6}^{9} e_9 & [e_3, e_7] &= \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_3, e_9] &= \alpha_{3,9}^{12} e_{12} \\
[e_3, e_{10}] &= \alpha_{3,10}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] &= \alpha_{4,6}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{11} e_{11} \\
[e_4, e_8] &= \alpha_{4,8}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{13} e_{13} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11} & [e_5, e_7] &= \alpha_{5,7}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{13} e_{13} & [e_6, e_7] &= \alpha_{6,7}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{7}-\alpha_{3,4}^{7}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{7}-\alpha_{2,6}^{8}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{7}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{8}-\alpha_{2,7}^{9}-\alpha_{3,6}^{9} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{8}-\alpha_{3,6}^{9}-\alpha_{4,5}^{9} &&=0\\
(e_2,e_3,e_4):&\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7}-\alpha_{3,6}^{9}+\alpha_{4,5}^{9} &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{9}-\alpha_{2,8}^{10}-\alpha_{3,7}^{10} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{9}-\alpha_{3,7}^{10}-\alpha_{4,6}^{10} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{9}-\alpha_{4,6}^{10} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10}+\alpha_{2,8}^{10}\alpha_{3,5}^{8} &&=0\\
(e_1,e_2,e_8):&\quad \alpha_{2,8}^{10}-\alpha_{2,9}^{11}-\alpha_{3,8}^{11} &&=0\\
(e_1,e_3,e_7):&\quad \alpha_{3,7}^{10}-\alpha_{3,8}^{11}-\alpha_{4,7}^{11} &&=0\\
(e_1,e_4,e_6):&\quad \alpha_{4,6}^{10}-\alpha_{4,7}^{11}-\alpha_{5,6}^{11} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11}+\alpha_{2,9}^{11}\alpha_{3,6}^{9}-\alpha_{5,6}^{11} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11}+\alpha_{2,9}^{11}\alpha_{4,5}^{9}+\alpha_{5,6}^{11} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha_{2,10}^{12}+\alpha_{2,9}^{11}-\alpha_{3,9}^{12} &&=0\\
(e_1,e_3,e_8):&\quad \alpha_{3,8}^{11}-\alpha_{3,9}^{12}-\alpha_{4,8}^{12} &&=0\\
(e_1,e_4,e_7):&\quad \alpha_{4,7}^{11}-\alpha_{4,8}^{12}-\alpha_{5,7}^{12} &&=0\\
(e_1,e_5,e_6):&\quad \alpha_{5,6}^{11}-\alpha_{5,7}^{12} &&=0\\
(e_2,e_3,e_7):&\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10}-\alpha_{2,7}^{9}\alpha_{3,9}^{12}-\alpha_{5,7}^{12} &&=0\\
(e_2,e_4,e_6):&\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10}-\alpha_{2,6}^{8}\alpha_{4,8}^{12} &&=0\\
(e_3,e_4,e_5):&\quad \alpha_{3,4}^{7}\alpha_{5,7}^{12}-\alpha_{3,5}^{8}\alpha_{4,8}^{12}+\alpha_{3,9}^{12}\alpha_{4,5}^{9} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha_{2,10}^{12}-\alpha_{2,11}^{13}-\alpha_{3,10}^{13} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{13}+\alpha_{3,9}^{12}-\alpha_{4,9}^{13} &&=0\\
(e_1,e_4,e_8):&\quad \alpha_{4,8}^{12}-\alpha_{4,9}^{13}-\alpha_{5,8}^{13} &&=0\\
(e_1,e_5,e_7):&\quad \alpha_{5,7}^{12}-\alpha_{5,8}^{13}-\alpha_{6,7}^{13} &&=0\\
(e_2,e_3,e_8):&\quad \alpha_{2,11}^{13}\alpha_{3,8}^{11}-\alpha_{2,8}^{10}\alpha_{3,10}^{13}-\alpha_{5,8}^{13} &&=0\\
(e_2,e_4,e_7):&\quad \alpha_{2,11}^{13}\alpha_{4,7}^{11}-\alpha_{2,7}^{9}\alpha_{4,9}^{13}-\alpha_{6,7}^{13} &&=0\\
(e_2,e_5,e_6):&\quad \alpha_{2,11}^{13}\alpha_{5,6}^{11}+\alpha_{2,5}^{7}\alpha_{6,7}^{13}-\alpha_{2,6}^{8}\alpha_{5,8}^{13} &&=0\\
(e_3,e_4,e_6):&\quad \alpha_{3,10}^{13}\alpha_{4,6}^{10}+\alpha_{3,4}^{7}\alpha_{6,7}^{13}-\alpha_{3,6}^{9}\alpha_{4,9}^{13} &&=0
\end{aligned}$$

Solution 1:

$$\alpha^{13}_{3,10} = 0$$
$$\alpha^{13}_{4,9} = 0$$
$$\alpha^{9}_{3,6} = 0$$
$$\alpha^{13}_{2,11} = 1$$
$$\alpha^{7}_{3,4} = 0$$
$$\alpha^{11}_{2,9} = 1$$
$$\alpha^{9}_{2,7} = 1$$
$$\alpha^{8}_{2,6} = 1$$
$$\alpha^{7}_{2,5} = 1$$
$$\alpha^{12}_{5,7} = 0$$
$$\alpha^{13}_{6,7} = 0$$
$$\alpha^{8}_{3,5} = 0$$
$$\alpha^{10}_{4,6} = 0$$
$$\alpha^{12}_{3,9} = 0$$
$$\alpha^{10}_{3,7} = 0$$
$$\alpha^{12}_{2,10} = 1$$
$$\alpha^{10}_{2,8} = 1$$
$$\alpha^{11}_{4,7} = 0$$
$$\alpha^{11}_{5,6} = 0$$
$$\alpha^{13}_{5,8} = 0$$
$$\alpha^{9}_{4,5} = 0$$
$$\alpha^{12}_{4,8} = 0$$
$$\alpha^{11}_{3,8} = 0$$

Solution 2:

$$\alpha^{13}_{3,10} = 7/165$$
$$\alpha^{13}_{4,9} = 1/132$$
$$\alpha^{9}_{3,6} = 3/35$$
$$\alpha^{13}_{2,11} = 27/55$$
$$\alpha^{7}_{3,4} = 1/10$$
$$\alpha^{11}_{2,9} = 7/12$$
$$\alpha^{9}_{2,7} = 5/7$$
$$\alpha^{8}_{2,6} = 4/5$$
$$\alpha^{7}_{2,5} = 9/10$$
$$\alpha^{12}_{5,7} = 1/420$$
$$\alpha^{13}_{6,7} = 1/2310$$
$$\alpha^{8}_{3,5} = 1/10$$
$$\alpha^{10}_{4,6} = 1/70$$
$$\alpha^{12}_{3,9} = 1/20$$
$$\alpha^{10}_{3,7} = 1/14$$
$$\alpha^{12}_{2,10} = 8/15$$
$$\alpha^{10}_{2,8} = 9/14$$
$$\alpha^{11}_{4,7} = 1/84$$
$$\alpha^{11}_{5,6} = 1/420$$
$$\alpha^{13}_{5,8} = 3/1540$$
$$\alpha^{9}_{4,5} = 1/70$$
$$\alpha^{12}_{4,8} = 1/105$$
$$\alpha^{11}_{3,8} = 5/84$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{3,10} \to x_1$$
$$\alpha^{13}_{4,9} \to x_2$$
$$\alpha^{9}_{3,6} \to x_3$$
$$\alpha^{13}_{2,11} \to x_4$$
$$\alpha^{7}_{3,4} \to x_5$$

$$\alpha_{2,9}^{11} \to x_6$$

$$\alpha_{2,7}^{9} \to x_7$$

$$\alpha_{2,6}^{8} \to x_8$$

$$\alpha_{2,5}^{7} \to x_9$$

$$\alpha_{5,7}^{12} \to x_{10}$$

$$\alpha_{6,7}^{13} \to x_{11}$$

$$\alpha_{3,5}^{8} \to x_{12}$$

$$\alpha_{4,6}^{10} \to x_{13}$$

$$\alpha_{3,9}^{12} \to x_{14}$$

$$\alpha_{3,7}^{10} \to x_{15}$$

$$\alpha_{2,10}^{12} \to x_{16}$$

$$\alpha_{2,8}^{10} \to x_{17}$$

$$\alpha_{4,7}^{11} \to x_{18}$$

$$\alpha_{5,6}^{11} \to x_{19}$$

$$\alpha_{5,8}^{13} \to x_{20}$$

$$\alpha_{4,5}^{9} \to x_{21}$$

$$\alpha_{4,8}^{12} \to x_{22}$$

$$\alpha_{3,8}^{11} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_5-x_9+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{12}-x_8+x_9 &&=0\\
(e_1,e_3,e_4):&\quad -x_{12}+x_5 &&=0\\
(e_1,e_2,e_6):&\quad -x_3-x_7+x_8 &&=0\\
(e_1,e_3,e_5):&\quad x_{12}-x_{21}-x_3 &&=0\\
(e_2,e_3,e_4):&\quad x_{21}-x_3+x_5x_7 &&=0\\
(e_1,e_2,e_7):&\quad -x_{15}-x_{17}+x_7 &&=0\\
(e_1,e_3,e_6):&\quad -x_{13}-x_{15}+x_3 &&=0\\
(e_1,e_4,e_5):&\quad -x_{13}+x_{21} &&=0\\
(e_2,e_3,e_5):&\quad x_{12}x_{17}-x_{15}x_9 &&=0\\
(e_1,e_2,e_8):&\quad x_{17}-x_{23}-x_6 &&=0\\
(e_1,e_3,e_7):&\quad x_{15}-x_{18}-x_{23} &&=0\\
(e_1,e_4,e_6):&\quad x_{13}-x_{18}-x_{19} &&=0\\
(e_2,e_3,e_6):&\quad -x_{19}-x_{23}x_8+x_3x_6 &&=0\\
(e_2,e_4,e_5):&\quad -x_{18}x_9+x_{19}+x_{21}x_6 &&=0\\
(e_1,e_2,e_9):&\quad -x_{14}-x_{16}+x_6 &&=0\\
(e_1,e_3,e_8):&\quad -x_{14}-x_{22}+x_{23} &&=0\\
(e_1,e_4,e_7):&\quad -x_{10}+x_{18}-x_{22} &&=0\\
(e_1,e_5,e_6):&\quad -x_{10}+x_{19} &&=0\\
(e_2,e_3,e_7):&\quad -x_{10}-x_{14}x_7+x_{15}x_{16} &&=0\\
(e_2,e_4,e_6):&\quad x_{13}x_{16}-x_{22}x_8 &&=0\\
(e_3,e_4,e_5):&\quad x_{10}x_5-x_{12}x_{22}+x_{14}x_{21} &&=0\\
(e_1,e_2,e_{10}):&\quad -x_1+x_{16}-x_4 &&=0\\
(e_1,e_3,e_9):&\quad -x_1+x_{14}-x_2 &&=0\\
(e_1,e_4,e_8):&\quad -x_2-x_{20}+x_{22} &&=0\\
(e_1,e_5,e_7):&\quad x_{10}-x_{11}-x_{20} &&=0\\
(e_2,e_3,e_8):&\quad -x_1x_{17}-x_{20}+x_{23}x_4 &&=0\\
(e_2,e_4,e_7):&\quad -x_{11}+x_{18}x_4-x_2x_7 &&=0\\
(e_2,e_5,e_6):&\quad x_{11}x_9+x_{19}x_4-x_{20}x_8 &&=0\\
(e_3,e_4,e_6):&\quad x_1x_{13}+x_{11}x_5-x_2x_3 &&=0
\end{aligned}
$$

Groebner basis (23 variables, 0 linear, 23 nonlinear)

$$343750x_1+1429436988x_{23}^5+31453125x_{23}^4+3609375x_{23}^3+1031250x_{23}^2-343750x_{23}=0$$

$$275000x_2-390368916x_{23}^5-8971875x_{23}^4-1031250x_{23}^3-412500x_{23}^2=0$$

$$-337363992x_{23}^5-6843750x_{23}^4-703125x_{23}^3-93750x_{23}^2-31250x_{23}+31250x_3=0$$
$$14633280144x_{23}^5+292875000x_{23}^4+29390625x_{23}^3+3093750x_{23}^2+2406250x_{23}+343750x_4-343750=0$$
$$-2134974996x_{23}^5-43171875x_{23}^4-4406250x_{23}^3-562500x_{23}^2-125000x_{23}+125000x_5=0$$
$$247333716x_{23}^5+5011875x_{23}^4+513750x_{23}^3+67500x_{23}^2+25000x_{23}+5000x_6-5000=0$$
$$561940596x_{23}^5+11371875x_{23}^4+1162500x_{23}^3+150000x_{23}^2+37500x_{23}+12500x_7-12500=0$$
$$2134974996x_{23}^5+43171875x_{23}^4+4406250x_{23}^3+562500x_{23}^2+125000x_{23}+62500x_8-62500=0$$
$$2134974996x_{23}^5+43171875x_{23}^4+4406250x_{23}^3+562500x_{23}^2+125000x_{23}+125000x_9-125000=0$$
$$62500x_{10}-110791044x_{23}^5-2109375x_{23}^4-187500x_{23}^3=0$$
$$343750x_{11}-155836044x_{23}^5-2578125x_{23}^4=0$$
$$125000x_{12}-2134974996x_{23}^5-43171875x_{23}^4-4406250x_{23}^3-562500x_{23}^2-125000x_{23}=0$$
$$125000x_{13}-785519028x_{23}^5-15796875x_{23}^4-1593750x_{23}^3-187500x_{23}^2=0$$
$$125000x_{14}+342354852x_{23}^5+7359375x_{23}^4+843750x_{23}^3+187500x_{23}^2-125000x_{23}=0$$
$$25000x_{15}-112787388x_{23}^5-2315625x_{23}^4-243750x_{23}^3-37500x_{23}^2-25000x_{23}=0$$
$$31250x_{16}+1460247012x_{23}^5+29484375x_{23}^4+3000000x_{23}^3+375000x_{23}^2+187500x_{23}-31250=0$$
$$5000x_{17}+247333716x_{23}^5+5011875x_{23}^4+513750x_{23}^3+67500x_{23}^2+20000x_{23}-5000=0$$
$$25000x_{18}-112787388x_{23}^5-2315625x_{23}^4-243750x_{23}^3-37500x_{23}^2=0$$
$$62500x_{19}-110791044x_{23}^5-2109375x_{23}^4-187500x_{23}^3=0$$
$$687500x_{20}-907029396x_{23}^5-18046875x_{23}^4-2062500x_{23}^3=0$$
$$125000x_{21}-785519028x_{23}^5-15796875x_{23}^4-1593750x_{23}^3-187500x_{23}^2=0$$
$$125000x_{22}-342354852x_{23}^5-7359375x_{23}^4-843750x_{23}^3-187500x_{23}^2=0$$
$$84x_{23}^6-5x_{23}^5=0$$

Solution 1:

$$x_1=0$$
$$x_2=0$$
$$x_3=0$$
$$x_4=1$$
$$x_5=0$$
$$x_6=1$$
$$x_7=1$$
$$x_8=1$$
$$x_9=1$$
$$x_10=0$$

$$x_11 = 0$$
$$x_12 = 0$$
$$x_13 = 0$$
$$x_14 = 0$$
$$x_15 = 0$$
$$x_16 = 1$$
$$x_17 = 1$$
$$x_18 = 0$$
$$x_19 = 0$$
$$x_20 = 0$$
$$x_21 = 0$$
$$x_22 = 0$$
$$x_23 = 0$$

Solution 2:

$$x_1 = 7/165$$
$$x_2 = 1/132$$
$$x_3 = 3/35$$
$$x_4 = 27/55$$
$$x_5 = 1/10$$
$$x_6 = 7/12$$
$$x_7 = 5/7$$
$$x_8 = 4/5$$
$$x_9 = 9/10$$
$$x_10 = 1/420$$
$$x_11 = 1/2310$$
$$x_12 = 1/10$$
$$x_13 = 1/70$$
$$x_14 = 1/20$$
$$x_15 = 1/14$$
$$x_16 = 8/15$$
$$x_17 = 9/14$$
$$x_18 = 1/84$$
$$x_19 = 1/420$$
$$x_20 = 3/1540$$
$$x_21 = 1/70$$
$$x_22 = 1/105$$
$$x_23 = 5/84$$

$\mathfrak{m}_{2A}(3,13)$

m2A313 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_9]=e_{12}$$
$$[e_2,e_{10}]=4e_{13} \qquad [e_3,e_8]=-e_{12}$$
$$[e_3,e_9]=-3e_{13} \qquad [e_4,e_7]=e_{12}$$
$$[e_4,e_8]=2e_{13} \qquad [e_5,e_6]=-e_{12}$$
$$[e_5,e_7]=-e_{13}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3,13)$

m4A313 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_7]=e_{10}$$
$$[e_2,e_8]=3e_{11} \qquad [e_2,e_9]=\frac{7e_{12}}{2}$$
$$[e_2,e_{10}]=0 \qquad [e_3,e_6]=-e_{10}$$
$$[e_3,e_7]=-2e_{11} \qquad [e_3,e_8]=-\frac{e_{12}}{2}$$
$$[e_3,e_9]=\frac{7e_{13}}{2} \qquad [e_4,e_5]=e_{10}$$
$$[e_4,e_6]=e_{11} \qquad [e_4,e_7]=-\frac{3e_{12}}{2}$$
$$[e_4,e_8]=-4e_{13} \qquad [e_5,e_6]=\frac{5e_{12}}{2}$$
$$[e_5,e_7]=\frac{5e_{13}}{2}$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_7]=e_{10}$$
$$[e_2,e_8]=3e_{11} \qquad [e_2,e_9]=\alpha_{2,9}^{12}e_{12}$$
$$[e_2,e_{10}]=\alpha_{2,10}^{13}e_{13} \qquad [e_3,e_6]=-e_{10}$$
$$[e_3,e_7]=-2e_{11} \qquad [e_3,e_8]=\alpha_{3,8}^{12}e_{12}$$
$$[e_3,e_9]=\alpha_{3,9}^{13}e_{13} \qquad [e_4,e_5]=e_{10}$$
$$[e_4,e_6]=e_{11} \qquad [e_4,e_7]=\alpha_{4,7}^{12}e_{12}$$
$$[e_4,e_8]=\alpha_{4,8}^{13}e_{13} \qquad [e_5,e_6]=\alpha_{5,6}^{12}e_{12}$$
$$[e_5,e_7]=\alpha_{5,7}^{13}e_{13}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_8):&\quad -\alpha_{2,9}^{12}-\alpha_{3,8}^{12}+3 &&=0\\
(e_1,e_3,e_7):&\quad -\alpha_{3,8}^{12}-\alpha_{4,7}^{12}-2 &&=0\\
(e_1,e_4,e_6):&\quad -\alpha_{4,7}^{12}-\alpha_{5,6}^{12}+1 &&=0\\
(e_1,e_2,e_9):&\quad -\alpha_{2,10}^{13}+\alpha_{2,9}^{12}-\alpha_{3,9}^{13} &&=0\\
(e_1,e_3,e_8):&\quad \alpha_{3,8}^{12}-\alpha_{3,9}^{13}-\alpha_{4,8}^{13} &&=0\\
(e_1,e_4,e_7):&\quad \alpha_{4,7}^{12}-\alpha_{4,8}^{13}-\alpha_{5,7}^{13} &&=0\\
(e_1,e_5,e_6):&\quad \alpha_{5,6}^{12}-\alpha_{5,7}^{13} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha_{2,10}^{13} &&=0\\
(e_2,e_4,e_5):&\quad \alpha_{2,10}^{13} &&=0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha^{13}_{4,8} &= -4\\
\alpha^{13}_{3,9} &= 7/2\\
\alpha^{12}_{3,8} &= -1/2\\
\alpha^{13}_{5,7} &= 5/2\\
\alpha^{13}_{2,10} &= 0\\
\alpha^{12}_{2,9} &= 7/2\\
\alpha^{12}_{4,7} &= -3/2\\
\alpha^{12}_{5,6} &= 5/2
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{4,8} \to x_1$$

$$\alpha^{13}_{3,9} \to x_2$$

$$\alpha^{12}_{3,8} \to x_3$$

$$\alpha^{13}_{5,7} \to x_4$$

$$\alpha^{13}_{2,10} \to x_5$$

$$\alpha^{12}_{2,9} \to x_6$$

$$\alpha^{12}_{4,7} \to x_7$$

$$\alpha^{12}_{5,6} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - x_6 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_3 - x_7 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_7 - x_8 + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -x_2 - x_5 + x_6 &&= 0\\
(e_1, e_3, e_8): &\quad -x_1 - x_2 + x_3 &&= 0\\
(e_1, e_4, e_7): &\quad -x_1 - x_4 + x_7 &&= 0\\
(e_1, e_5, e_6): &\quad -x_4 + x_8 &&= 0\\
(e_2, e_3, e_6): &\quad -x_5 &&= 0\\
(e_2, e_4, e_5): &\quad x_5 &&= 0
\end{aligned}$$

Groebner basis (8 variables, 8 linear, 0 nonlinear)

$$x_1 + 4 = 0$$
$$2x_2 - 7 = 0$$
$$2x_3 + 1 = 0$$
$$2x_4 - 5 = 0$$
$$x_5 = 0$$
$$2x_6 - 7 = 0$$
$$2x_7 + 3 = 0$$
$$2x_8 - 5 = 0$$

Solution 1:

$$x_1 = -4$$
$$x_2 = 7/2$$
$$x_3 = -1/2$$
$$x_4 = 5/2$$
$$x_5 = 0$$
$$x_6 = 7/2$$
$$x_7 = -3/2$$
$$x_8 = 5/2$$

$\mathfrak{m}_{6A}(3, 13)$

m6A313 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_5] = e_8$$
$$[e_2, e_6] = 2e_9 \qquad [e_2, e_7] = \frac{5e_{10}}{3}$$
$$[e_2, e_8] = 0 \qquad [e_2, e_9] = -\frac{49e_{12}}{33}$$
$$[e_2, e_{10}] = -\frac{14e_{13}}{11} \qquad [e_3, e_4] = -e_8$$
$$[e_3, e_5] = -e_9 \qquad [e_3, e_6] = \frac{e_{10}}{3}$$
$$[e_3, e_7] = \frac{5e_{11}}{3} \qquad [e_3, e_8] = \frac{49e_{12}}{33}$$
$$[e_3, e_9] = -\frac{7e_{13}}{33} \qquad [e_4, e_5] = -\frac{4e_{10}}{3}$$
$$[e_4, e_6] = -\frac{4e_{11}}{3} \qquad [e_4, e_7] = \frac{2e_{12}}{11}$$
$$[e_4, e_8] = \frac{56e_{13}}{33} \qquad [e_5, e_6] = -\frac{50e_{12}}{33}$$
$$[e_5, e_7] = -\frac{50e_{13}}{33}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_5] = e_8 \\
[e_2, e_6] = 2e_9 & [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] = \alpha_{2,8}^{11} e_{11} & [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} & [e_3, e_4] = -e_8 \\
[e_3, e_5] = -e_9 & [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] = \alpha_{3,7}^{11} e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] = \alpha_{4,6}^{11} e_{11} & [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] = \alpha_{4,8}^{13} e_{13} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6):\quad & -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0\\
(e_1, e_3, e_5):\quad & -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0\\
(e_1, e_2, e_7):\quad & \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6):\quad & \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5):\quad & \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4):\quad & -\alpha_{2,8}^{11} &&= 0\\
(e_1, e_2, e_8):\quad & \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_3, e_7):\quad & \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0\\
(e_1, e_4, e_6):\quad & \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0\\
(e_2, e_3, e_5):\quad & -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_2, e_9):\quad & -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8):\quad & \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7):\quad & \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6):\quad & \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6):\quad & \alpha_{2,10}^{13}\alpha_{3,6}^{10} - 2\alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_5):\quad & \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{4,8}^{13} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{13}_{4,8} = 56/33$$
$$\alpha^{11}_{2,8} = 0$$
$$\alpha^{13}_{3,9} = -7/33$$
$$\alpha^{10}_{3,6} = 1/3$$
$$\alpha^{11}_{4,6} = -4/3$$
$$\alpha^{12}_{3,8} = 49/33$$
$$\alpha^{13}_{5,7} = -50/33$$
$$\alpha^{11}_{3,7} = 5/3$$
$$\alpha^{12}_{2,9} = -49/33$$
$$\alpha^{10}_{4,5} = -4/3$$
$$\alpha^{13}_{2,10} = -14/11$$
$$\alpha^{10}_{2,7} = 5/3$$
$$\alpha^{12}_{4,7} = 2/11$$
$$\alpha^{12}_{5,6} = -50/33$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{4,8} \to x_1$$
$$\alpha^{11}_{2,8} \to x_2$$
$$\alpha^{13}_{3,9} \to x_3$$
$$\alpha^{10}_{3,6} \to x_4$$
$$\alpha^{11}_{4,6} \to x_5$$
$$\alpha^{12}_{3,8} \to x_6$$
$$\alpha^{13}_{5,7} \to x_7$$
$$\alpha^{11}_{3,7} \to x_8$$
$$\alpha^{12}_{2,9} \to x_9$$
$$\alpha^{10}_{4,5} \to x_{10}$$
$$\alpha^{13}_{2,10} \to x_{11}$$
$$\alpha^{10}_{2,7} \to x_{12}$$
$$\alpha^{12}_{4,7} \to x_{13}$$

$$\alpha_{5,6}^{12} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{12} - x_4 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{10} - x_4 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad x_{12} - x_2 - x_8 &&= 0\\
(e_1, e_3, e_6): &\quad x_4 - x_5 - x_8 &&= 0\\
(e_1, e_4, e_5): &\quad x_{10} - x_5 &&= 0\\
(e_2, e_3, e_4): &\quad -x_2 &&= 0\\
(e_1, e_2, e_8): &\quad x_2 - x_6 - x_9 &&= 0\\
(e_1, e_3, e_7): &\quad -x_{13} - x_6 + x_8 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{13} - x_{14} + x_5 &&= 0\\
(e_2, e_3, e_5): &\quad -x_6 - x_9 &&= 0\\
(e_1, e_2, e_9): &\quad -x_{11} - x_3 + x_9 &&= 0\\
(e_1, e_3, e_8): &\quad -x_1 - x_3 + x_6 &&= 0\\
(e_1, e_4, e_7): &\quad -x_1 + x_{13} - x_7 &&= 0\\
(e_1, e_5, e_6): &\quad x_{14} - x_7 &&= 0\\
(e_2, e_3, e_6): &\quad x_{11}x_4 - 2x_3 &&= 0\\
(e_2, e_4, e_5): &\quad -x_1 + x_{10}x_{11} &&= 0
\end{aligned}$$

Groebner basis (14 variables, 14 linear, 0 nonlinear)

$$33x_1 - 56 = 0$$

$$x_2 = 0$$

$$33x_3 + 7 = 0$$

$$3x_4 - 1 = 0$$

$$3x_5 + 4 = 0$$

$$33x_6 - 49 = 0$$

$$33x_7 + 50 = 0$$

$$3x_8 - 5 = 0$$

$$33x_9 + 49 = 0$$

$$3x_{10} + 4 = 0$$

$$11x_{11} + 14 = 0$$

$$3x_{12} - 5 = 0$$

$$11x_{13} - 2 = 0$$

$$33x_{14} + 50 = 0$$

Solution 1:

$$x_1 = 56/33$$

$$x_2 = 0$$

$$x_3 = -7/33$$

$$x_4 = 1/3$$

$$x_5 = -4/3$$

$$x_6 = 49/33$$

$$x_7 = -50/33$$

$$x_8 = 5/3$$

$$x_9 = -49/33$$

$$x_10 = -4/3$$

$$x_11 = -14/11$$

$$x_12 = 5/3$$

$$x_13 = 2/11$$

$$x_14 = -50/33$$

$\mathfrak{m}_{8A}(3, 13)$

m8A313 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] &= \alpha_{2,6}^{9} e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_2, e_9] &= \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] &= \alpha_{2,10}^{13} e_{13} & [e_3, e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] &= \alpha_{3,5}^{9} e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] &= \alpha_{3,9}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11} & [e_4, e_7] &= \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] &= \alpha_{4,8}^{13} e_{13} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] &= \alpha_{5,7}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4) &: \quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &&= 0\\
(e_1, e_2, e_5) &: \quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &&= 0\\
(e_1, e_3, e_4) &: \quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &&= 0\\
(e_1, e_2, e_6) &: \quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &&= 0\\
(e_1, e_3, e_5) &: \quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &&= 0\\
(e_1, e_2, e_7) &: \quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6) &: \quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5) &: \quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4) &: \quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &&= 0\\
(e_1, e_2, e_8) &: \quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_3, e_7) &: \quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0\\
(e_1, e_4, e_6) &: \quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0\\
(e_2, e_3, e_5) &: \quad -\alpha_{2,5}^{8}\alpha_{3,8}^{12} + \alpha_{2,9}^{12}\alpha_{3,5}^{9} + \alpha_{5,6}^{12} &&= 0\\
(e_1, e_2, e_9) &: \quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8) &: \quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7) &: \quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6) &: \quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6) &: \quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - \alpha_{2,6}^{9}\alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_5) &: \quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{2,5}^{8}\alpha_{4,8}^{13} + \alpha_{5,7}^{13} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,8}^{13} \to x_1$$
$$\alpha_{2,8}^{11} \to x_2$$
$$\alpha_{5,6}^{12} \to x_3$$
$$\alpha_{3,9}^{13} \to x_4$$
$$\alpha_{3,6}^{10} \to x_5$$
$$\alpha_{4,6}^{11} \to x_6$$
$$\alpha_{3,8}^{12} \to x_7$$
$$\alpha_{5,7}^{13} \to x_8$$

$$\alpha_{3,7}^{11} \to x_9$$

$$\alpha_{2,9}^{12} \to x_{10}$$

$$\alpha_{2,6}^{9} \to x_{11}$$

$$\alpha_{4,5}^{10} \to x_{12}$$

$$\alpha_{3,4}^{8} \to x_{13}$$

$$\alpha_{2,10}^{13} \to x_{14}$$

$$\alpha_{2,7}^{10} \to x_{15}$$

$$\alpha_{4,7}^{12} \to x_{16}$$

$$\alpha_{2,5}^{8} \to x_{17}$$

$$\alpha_{3,5}^{9} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{13} - x_{17} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{11} + x_{17} - x_{18} &&= 0\\
(e_1, e_3, e_4): &\quad x_{13} - x_{18} &&= 0\\
(e_1, e_2, e_6): &\quad x_{11} - x_{15} - x_5 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{12} + x_{18} - x_5 &&= 0\\
(e_1, e_2, e_7): &\quad x_{15} - x_2 - x_9 &&= 0\\
(e_1, e_3, e_6): &\quad x_5 - x_6 - x_9 &&= 0\\
(e_1, e_4, e_5): &\quad x_{12} - x_6 &&= 0\\
(e_2, e_3, e_4): &\quad x_{13}x_2 + x_6 - x_9 &&= 0\\
(e_1, e_2, e_8): &\quad -x_{10} + x_2 - x_7 &&= 0\\
(e_1, e_3, e_7): &\quad -x_{16} - x_7 + x_9 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{16} - x_3 + x_6 &&= 0\\
(e_2, e_3, e_5): &\quad x_{10}x_{18} - x_{17}x_7 + x_3 &&= 0\\
(e_1, e_2, e_9): &\quad x_{10} - x_{14} - x_4 &&= 0\\
(e_1, e_3, e_8): &\quad -x_1 - x_4 + x_7 &&= 0\\
(e_1, e_4, e_7): &\quad -x_1 + x_{16} - x_8 &&= 0\\
(e_1, e_5, e_6): &\quad x_3 - x_8 &&= 0\\
(e_2, e_3, e_6): &\quad -x_{11}x_4 + x_{14}x_5 &&= 0\\
(e_2, e_4, e_5): &\quad -x_1x_{17} + x_{12}x_{14} + x_8 &&= 0
\end{aligned}$$

Groebner basis (18 variables, 3 linear, 14 nonlinear)

$$24x_1 - 22x_{16}x_{18}^2 + 3x_{16}x_{18} - 39x_{16} - 4x_{18}^3 + 20x_{18}^2 = 0$$
$$22x_{16}x_{18}^2 - 3x_{16}x_{18} - 9x_{16} + 4x_{18}^3 - 20x_{18}^2 + 32x_{18} + 8x_2 - 8 = 0$$
$$22x_{16}x_{18}^2 - 3x_{16}x_{18} + 15x_{16} + 4x_{18}^3 - 20x_{18}^2 + 24x_3 = 0$$
$$-22x_{16}x_{18}^2 + 3x_{16}x_{18} + 81x_{16} - 4x_{18}^3 + 20x_{18}^2 - 24x_{18} + 24x_4 = 0$$
$$-22x_{16}x_{18}^2 + 3x_{16}x_{18} + 9x_{16} - 4x_{18}^3 + 20x_{18}^2 - 24x_{18} + 24x_5 = 0$$
$$22x_{16}x_{18}^2 - 3x_{16}x_{18} - 9x_{16} + 4x_{18}^3 - 20x_{18}^2 + 24x_6 = 0$$
$$-22x_{16}x_{18}^2 + 3x_{16}x_{18} + 21x_{16} - 4x_{18}^3 + 20x_{18}^2 - 12x_{18} + 12x_7 = 0$$
$$22x_{16}x_{18}^2 - 3x_{16}x_{18} + 15x_{16} + 4x_{18}^3 - 20x_{18}^2 + 24x_8 = 0$$
$$-22x_{16}x_{18}^2 + 3x_{16}x_{18} + 9x_{16} - 4x_{18}^3 + 20x_{18}^2 - 12x_{18} + 12x_9 = 0$$
$$24x_{10} + 110x_{16}x_{18}^2 - 15x_{16}x_{18} - 69x_{16} + 20x_{18}^3 - 100x_{18}^2 + 120x_{18} - 24 = 0$$
$$x_{11} + 2x_{18} - 1 = 0$$
$$24x_{12} + 22x_{16}x_{18}^2 - 3x_{16}x_{18} - 9x_{16} + 4x_{18}^3 - 20x_{18}^2 = 0$$
$$x_{13} - x_{18} = 0$$
$$4x_{14} + 22x_{16}x_{18}^2 - 3x_{16}x_{18} - 25x_{16} + 4x_{18}^3 - 20x_{18}^2 + 24x_{18} - 4 = 0$$
$$24x_{15} + 22x_{16}x_{18}^2 - 3x_{16}x_{18} - 9x_{16} + 4x_{18}^3 - 20x_{18}^2 + 72x_{18} - 24 = 0$$
$$22x_{16}x_{18}^3 + 19x_{16}x_{18}^2 - 12x_{16}x_{18} - 9x_{16} + 4x_{18}^4 - 16x_{18}^3 + 12x_{18}^2 = 0$$
$$x_{17} + x_{18} - 1 = 0$$

$\mathfrak{m}_{1A}(4, 13)$

m1A413 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_9] = e_{13}$$
$$[e_3, e_8] = -e_{13} \qquad [e_4, e_7] = e_{13}$$
$$[e_5, e_6] = -e_{13}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4,13)$

m3A413 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_2,e_7] &= e_{11} \\
[e_2,e_8] &= 3e_{12} & [e_2,e_9] &= \alpha_{2,9}^{13} e_{13} \\
[e_3,e_6] &= -e_{11} & [e_3,e_7] &= -2e_{12} \\
[e_3,e_8] &= \alpha_{3,8}^{13} e_{13} & [e_4,e_5] &= e_{11} \\
[e_4,e_6] &= e_{12} & [e_4,e_7] &= \alpha_{4,7}^{13} e_{13} \\
[e_5,e_6] &= \alpha_{5,6}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8): &\quad -\alpha_{2,9}^{13} - \alpha_{3,8}^{13} + 3 &= 0 \\
(e_1,e_3,e_7): &\quad -\alpha_{3,8}^{13} - \alpha_{4,7}^{13} - 2 &= 0 \\
(e_1,e_4,e_6): &\quad -\alpha_{4,7}^{13} - \alpha_{5,6}^{13} + 1 &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,9}^{13} &\to x_1 \\
\alpha_{5,6}^{13} &\to x_2 \\
\alpha_{3,8}^{13} &\to x_3 \\
\alpha_{4,7}^{13} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_8): &\quad -x_1 - x_3 + 3 &= 0 \\
(e_1,e_3,e_7): &\quad -x_3 - x_4 - 2 &= 0 \\
(e_1,e_4,e_6): &\quad -x_2 - x_4 + 1 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 - 5 = 0$$
$$x_2 + x_4 - 1 = 0$$
$$x_3 + x_4 + 2 = 0$$

$\mathfrak{m}_{5A}(4, 13)$

m5A413 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_9 \\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_2, e_8] &= \alpha_{2,8}^{12} e_{12} & [e_2, e_9] &= \alpha_{2,9}^{13} e_{13} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11} \\
[e_4, e_6] &= \alpha_{4,6}^{12} e_{12} & [e_4, e_7] &= \alpha_{4,7}^{13} e_{13} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6) &: \quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &&= 0 \\
(e_1, e_3, e_5) &: \quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &&= 0 \\
(e_1, e_2, e_7) &: \quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &&= 0 \\
(e_1, e_3, e_6) &: \quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &&= 0 \\
(e_1, e_4, e_5) &: \quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &&= 0 \\
(e_1, e_2, e_8) &: \quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &&= 0 \\
(e_1, e_3, e_7) &: \quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &&= 0 \\
(e_1, e_4, e_6) &: \quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &&= 0 \\
(e_2, e_3, e_4) &: \quad -\alpha_{2,9}^{13} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,7}^{12} \to x_1$$
$$\alpha_{3,8}^{13} \to x_2$$
$$\alpha_{2,8}^{12} \to x_3$$
$$\alpha_{4,6}^{12} \to x_4$$
$$\alpha_{3,6}^{11} \to x_5$$
$$\alpha_{4,7}^{13} \to x_6$$
$$\alpha_{2,9}^{13} \to x_7$$
$$\alpha_{5,6}^{13} \to x_8$$
$$\alpha_{4,5}^{11} \to x_9$$
$$\alpha_{2,7}^{11} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{10} - x_5 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_5 - x_9 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad -x_1 + x_{10} - x_3 &&= 0\\
(e_1, e_3, e_6): &\quad -x_1 - x_4 + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad -x_4 + x_9 &&= 0\\
(e_1, e_2, e_8): &\quad -x_2 + x_3 - x_7 &&= 0\\
(e_1, e_3, e_7): &\quad x_1 - x_2 - x_6 &&= 0\\
(e_1, e_4, e_6): &\quad x_4 - x_6 - x_8 &&= 0\\
(e_2, e_3, e_4): &\quad -x_7 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 9 linear, 0 nonlinear)

$$x_1 + 2x_{10} - 5 = 0$$
$$-3x_{10} + x_2 + 5 = 0$$
$$-3x_{10} + x_3 + 5 = 0$$
$$-x_{10} + x_4 + 3 = 0$$
$$x_{10} + x_5 - 2 = 0$$
$$5x_{10} + x_6 - 10 = 0$$
$$x_7 = 0$$
$$-6x_{10} + x_8 + 13 = 0$$
$$-x_{10} + x_9 + 3 = 0$$

$\mathfrak{m}_{7A}(4, 13)$

m7A413 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_7 \\
[e_2, e_4] &= e_8 & [e_2, e_5] &= \alpha_{2,5}^{9} e_9 \\
[e_2, e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_2, e_8] &= \alpha_{2,8}^{12} e_{12} & [e_2, e_9] &= \alpha_{2,9}^{13} e_{13} \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11} \\
[e_4, e_6] &= \alpha_{4,6}^{12} e_{12} & [e_4, e_7] &= \alpha_{4,7}^{13} e_{13} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0 \\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0 \\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0 \\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0 \\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0 \\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0 \\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0 \\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0 \\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &= 0 \\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &= 0 \\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &= 0 \\
(e_2, e_3, e_4):&\quad \alpha_{2,9}^{13}\alpha_{3,4}^{9} - \alpha_{3,8}^{13} + \alpha_{4,7}^{13} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{3,7}^{12} \to x_1$$
$$\alpha_{3,8}^{13} \to x_2$$
$$\alpha_{2,6}^{10} \to x_3$$
$$\alpha_{2,8}^{12} \to x_4$$
$$\alpha_{4,6}^{12} \to x_5$$
$$\alpha_{3,5}^{10} \to x_6$$
$$\alpha_{3,6}^{11} \to x_7$$
$$\alpha_{2,5}^{9} \to x_8$$
$$\alpha_{4,7}^{13} \to x_9$$
$$\alpha_{2,9}^{13} \to x_{10}$$
$$\alpha_{3,4}^{9} \to x_{11}$$
$$\alpha_{5,6}^{13} \to x_{12}$$
$$\alpha_{4,5}^{11} \to x_{13}$$
$$\alpha_{2,7}^{11} \to x_{14}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -x_{11} - x_8 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_3 - x_6 + x_8 &&= 0\\
(e_1, e_3, e_4):&\quad x_{11} - x_6 &&= 0\\
(e_1, e_2, e_6):&\quad -x_{14} + x_3 - x_7 &&= 0\\
(e_1, e_3, e_5):&\quad -x_{13} + x_6 - x_7 &&= 0\\
(e_1, e_2, e_7):&\quad -x_1 + x_{14} - x_4 &&= 0\\
(e_1, e_3, e_6):&\quad -x_1 - x_5 + x_7 &&= 0\\
(e_1, e_4, e_5):&\quad x_{13} - x_5 &&= 0\\
(e_1, e_2, e_8):&\quad -x_{10} - x_2 + x_4 &&= 0\\
(e_1, e_3, e_7):&\quad x_1 - x_2 - x_9 &&= 0\\
(e_1, e_4, e_6):&\quad -x_{12} + x_5 - x_9 &&= 0\\
(e_2, e_3, e_4):&\quad x_{10}x_{11} - x_2 + x_9 &&= 0
\end{aligned}
$$

Groebner basis (14 variables, 11 linear, 1 nonlinear)

$$3x_1 + 5x_{13} + x_{14} - 1 = 0$$

$$-3x_{12}+8x_{13}+x_{14}+3x_2-1=0$$
$$2x_{13}-2x_{14}+3x_3-1=0$$
$$-5x_{13}-4x_{14}+3x_4+1=0$$
$$-x_{13}+x_5=0$$
$$-x_{13}+x_{14}+3x_6-1=0$$
$$2x_{13}+x_{14}+3x_7-1=0$$
$$x_{13}-x_{14}+3x_8-2=0$$
$$x_{12}-x_{13}+x_9=0$$
$$3x_{10}+3x_{12}-13x_{13}-5x_{14}+2=0$$
$$3x_{11}-x_{13}+x_{14}-1=0$$
$$3x_{12}x_{13}-3x_{12}x_{14}+21x_{12}-13x_{13}^2+8x_{13}x_{14}-44x_{13}+5x_{14}^2-10x_{14}+5=0$$

$\mathfrak{m}_{2A}(5,13)$

m2A513 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_1,e_{12}]&=e_{13} & [e_2,e_7]&=e_{12}\\
[e_2,e_8]&=3e_{13} & [e_3,e_6]&=-e_{12}\\
[e_3,e_7]&=-2e_{13} & [e_4,e_5]&=e_{12}\\
[e_4,e_6]&=e_{13} & &
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(5,13)$

m4A513 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_2,e_5] &= e_{10}\\
[e_2,e_6] &= 2e_{11} & [e_2,e_7] &= \alpha_{2,7}^{12}e_{12}\\
[e_2,e_8] &= \alpha_{2,8}^{13}e_{13} & [e_3,e_4] &= -e_{10}\\
[e_3,e_5] &= -e_{11} & [e_3,e_6] &= \alpha_{3,6}^{12}e_{12}\\
[e_3,e_7] &= \alpha_{3,7}^{13}e_{13} & [e_4,e_5] &= \alpha_{4,5}^{12}e_{12}\\
[e_4,e_6] &= \alpha_{4,6}^{13}e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -\alpha_{2,7}^{12}-\alpha_{3,6}^{12}+2 &&= 0\\
(e_1,e_3,e_5):&\quad -\alpha_{3,6}^{12}-\alpha_{4,5}^{12}-1 &&= 0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{12}-\alpha_{2,8}^{13}-\alpha_{3,7}^{13} &&= 0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{12}-\alpha_{3,7}^{13}-\alpha_{4,6}^{13} &&= 0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{12}-\alpha_{4,6}^{13} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{12} \to x_1$$

$$\alpha_{4,6}^{13} \to x_2$$

$$\alpha_{3,6}^{12} \to x_3$$

$$\alpha_{2,8}^{13} \to x_4$$

$$\alpha_{4,5}^{12} \to x_5$$

$$\alpha_{3,7}^{13} \to x_6$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -x_1-x_3+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_3-x_5-1 &&=0\\
(e_1,e_2,e_7):&\quad x_1-x_4-x_6 &&=0\\
(e_1,e_3,e_6):&\quad -x_2+x_3-x_6 &&=0\\
(e_1,e_4,e_5):&\quad -x_2+x_5 &&=0
\end{aligned}
$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$2x_1+x_6-5=0$$
$$2x_2+x_6+1=0$$
$$2x_3-x_6+1=0$$
$$2x_4+3x_6-5=0$$
$$2x_5+x_6+1=0$$

$\mathfrak{m}_{6A}(5,13)$

m6A513 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_1,e_{12}]&=e_{13} & [e_2,e_3]&=e_8\\
[e_2,e_4]&=e_9 & [e_2,e_5]&=\alpha_{2,5}^{10}e_{10}\\
[e_2,e_6]&=\alpha_{2,6}^{11}e_{11} & [e_2,e_7]&=\alpha_{2,7}^{12}e_{12}\\
[e_2,e_8]&=\alpha_{2,8}^{13}e_{13} & [e_3,e_4]&=\alpha_{3,4}^{10}e_{10}\\
[e_3,e_5]&=\alpha_{3,5}^{11}e_{11} & [e_3,e_6]&=\alpha_{3,6}^{12}e_{12}\\
[e_3,e_7]&=\alpha_{3,7}^{13}e_{13} & [e_4,e_5]&=\alpha_{4,5}^{12}e_{12}\\
[e_4,e_6]&=\alpha_{4,6}^{13}e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{10}-\alpha_{3,4}^{10}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{10}-\alpha_{2,6}^{11}-\alpha_{3,5}^{11} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{10}-\alpha_{3,5}^{11} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{11}-\alpha_{2,7}^{12}-\alpha_{3,6}^{12} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{11}-\alpha_{3,6}^{12}-\alpha_{4,5}^{12} &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{12}-\alpha_{2,8}^{13}-\alpha_{3,7}^{13} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{12}-\alpha_{3,7}^{13}-\alpha_{4,6}^{13} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{12}-\alpha_{4,6}^{13} &&=0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{2,6}^{11}\to x_1\\
\alpha_{2,7}^{12}\to x_2\\
\alpha_{3,4}^{10}\to x_3\\
\alpha_{2,5}^{10}\to x_4\\
\alpha_{4,6}^{13}\to x_5\\
\alpha_{3,6}^{12}\to x_6\\
\alpha_{2,8}^{13}\to x_7\\
\alpha_{3,5}^{11}\to x_8\\
\alpha_{4,5}^{12}\to x_9\\
\alpha_{3,7}^{13}\to x_{10}
\end{gathered}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_3-x_4+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_1+x_4-x_8 &&=0\\
(e_1,e_3,e_4):&\quad x_3-x_8 &&=0\\
(e_1,e_2,e_6):&\quad x_1-x_2-x_6 &&=0\\
(e_1,e_3,e_5):&\quad -x_6+x_8-x_9 &&=0\\
(e_1,e_2,e_7):&\quad -x_{10}+x_2-x_7 &&=0\\
(e_1,e_3,e_6):&\quad -x_{10}-x_5+x_6 &&=0\\
(e_1,e_4,e_5):&\quad -x_5+x_9 &&=0
\end{aligned}
$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + 2x_{10} + 4x_9 - 1 = 0$$
$$3x_{10} + x_2 + 5x_9 - 1 = 0$$
$$-x_{10} + x_3 - 2x_9 = 0$$
$$x_{10} + x_4 + 2x_9 - 1 = 0$$
$$x_5 - x_9 = 0$$
$$-x_{10} + x_6 - x_9 = 0$$
$$4x_{10} + x_7 + 5x_9 - 1 = 0$$
$$-x_{10} + x_8 - 2x_9 = 0$$

$\mathfrak{m}_{1A}(6, 13)$

m1A613 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_7] = e_{13}$$
$$[e_3, e_6] = -e_{13} \qquad [e_4, e_5] = e_{13}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(6, 13)$

m3A613 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{11} \\
[e_2, e_6] &= 2e_{12} & [e_2, e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_3, e_4] &= -e_{11} & [e_3, e_5] &= -e_{12} \\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{13} e_{13}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{13} - \alpha_{3,6}^{13} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{13} - \alpha_{4,5}^{13} - 1 &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,6}^{13} \to x_1$$

$$\alpha_{4,5}^{13} \to x_2$$

$$\alpha_{2,7}^{13} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_3 + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -x_1 - x_2 - 1 &= 0
\end{aligned}$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 - 2 = 0$$

$$x_2 - x_3 + 3 = 0$$

$\mathfrak{m}_{5A}(6, 13)$

m5A613 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_9 \\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= \alpha_{2,5}^{11} e_{11} \\
[e_2, e_6] &= \alpha_{2,6}^{12} e_{12} & [e_2, e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_3, e_4] &= \alpha_{3,4}^{11} e_{11} & [e_3, e_5] &= \alpha_{3,5}^{12} e_{12} \\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{12} - \alpha_{2,7}^{13} - \alpha_{3,6}^{13} &= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{12} - \alpha_{3,6}^{13} - \alpha_{4,5}^{13} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{4,5}^{13} &\to x_1 \\
\alpha_{2,7}^{13} &\to x_2 \\
\alpha_{2,5}^{11} &\to x_3 \\
\alpha_{3,4}^{11} &\to x_4 \\
\alpha_{2,6}^{12} &\to x_5 \\
\alpha_{3,5}^{12} &\to x_6 \\
\alpha_{3,6}^{13} &\to x_7
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): \quad & -x_3 - x_4 + 1 & = 0 \\
(e_1, e_2, e_5): \quad & x_3 - x_5 - x_6 & = 0 \\
(e_1, e_3, e_4): \quad & x_4 - x_6 & = 0 \\
(e_1, e_2, e_6): \quad & -x_2 + x_5 - x_7 & = 0 \\
(e_1, e_3, e_5): \quad & -x_1 + x_6 - x_7 & = 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - x_6 + x_7 = 0$$

$$x_2 + 2x_6 + x_7 - 1 = 0$$

$$x_3 + x_6 - 1 = 0$$

$$x_4 - x_6 = 0$$

$$x_5 + 2x_6 - 1 = 0$$

$\mathfrak{m}_{2A}(7, 13)$

m2A713 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{12} \\
[e_2, e_6] &= 2e_{13} & [e_3, e_4] &= -e_{12} \\
[e_3, e_5] &= -e_{13}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(7, 13)$

m4A713 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_5] &= \alpha_{2,5}^{12} e_{12} \\
[e_2, e_6] &= \alpha_{2,6}^{13} e_{13} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12} \\
[e_3, e_5] &= \alpha_{3,5}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{12} - \alpha_{3,5}^{13} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,6}^{13} &\to x_1 \\
\alpha_{3,4}^{12} &\to x_2 \\
\alpha_{2,5}^{12} &\to x_3 \\
\alpha_{3,5}^{13} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_3 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad -x_1 + x_3 - x_4 &= 0 \\
(e_1, e_3, e_4): &\quad x_2 - x_4 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 + 2x_4 - 1 &= 0 \\
x_2 - x_4 &= 0 \\
x_3 + x_4 - 1 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(8, 13)$

m1A813 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{13} \\
[e_3, e_4] &= -e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(8, 13)$

m3A813 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{11} \\
[e_2, e_4] &= e_{12} & [e_2, e_5] &= \alpha_{2,5}^{13} e_{13} \\
[e_3, e_4] &= \alpha_{3,4}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
(e_1, e_2, e_4): \quad -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 = 0
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,5}^{13} &\to x_1 \\
\alpha_{3,4}^{13} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(9, 13)$

m2A913 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{12} \\
[e_2, e_4] &= e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(10, 13)$

m1A1013 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2, 14)$

m2A214 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_{11}] &= e_{13} & [e_2, e_{12}] &= 5e_{14} \\
[e_3, e_{10}] &= -e_{13} & [e_3, e_{11}] &= -4e_{14} \\
[e_4, e_9] &= e_{13} & [e_4, e_{10}] &= 3e_{14} \\
[e_5, e_8] &= -e_{13} & [e_5, e_9] &= -2e_{14} \\
[e_6, e_7] &= e_{13} & [e_6, e_8] &= e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2, 14)$

m4A214 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_9] = e_{11} & [e_2, e_{10}] = 4e_{12} \\
[e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} & [e_2, e_{12}] = \alpha_{2,12}^{14} e_{14} \\
[e_3, e_8] = -e_{11} & [e_3, e_9] = -3e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_3, e_{11}] = \alpha_{3,11}^{14} e_{14} \\
[e_4, e_7] = e_{11} & [e_4, e_8] = 2e_{12} \\
[e_4, e_9] = \alpha_{4,9}^{13} e_{13} & [e_4, e_{10}] = \alpha_{4,10}^{14} e_{14} \\
[e_5, e_6] = -e_{11} & [e_5, e_7] = -e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_5, e_9] = \alpha_{5,9}^{14} e_{14} \\
[e_6, e_7] = \alpha_{6,7}^{13} e_{13} & [e_6, e_8] = \alpha_{6,8}^{14} e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{2,11}^{13} - \alpha_{3,10}^{13} + 4 &= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{13} - \alpha_{4,9}^{13} - 3 &= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{13} - \alpha_{5,8}^{13} + 2 &= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{13} - \alpha_{6,7}^{13} - 1 &= 0\\
(e_2, e_3, e_8): &\quad -\alpha_{2,11}^{13} &= 0\\
(e_2, e_4, e_7): &\quad \alpha_{2,11}^{13} &= 0\\
(e_2, e_5, e_6): &\quad -\alpha_{2,11}^{13} &= 0\\
(e_1, e_2, e_{11}): &\quad \alpha_{2,11}^{13} - \alpha_{2,12}^{14} - \alpha_{3,11}^{14} &= 0\\
(e_1, e_3, e_{10}): &\quad \alpha_{3,10}^{13} - \alpha_{3,11}^{14} - \alpha_{4,10}^{14} &= 0\\
(e_1, e_4, e_9): &\quad -\alpha_{4,10}^{14} + \alpha_{4,9}^{13} - \alpha_{5,9}^{14} &= 0\\
(e_1, e_5, e_8): &\quad \alpha_{5,8}^{13} - \alpha_{5,9}^{14} - \alpha_{6,8}^{14} &= 0\\
(e_1, e_6, e_7): &\quad \alpha_{6,7}^{13} - \alpha_{6,8}^{14} &= 0\\
(e_2, e_3, e_9): &\quad -3\alpha_{2,12}^{14} - \alpha_{3,11}^{14} &= 0\\
(e_2, e_4, e_8): &\quad 2\alpha_{2,12}^{14} &= 0\\
(e_2, e_5, e_7): &\quad -\alpha_{2,12}^{14} &= 0\\
(e_3, e_4, e_7): &\quad \alpha_{3,11}^{14} &= 0\\
(e_3, e_5, e_6): &\quad -\alpha_{3,11}^{14} &= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,10}^{14} \to x_1$$

$$\alpha_{6,8}^{14} \to x_2$$

$$\alpha_{5,8}^{13} \to x_3$$

$$\alpha_{3,10}^{13} \to x_4$$

$$\alpha_{4,9}^{13} \to x_5$$

$$\alpha_{2,11}^{13} \to x_6$$

$$\alpha_{6,7}^{13} \to x_7$$

$$\alpha_{2,12}^{14} \to x_8$$

$$\alpha_{3,11}^{14} \to x_9$$

$$\alpha_{5,9}^{14} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_{10}): \quad & -x_4 - x_6 + 4 && = 0 \\
(e_1, e_3, e_9): \quad & -x_4 - x_5 - 3 && = 0 \\
(e_1, e_4, e_8): \quad & -x_3 - x_5 + 2 && = 0 \\
(e_1, e_5, e_7): \quad & -x_3 - x_7 - 1 && = 0 \\
(e_2, e_3, e_8): \quad & -x_6 && = 0 \\
(e_2, e_4, e_7): \quad & x_6 && = 0 \\
(e_2, e_5, e_6): \quad & -x_6 && = 0 \\
(e_1, e_2, e_{11}): \quad & x_6 - x_8 - x_9 && = 0 \\
(e_1, e_3, e_{10}): \quad & -x_1 + x_4 - x_9 && = 0 \\
(e_1, e_4, e_9): \quad & -x_1 - x_{10} + x_5 && = 0 \\
(e_1, e_5, e_8): \quad & -x_{10} - x_2 + x_3 && = 0 \\
(e_1, e_6, e_7): \quad & -x_2 + x_7 && = 0 \\
(e_2, e_3, e_9): \quad & -3x_8 - x_9 && = 0 \\
(e_2, e_4, e_8): \quad & 2x_8 && = 0 \\
(e_2, e_5, e_7): \quad & -x_8 && = 0 \\
(e_3, e_4, e_7): \quad & x_9 && = 0 \\
(e_3, e_5, e_6): \quad & -x_9 && = 0
\end{aligned}
$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{10A}(2, 14)$

m10A214 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_3] = e_5 \qquad [e_2, e_4] = e_6$$
$$[e_2, e_5] = e_7 \qquad [e_2, e_6] = e_8$$
$$[e_2, e_7] = e_9 \qquad [e_2, e_8] = e_{10}$$
$$[e_2, e_9] = e_{11} \qquad [e_2, e_{10}] = e_{12}$$
$$[e_2, e_{11}] = e_{13} \qquad [e_2, e_{12}] = e_{14}$$
$$[e_3, e_4] = 0 \qquad [e_3, e_5] = 0$$
$$[e_3, e_6] = 0 \qquad [e_3, e_7] = 0$$
$$[e_3, e_8] = 0 \qquad [e_3, e_9] = 0$$
$$[e_3, e_{10}] = 0 \qquad [e_3, e_{11}] = 0$$
$$[e_4, e_5] = 0 \qquad [e_4, e_6] = 0$$
$$[e_4, e_7] = 0 \qquad [e_4, e_8] = 0$$
$$[e_4, e_9] = 0 \qquad [e_4, e_{10}] = 0$$
$$[e_5, e_6] = 0 \qquad [e_5, e_7] = 0$$
$$[e_5, e_8] = 0 \qquad [e_5, e_9] = 0$$
$$[e_6, e_7] = 0 \qquad [e_6, e_8] = 0$$

Solution 2

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_3] = e_5 \qquad [e_2, e_4] = e_6$$
$$[e_2, e_5] = \frac{9e_7}{10} \qquad [e_2, e_6] = \frac{4e_8}{5}$$
$$[e_2, e_7] = \frac{5e_9}{7} \qquad [e_2, e_8] = \frac{9e_{10}}{14}$$
$$[e_2, e_9] = \frac{7e_{11}}{12} \qquad [e_2, e_{10}] = \frac{8e_{12}}{15}$$
$$[e_2, e_{11}] = \frac{27e_{13}}{55} \qquad [e_2, e_{12}] = \frac{5e_{14}}{11}$$
$$[e_3, e_4] = \frac{e_7}{10} \qquad [e_3, e_5] = \frac{e_8}{10}$$
$$[e_3, e_6] = \frac{3e_9}{35} \qquad [e_3, e_7] = \frac{e_{10}}{14}$$
$$[e_3, e_8] = \frac{5e_{11}}{84} \qquad [e_3, e_9] = \frac{e_{12}}{20}$$
$$[e_3, e_{10}] = \frac{7e_{13}}{165} \qquad [e_3, e_{11}] = \frac{2e_{14}}{55}$$
$$[e_4, e_5] = \frac{e_9}{70} \qquad [e_4, e_6] = \frac{e_{10}}{70}$$
$$[e_4, e_7] = \frac{e_{11}}{84} \qquad [e_4, e_8] = \frac{e_{12}}{105}$$
$$[e_4, e_9] = \frac{e_{13}}{132} \qquad [e_4, e_{10}] = \frac{e_{14}}{165}$$
$$[e_5, e_6] = \frac{e_{11}}{420} \qquad [e_5, e_7] = \frac{e_{12}}{420}$$
$$[e_5, e_8] = \frac{3e_{13}}{1540} \qquad [e_5, e_9] = \frac{e_{14}}{660}$$
$$[e_6, e_7] = \frac{e_{13}}{2310} \qquad [e_6, e_8] = \frac{e_{14}}{2310}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_3] = e_5 & [e_2, e_4] = e_6 \\
[e_2, e_5] = \alpha_{2,5}^{7} e_7 & [e_2, e_6] = \alpha_{2,6}^{8} e_8 \\
[e_2, e_7] = \alpha_{2,7}^{9} e_9 & [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
[e_2, e_9] = \alpha_{2,9}^{11} e_{11} & [e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} \\
[e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} & [e_2, e_{12}] = \alpha_{2,12}^{14} e_{14} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] = \alpha_{3,8}^{11} e_{11} & [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_3, e_{11}] = \alpha_{3,11}^{14} e_{14} \\
[e_4, e_5] = \alpha_{4,5}^{9} e_9 & [e_4, e_6] = \alpha_{4,6}^{10} e_{10} \\
[e_4, e_7] = \alpha_{4,7}^{11} e_{11} & [e_4, e_8] = \alpha_{4,8}^{12} e_{12} \\
[e_4, e_9] = \alpha_{4,9}^{13} e_{13} & [e_4, e_{10}] = \alpha_{4,10}^{14} e_{14} \\
[e_5, e_6] = \alpha_{5,6}^{11} e_{11} & [e_5, e_7] = \alpha_{5,7}^{12} e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_5, e_9] = \alpha_{5,9}^{14} e_{14} \\
[e_6, e_7] = \alpha_{6,7}^{13} e_{13} & [e_6, e_8] = \alpha_{6,8}^{14} e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &&= 0\\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &&= 0\\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &&= 0\\
(e_2, e_3, e_4):&\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &&= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5):&\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10} + \alpha_{2,8}^{10}\alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6):&\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11} + \alpha_{2,9}^{11}\alpha_{3,6}^{9} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_4, e_5):&\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11} + \alpha_{2,9}^{11}\alpha_{4,5}^{9} + \alpha_{5,6}^{11} &&= 0\\
(e_1, e_2, e_9):&\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8):&\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7):&\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6):&\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7):&\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10} - \alpha_{2,7}^{9}\alpha_{3,9}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_4, e_6):&\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10} - \alpha_{2,6}^{8}\alpha_{4,8}^{12} &&= 0\\
(e_3, e_4, e_5):&\quad \alpha_{3,4}^{7}\alpha_{5,7}^{12} - \alpha_{3,5}^{8}\alpha_{4,8}^{12} + \alpha_{3,9}^{12}\alpha_{4,5}^{9} &&= 0\\
(e_1, e_2, e_{10}):&\quad \alpha_{2,10}^{12} - \alpha_{2,11}^{13} - \alpha_{3,10}^{13} &&= 0\\
(e_1, e_3, e_9):&\quad -\alpha_{3,10}^{13} + \alpha_{3,9}^{12} - \alpha_{4,9}^{13} &&= 0\\
(e_1, e_4, e_8):&\quad \alpha_{4,8}^{12} - \alpha_{4,9}^{13} - \alpha_{5,8}^{13} &&= 0\\
(e_1, e_5, e_7):&\quad \alpha_{5,7}^{12} - \alpha_{5,8}^{13} - \alpha_{6,7}^{13} &&= 0\\
(e_2, e_3, e_8):&\quad \alpha_{2,11}^{13}\alpha_{3,8}^{11} - \alpha_{2,8}^{10}\alpha_{3,10}^{13} - \alpha_{5,8}^{13} &&= 0\\
(e_2, e_4, e_7):&\quad \alpha_{2,11}^{13}\alpha_{4,7}^{11} - \alpha_{2,7}^{9}\alpha_{4,9}^{13} - \alpha_{6,7}^{13} &&= 0\\
(e_2, e_5, e_6):&\quad \alpha_{2,11}^{13}\alpha_{5,6}^{11} + \alpha_{2,5}^{7}\alpha_{6,7}^{13} - \alpha_{2,6}^{8}\alpha_{5,8}^{13} &&= 0\\
(e_3, e_4, e_6):&\quad \alpha_{3,10}^{13}\alpha_{4,6}^{10} + \alpha_{3,4}^{7}\alpha_{6,7}^{13} - \alpha_{3,6}^{9}\alpha_{4,9}^{13} &&= 0\\
(e_1, e_2, e_{11}):&\quad \alpha_{2,11}^{13} - \alpha_{2,12}^{14} - \alpha_{3,11}^{14} &&= 0\\
(e_1, e_3, e_{10}):&\quad \alpha_{3,10}^{13} - \alpha_{3,11}^{14} - \alpha_{4,10}^{14} &&= 0\\
(e_1, e_4, e_9):&\quad -\alpha_{4,10}^{14} + \alpha_{4,9}^{13} - \alpha_{5,9}^{14} &&= 0\\
(e_1, e_5, e_8):&\quad \alpha_{5,8}^{13} - \alpha_{5,9}^{14} - \alpha_{6,8}^{14} &&= 0\\
(e_1, e_6, e_7):&\quad \alpha_{6,7}^{13} - \alpha_{6,8}^{14} &&= 0\\
(e_2, e_3, e_9):&\quad \alpha_{2,12}^{14}\alpha_{3,9}^{12} - \alpha_{2,9}^{11}\alpha_{3,11}^{14} - \alpha_{5,9}^{14} &&= 0\\
(e_2, e_4, e_8):&\quad \alpha_{2,12}^{14}\alpha_{4,8}^{12} - \alpha_{2,8}^{10}\alpha_{4,10}^{14} - \alpha_{6,8}^{14} &&= 0\\
(e_2, e_5, e_7):&\quad \alpha_{2,12}^{14}\alpha_{5,7}^{12} - \alpha_{2,7}^{9}\alpha_{5,9}^{14} &&= 0\\
(e_3, e_4, e_7):&\quad \alpha_{3,11}^{14}\alpha_{4,7}^{11} - \alpha_{3,7}^{10}\alpha_{4,10}^{14} &&= 0\\
(e_3, e_5, e_6):&\quad \alpha_{3,11}^{14}\alpha_{5,6}^{11} + \alpha_{3,5}^{8}\alpha_{6,8}^{14} - \alpha_{3,6}^{9}\alpha_{5,9}^{14} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,10}^{14} &= 0 \\
\alpha_{3,10}^{13} &= 0 \\
\alpha_{4,9}^{13} &= 0 \\
\alpha_{3,6}^{9} &= 0 \\
\alpha_{2,11}^{13} &= 1 \\
\alpha_{3,4}^{7} &= 0 \\
\alpha_{2,9}^{11} &= 1 \\
\alpha_{6,8}^{14} &= 0 \\
\alpha_{2,7}^{9} &= 1 \\
\alpha_{2,6}^{8} &= 1 \\
\alpha_{2,5}^{7} &= 1 \\
\alpha_{5,7}^{12} &= 0 \\
\alpha_{6,7}^{13} &= 0 \\
\alpha_{3,5}^{8} &= 0 \\
\alpha_{4,6}^{10} &= 0 \\
\alpha_{3,11}^{14} &= 0 \\
\alpha_{5,9}^{14} &= 0 \\
\alpha_{3,9}^{12} &= 0 \\
\alpha_{3,7}^{10} &= 0 \\
\alpha_{2,10}^{12} &= 1 \\
\alpha_{2,8}^{10} &= 1 \\
\alpha_{4,7}^{11} &= 0 \\
\alpha_{5,6}^{11} &= 0 \\
\alpha_{2,12}^{14} &= 1 \\
\alpha_{5,8}^{13} &= 0 \\
\alpha_{4,5}^{9} &= 0 \\
\alpha_{4,8}^{12} &= 0 \\
\alpha_{3,8}^{11} &= 0
\end{aligned}
$$

Solution 2:

$$\alpha^{14}_{4,10} = 1/165$$
$$\alpha^{13}_{3,10} = 7/165$$
$$\alpha^{13}_{4,9} = 1/132$$
$$\alpha^{9}_{3,6} = 3/35$$
$$\alpha^{13}_{2,11} = 27/55$$
$$\alpha^{7}_{3,4} = 1/10$$
$$\alpha^{11}_{2,9} = 7/12$$
$$\alpha^{14}_{6,8} = 1/2310$$
$$\alpha^{9}_{2,7} = 5/7$$
$$\alpha^{8}_{2,6} = 4/5$$
$$\alpha^{7}_{2,5} = 9/10$$
$$\alpha^{12}_{5,7} = 1/420$$
$$\alpha^{13}_{6,7} = 1/2310$$
$$\alpha^{8}_{3,5} = 1/10$$
$$\alpha^{10}_{4,6} = 1/70$$
$$\alpha^{14}_{3,11} = 2/55$$
$$\alpha^{14}_{5,9} = 1/660$$
$$\alpha^{12}_{3,9} = 1/20$$
$$\alpha^{10}_{3,7} = 1/14$$
$$\alpha^{12}_{2,10} = 8/15$$
$$\alpha^{10}_{2,8} = 9/14$$
$$\alpha^{11}_{4,7} = 1/84$$
$$\alpha^{11}_{5,6} = 1/420$$
$$\alpha^{14}_{2,12} = 5/11$$
$$\alpha^{13}_{5,8} = 3/1540$$
$$\alpha^{9}_{4,5} = 1/70$$
$$\alpha^{12}_{4,8} = 1/105$$
$$\alpha^{11}_{3,8} = 5/84$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{14}_{4,10} \to x_1$$

$$\alpha^{13}_{3,10} \to x_2$$

$$\alpha^{13}_{4,9} \to x_3$$

$$\alpha^{9}_{3,6} \to x_4$$

$$\alpha^{13}_{2,11} \to x_5$$

$$\alpha^{7}_{3,4} \to x_6$$

$$\alpha^{11}_{2,9} \to x_7$$

$$\alpha^{14}_{6,8} \to x_8$$

$$\alpha^{9}_{2,7} \to x_9$$

$$\alpha^{8}_{2,6} \to x_{10}$$

$$\alpha^{7}_{2,5} \to x_{11}$$

$$\alpha^{12}_{5,7} \to x_{12}$$

$$\alpha^{13}_{6,7} \to x_{13}$$

$$\alpha^{8}_{3,5} \to x_{14}$$

$$\alpha^{10}_{4,6} \to x_{15}$$

$$\alpha^{14}_{3,11} \to x_{16}$$

$$\alpha^{14}_{5,9} \to x_{17}$$

$$\alpha^{12}_{3,9} \to x_{18}$$

$$\alpha^{10}_{3,7} \to x_{19}$$

$$\alpha^{12}_{2,10} \to x_{20}$$

$$\alpha^{10}_{2,8} \to x_{21}$$

$$\alpha^{11}_{4,7} \to x_{22}$$

$$\alpha^{11}_{5,6} \to x_{23}$$

$$\alpha^{14}_{2,12} \to x_{24}$$

$$\alpha^{13}_{5,8} \to x_{25}$$

$$\alpha^{9}_{4,5} \to x_{26}$$

$$\alpha^{12}_{4,8} \to x_{27}$$

$$\alpha^{11}_{3,8} \to x_{28}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -x_{11} - x_6 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_{10} + x_{11} - x_{14} &&= 0\\
(e_1, e_3, e_4):&\quad -x_{14} + x_6 &&= 0\\
(e_1, e_2, e_6):&\quad x_{10} - x_4 - x_9 &&= 0\\
(e_1, e_3, e_5):&\quad x_{14} - x_{26} - x_4 &&= 0\\
(e_2, e_3, e_4):&\quad x_{26} - x_4 + x_6 x_9 &&= 0\\
(e_1, e_2, e_7):&\quad -x_{19} - x_{21} + x_9 &&= 0\\
(e_1, e_3, e_6):&\quad -x_{15} - x_{19} + x_4 &&= 0\\
(e_1, e_4, e_5):&\quad -x_{15} + x_{26} &&= 0\\
(e_2, e_3, e_5):&\quad -x_{11} x_{19} + x_{14} x_{21} &&= 0\\
(e_1, e_2, e_8):&\quad x_{21} - x_{28} - x_7 &&= 0\\
(e_1, e_3, e_7):&\quad x_{19} - x_{22} - x_{28} &&= 0\\
(e_1, e_4, e_6):&\quad x_{15} - x_{22} - x_{23} &&= 0\\
(e_2, e_3, e_6):&\quad -x_{10} x_{28} - x_{23} + x_4 x_7 &&= 0\\
(e_2, e_4, e_5):&\quad -x_{11} x_{22} + x_{23} + x_{26} x_7 &&= 0\\
(e_1, e_2, e_9):&\quad -x_{18} - x_{20} + x_7 &&= 0\\
(e_1, e_3, e_8):&\quad -x_{18} - x_{27} + x_{28} &&= 0\\
(e_1, e_4, e_7):&\quad -x_{12} + x_{22} - x_{27} &&= 0\\
(e_1, e_5, e_6):&\quad -x_{12} + x_{23} &&= 0\\
(e_2, e_3, e_7):&\quad -x_{12} - x_{18} x_9 + x_{19} x_{20} &&= 0\\
(e_2, e_4, e_6):&\quad -x_{10} x_{27} + x_{15} x_{20} &&= 0\\
(e_3, e_4, e_5):&\quad x_{12} x_6 - x_{14} x_{27} + x_{18} x_{26} &&= 0\\
(e_1, e_2, e_{10}):&\quad -x_2 + x_{20} - x_5 &&= 0\\
(e_1, e_3, e_9):&\quad x_{18} - x_2 - x_3 &&= 0\\
(e_1, e_4, e_8):&\quad -x_{25} + x_{27} - x_3 &&= 0\\
(e_1, e_5, e_7):&\quad x_{12} - x_{13} - x_{25} &&= 0\\
(e_2, e_3, e_8):&\quad -x_2 x_{21} - x_{25} + x_{28} x_5 &&= 0\\
(e_2, e_4, e_7):&\quad -x_{13} + x_{22} x_5 - x_3 x_9 &&= 0\\
(e_2, e_5, e_6):&\quad -x_{10} x_{25} + x_{11} x_{13} + x_{23} x_5 &&= 0\\
(e_3, e_4, e_6):&\quad x_{13} x_6 + x_{15} x_2 - x_3 x_4 &&= 0\\
(e_1, e_2, e_{11}):&\quad -x_{16} - x_{24} + x_5 &&= 0\\
(e_1, e_3, e_{10}):&\quad -x_1 - x_{16} + x_2 &&= 0\\
(e_1, e_4, e_9):&\quad -x_1 - x_{17} + x_3 &&= 0\\
(e_1, e_5, e_8):&\quad -x_{17} + x_{25} - x_8 &&= 0\\
(e_1, e_6, e_7):&\quad x_{13} - x_8 &&= 0\\
(e_2, e_3, e_9):&\quad -x_{16} x_7 - x_{17} + x_{18} x_{24} &&= 0\\
(e_2, e_4, e_8):&\quad -x_1 x_{21} + x_{24} x_{27} - x_8 &&= 0\\
(e_2, e_5, e_7):&\quad x_{12} x_{24} - x_{17} x_9 &&= 0\\
(e_3, e_4, e_7):&\quad -x_1 x_{19} + x_{16} x_{22} &&= 0\\
(e_3, e_5, e_6):&\quad x_{14} x_8 + x_{16} x_{23} - x_{17} x_4 &&= 0
\end{aligned}
$$

Groebner basis (28 variables, 0 linear, 28 nonlinear)

$$1375000x_1 - 761129964x_{28}^5 - 19078125x_{28}^4 - 1031250x_{28}^3 - 2062500x_{28}^2 = 0$$

$$343750x_2+1429436988x_{28}^5+31453125x_{28}^4+3609375x_{28}^3+1031250x_{28}^2-343750x_{28} = 0$$

$$-390368916x_{28}^5 - 8971875x_{28}^4 - 1031250x_{28}^3 - 412500x_{28}^2 + 275000x_3 = 0$$

$$-337363992x_{28}^5-6843750x_{28}^4-703125x_{28}^3-93750x_{28}^2-31250x_{28}+31250x_4 = 0$$

$$14633280144x_{28}^5+292875000x_{28}^4+29390625x_{28}^3+3093750x_{28}^2+2406250x_{28}+343750x_5-343750 = 0$$

$$-2134974996x_{28}^5-43171875x_{28}^4-4406250x_{28}^3-562500x_{28}^2-125000x_{28}+125000x_6 = 0$$

$$247333716x_{28}^5+5011875x_{28}^4+513750x_{28}^3+67500x_{28}^2+25000x_{28}+5000x_7-5000 = 0$$

$$-155836044x_{28}^5 - 2578125x_{28}^4 + 343750x_8 = 0$$

$$561940596x_{28}^5+11371875x_{28}^4+1162500x_{28}^3+150000x_{28}^2+37500x_{28}+12500x_9-12500 = 0$$

$$62500x_{10}+2134974996x_{28}^5+43171875x_{28}^4+4406250x_{28}^3+562500x_{28}^2+125000x_{28}-62500 = 0$$

$$125000x_{11}+2134974996x_{28}^5+43171875x_{28}^4+4406250x_{28}^3+562500x_{28}^2+125000x_{28}-125000 = 0$$

$$62500x_{12} - 110791044x_{28}^5 - 2109375x_{28}^4 - 187500x_{28}^3 = 0$$

$$343750x_{13} - 155836044x_{28}^5 - 2578125x_{28}^4 = 0$$

$$125000x_{14}-2134974996x_{28}^5-43171875x_{28}^4-4406250x_{28}^3-562500x_{28}^2-125000x_{28} = 0$$

$$125000x_{15} - 785519028x_{28}^5 - 15796875x_{28}^4 - 1593750x_{28}^3 - 187500x_{28}^2 = 0$$

$$1375000x_{16}+6478877916x_{28}^5+144890625x_{28}^4+15468750x_{28}^3+6187500x_{28}^2-1375000x_{28} = 0$$

$$687500x_{17} - 595357308x_{28}^5 - 12890625x_{28}^4 - 2062500x_{28}^3 = 0$$

$$125000x_{18}+342354852x_{28}^5+7359375x_{28}^4+843750x_{28}^3+187500x_{28}^2-125000x_{28} = 0$$

$$25000x_{19} - 112787388x_{28}^5 - 2315625x_{28}^4 - 243750x_{28}^3 - 37500x_{28}^2 - 25000x_{28} = 0$$

$$31250x_{20}+1460247012x_{28}^5+29484375x_{28}^4+3000000x_{28}^3+375000x_{28}^2+187500x_{28}-31250 = 0$$

$$5000x_{21}+247333716x_{28}^5+5011875x_{28}^4+513750x_{28}^3+67500x_{28}^2+20000x_{28}-5000 = 0$$

$$25000x_{22} - 112787388x_{28}^5 - 2315625x_{28}^4 - 243750x_{28}^3 - 37500x_{28}^2 = 0$$

$$62500x_{23} - 110791044x_{28}^5 - 2109375x_{28}^4 - 187500x_{28}^3 = 0$$

$$275000x_{24}+10410848532x_{28}^5+205321875x_{28}^4+20418750x_{28}^3+1237500x_{28}^2+2200000x_{28}-275000 = 0$$

$$687500x_{25} - 907029396x_{28}^5 - 18046875x_{28}^4 - 2062500x_{28}^3 = 0$$

$$125000x_{26} - 785519028x_{28}^5 - 15796875x_{28}^4 - 1593750x_{28}^3 - 187500x_{28}^2 = 0$$

$$125000x_{27} - 342354852x_{28}^5 - 7359375x_{28}^4 - 843750x_{28}^3 - 187500x_{28}^2 = 0$$

$$84x_{28}^6 - 5x_{28}^5 = 0$$

Solution 1:

$$x_1 = 0$$

$$x_2 = 0$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 1$$
$$x_6 = 0$$
$$x_7 = 1$$
$$x_8 = 0$$
$$x_9 = 1$$
$$x_10 = 1$$
$$x_11 = 1$$
$$x_12 = 0$$
$$x_13 = 0$$
$$x_14 = 0$$
$$x_15 = 0$$
$$x_16 = 0$$
$$x_17 = 0$$
$$x_18 = 0$$
$$x_19 = 0$$
$$x_20 = 1$$
$$x_21 = 1$$
$$x_22 = 0$$
$$x_23 = 0$$
$$x_24 = 1$$
$$x_25 = 0$$
$$x_26 = 0$$
$$x_27 = 0$$
$$x_28 = 0$$

Solution 2:

$$x_1 = 1/165$$
$$x_2 = 7/165$$
$$x_3 = 1/132$$
$$x_4 = 3/35$$

$$x_5 = 27/55$$

$$x_6 = 1/10$$

$$x_7 = 7/12$$

$$x_8 = 1/2310$$

$$x_9 = 5/7$$

$$x_1 0 = 4/5$$

$$x_1 1 = 9/10$$

$$x_1 2 = 1/420$$

$$x_1 3 = 1/2310$$

$$x_1 4 = 1/10$$

$$x_1 5 = 1/70$$

$$x_1 6 = 2/55$$

$$x_1 7 = 1/660$$

$$x_1 8 = 1/20$$

$$x_1 9 = 1/14$$

$$x_2 0 = 8/15$$

$$x_2 1 = 9/14$$

$$x_2 2 = 1/84$$

$$x_2 3 = 1/420$$

$$x_2 4 = 5/11$$

$$x_2 5 = 3/1540$$

$$x_2 6 = 1/70$$

$$x_2 7 = 1/105$$

$$x_2 8 = 5/84$$

$\mathfrak{m}_{1A}(3,14)$

m1A314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_1,e_{11}]=e_{12}\\
&[e_1,e_{12}]=e_{13} && [e_1,e_{13}]=e_{14}\\
&[e_2,e_{11}]=e_{14} && [e_3,e_{10}]=-e_{14}\\
&[e_4,e_9]=e_{14} && [e_5,e_8]=-e_{14}\\
&[e_6,e_7]=e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3,14)$

m3A314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_1,e_{11}]=e_{12}\\
&[e_1,e_{12}]=e_{13} && [e_1,e_{13}]=e_{14}\\
&[e_2,e_9]=e_{12} && [e_2,e_{10}]=4e_{13}\\
&[e_2,e_{11}]=\alpha_{2,11}^{14}e_{14} && [e_3,e_8]=-e_{12}\\
&[e_3,e_9]=-3e_{13} && [e_3,e_{10}]=\alpha_{3,10}^{14}e_{14}\\
&[e_4,e_7]=e_{12} && [e_4,e_8]=2e_{13}\\
&[e_4,e_9]=\alpha_{4,9}^{14}e_{14} && [e_5,e_6]=-e_{12}\\
&[e_5,e_7]=-e_{13} && [e_5,e_8]=\alpha_{5,8}^{14}e_{14}\\
&[e_6,e_7]=\alpha_{6,7}^{14}e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}): \quad & -\alpha_{2,11}^{14} - \alpha_{3,10}^{14} + 4 && = 0 \\
(e_1, e_3, e_9): \quad & -\alpha_{3,10}^{14} - \alpha_{4,9}^{14} - 3 && = 0 \\
(e_1, e_4, e_8): \quad & -\alpha_{4,9}^{14} - \alpha_{5,8}^{14} + 2 && = 0 \\
(e_1, e_5, e_7): \quad & -\alpha_{5,8}^{14} - \alpha_{6,7}^{14} - 1 && = 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,10}^{14} \to x_1$$

$$\alpha_{6,7}^{14} \to x_2$$

$$\alpha_{5,8}^{14} \to x_3$$

$$\alpha_{4,9}^{14} \to x_4$$

$$\alpha_{2,11}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): \quad & -x_1 - x_5 + 4 && = 0 \\
(e_1, e_3, e_9): \quad & -x_1 - x_4 - 3 && = 0 \\
(e_1, e_4, e_8): \quad & -x_3 - x_4 + 2 && = 0 \\
(e_1, e_5, e_7): \quad & -x_2 - x_3 - 1 && = 0
\end{aligned}$$

Groebner basis (5 variables, 4 linear, 0 nonlinear)

$$x_1 + x_5 - 4 = 0$$

$$x_2 - x_5 + 10 = 0$$

$$x_3 + x_5 - 9 = 0$$

$$x_4 - x_5 + 7 = 0$$

$\mathfrak{m}_{5A}(3,14)$

m5A314 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_2,e_7]=e_{10} \qquad [e_2,e_8]=3e_{11}$$
$$[e_2,e_9]=\frac{7e_{12}}{2} \qquad [e_2,e_{10}]=0$$
$$[e_2,e_{11}]=0 \qquad [e_3,e_6]=-e_{10}$$
$$[e_3,e_7]=-2e_{11} \qquad [e_3,e_8]=-\frac{e_{12}}{2}$$
$$[e_3,e_9]=\frac{7e_{13}}{2} \qquad [e_3,e_{10}]=0$$
$$[e_4,e_5]=e_{10} \qquad [e_4,e_6]=e_{11}$$
$$[e_4,e_7]=-\frac{3e_{12}}{2} \qquad [e_4,e_8]=-4e_{13}$$
$$[e_4,e_9]=\frac{7e_{14}}{2} \qquad [e_5,e_6]=\frac{5e_{12}}{2}$$
$$[e_5,e_7]=\frac{5e_{13}}{2} \qquad [e_5,e_8]=-\frac{15e_{14}}{2}$$
$$[e_6,e_7]=10e_{14}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_7] = e_{10} & [e_2, e_8] = 3e_{11} \\
[e_2, e_9] = \alpha_{2,9}^{12} e_{12} & [e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} \\
[e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} & [e_3, e_6] = -e_{10} \\
[e_3, e_7] = -2e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14} \\
[e_4, e_5] = e_{10} & [e_4, e_6] = e_{11} \\
[e_4, e_7] = \alpha_{4,7}^{12} e_{12} & [e_4, e_8] = \alpha_{4,8}^{13} e_{13} \\
[e_4, e_9] = \alpha_{4,9}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} & [e_5, e_8] = \alpha_{5,8}^{14} e_{14} \\
[e_6, e_7] = \alpha_{6,7}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,10}^{13} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13} &= 0\\
(e_1, e_2, e_{10}): &\quad \alpha_{2,10}^{13} - \alpha_{2,11}^{14} - \alpha_{3,10}^{14} &= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{14} + \alpha_{3,9}^{13} - \alpha_{4,9}^{14} &= 0\\
(e_1, e_4, e_8): &\quad \alpha_{4,8}^{13} - \alpha_{4,9}^{14} - \alpha_{5,8}^{14} &= 0\\
(e_1, e_5, e_7): &\quad \alpha_{5,7}^{13} - \alpha_{5,8}^{14} - \alpha_{6,7}^{14} &= 0\\
(e_2, e_3, e_7): &\quad -2\alpha_{2,11}^{14} - \alpha_{3,10}^{14} &= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,11}^{14} &= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{14} &= 0
\end{aligned}$$

Solution 1:

$$\alpha^{13}_{4,8} = -4$$
$$\alpha^{14}_{3,10} = 0$$
$$\alpha^{14}_{6,7} = 10$$
$$\alpha^{14}_{4,9} = 7/2$$
$$\alpha^{13}_{3,9} = 7/2$$
$$\alpha^{12}_{3,8} = -1/2$$
$$\alpha^{14}_{5,8} = -15/2$$
$$\alpha^{13}_{5,7} = 5/2$$
$$\alpha^{13}_{2,10} = 0$$
$$\alpha^{12}_{2,9} = 7/2$$
$$\alpha^{14}_{2,11} = 0$$
$$\alpha^{12}_{4,7} = -3/2$$
$$\alpha^{12}_{5,6} = 5/2$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{4,8} \to x_1$$
$$\alpha^{14}_{3,10} \to x_2$$
$$\alpha^{14}_{6,7} \to x_3$$
$$\alpha^{14}_{4,9} \to x_4$$
$$\alpha^{13}_{3,9} \to x_5$$
$$\alpha^{12}_{3,8} \to x_6$$
$$\alpha^{14}_{5,8} \to x_7$$
$$\alpha^{13}_{5,7} \to x_8$$
$$\alpha^{13}_{2,10} \to x_9$$
$$\alpha^{12}_{2,9} \to x_{10}$$
$$\alpha^{14}_{2,11} \to x_{11}$$
$$\alpha^{12}_{4,7} \to x_{12}$$
$$\alpha^{12}_{5,6} \to x_{13}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8) &: \quad -x_{10} - x_6 + 3 &&= 0 \\
(e_1, e_3, e_7) &: \quad -x_{12} - x_6 - 2 &&= 0 \\
(e_1, e_4, e_6) &: \quad -x_{12} - x_{13} + 1 &&= 0 \\
(e_1, e_2, e_9) &: \quad x_{10} - x_5 - x_9 &&= 0 \\
(e_1, e_3, e_8) &: \quad -x_1 - x_5 + x_6 &&= 0 \\
(e_1, e_4, e_7) &: \quad -x_1 + x_{12} - x_8 &&= 0 \\
(e_1, e_5, e_6) &: \quad x_{13} - x_8 &&= 0 \\
(e_2, e_3, e_6) &: \quad -x_9 &&= 0 \\
(e_2, e_4, e_5) &: \quad x_9 &&= 0 \\
(e_1, e_2, e_{10}) &: \quad -x_{11} - x_2 + x_9 &&= 0 \\
(e_1, e_3, e_9) &: \quad -x_2 - x_4 + x_5 &&= 0 \\
(e_1, e_4, e_8) &: \quad x_1 - x_4 - x_7 &&= 0 \\
(e_1, e_5, e_7) &: \quad -x_3 - x_7 + x_8 &&= 0 \\
(e_2, e_3, e_7) &: \quad -2x_{11} - x_2 &&= 0 \\
(e_2, e_4, e_6) &: \quad x_{11} &&= 0 \\
(e_3, e_4, e_5) &: \quad x_2 &&= 0
\end{aligned}$$

Groebner basis (13 variables, 13 linear, 0 nonlinear)

$$x_1 + 4 = 0$$

$$x_2 = 0$$

$$x_3 - 10 = 0$$

$$2x_4 - 7 = 0$$

$$2x_5 - 7 = 0$$

$$2x_6 + 1 = 0$$

$$2x_7 + 15 = 0$$

$$2x_8 - 5 = 0$$

$$x_9 = 0$$

$$2x_{10} - 7 = 0$$

$$x_{11} = 0$$

$$2x_{12} + 3 = 0$$

$$2x_{13} - 5 = 0$$

Solution 1:

$$x_1 = -4$$
$$x_2 = 0$$
$$x_3 = 10$$
$$x_4 = 7/2$$
$$x_5 = 7/2$$
$$x_6 = -1/2$$
$$x_7 = -15/2$$
$$x_8 = 5/2$$
$$x_9 = 0$$
$$x_10 = 7/2$$
$$x_11 = 0$$
$$x_12 = -3/2$$
$$x_13 = 5/2$$

$\mathfrak{m}_{7A}(3, 14)$

m7A314 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_5] = e_8 \qquad [e_2, e_6] = 2e_9$$
$$[e_2, e_7] = \frac{5e_{10}}{3} \qquad [e_2, e_8] = 0$$
$$[e_2, e_9] = -\frac{49e_{12}}{33} \qquad [e_2, e_{10}] = -\frac{14e_{13}}{11}$$
$$[e_2, e_{11}] = -\frac{7e_{14}}{11} \qquad [e_3, e_4] = -e_8$$
$$[e_3, e_5] = -e_9 \qquad [e_3, e_6] = \frac{e_{10}}{3}$$
$$[e_3, e_7] = \frac{5e_{11}}{3} \qquad [e_3, e_8] = \frac{49e_{12}}{33}$$
$$[e_3, e_9] = -\frac{7e_{13}}{33} \qquad [e_3, e_{10}] = -\frac{7e_{14}}{11}$$
$$[e_4, e_5] = -\frac{4e_{10}}{3} \qquad [e_4, e_6] = -\frac{4e_{11}}{3}$$
$$[e_4, e_7] = \frac{2e_{12}}{11} \qquad [e_4, e_8] = \frac{56e_{13}}{33}$$
$$[e_4, e_9] = \frac{14e_{14}}{33} \qquad [e_5, e_6] = -\frac{50e_{12}}{33}$$
$$[e_5, e_7] = -\frac{50e_{13}}{33} \qquad [e_5, e_8] = \frac{14e_{14}}{11}$$
$$[e_6, e_7] = -\frac{92e_{14}}{33}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_5] = e_8 & [e_2, e_6] = 2e_9 \\
[e_2, e_7] = \alpha_{2,7}^{10} e_{10} & [e_2, e_8] = \alpha_{2,8}^{11} e_{11} \\
[e_2, e_9] = \alpha_{2,9}^{12} e_{12} & [e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} \\
[e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} & [e_3, e_4] = -e_8 \\
[e_3, e_5] = -e_9 & [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] = \alpha_{3,7}^{11} e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14} \\
[e_4, e_5] = \alpha_{4,5}^{10} e_{10} & [e_4, e_6] = \alpha_{4,6}^{11} e_{11} \\
[e_4, e_7] = \alpha_{4,7}^{12} e_{12} & [e_4, e_8] = \alpha_{4,8}^{13} e_{13} \\
[e_4, e_9] = \alpha_{4,9}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} & [e_5, e_8] = \alpha_{5,8}^{14} e_{14} \\
[e_6, e_7] = \alpha_{6,7}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 && = 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 && = 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} && = 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} && = 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} && = 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} && = 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} && = 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} && = 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} && = 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} && = 0 \\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} && = 0 \\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} && = 0 \\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} && = 0 \\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} && = 0 \\
(e_2, e_3, e_6): &\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - 2\alpha_{3,9}^{13} && = 0 \\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{4,8}^{13} && = 0 \\
(e_1, e_2, e_{10}): &\quad \alpha_{2,10}^{13} - \alpha_{2,11}^{14} - \alpha_{3,10}^{14} && = 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{14} + \alpha_{3,9}^{13} - \alpha_{4,9}^{14} && = 0 \\
(e_1, e_4, e_8): &\quad \alpha_{4,8}^{13} - \alpha_{4,9}^{14} - \alpha_{5,8}^{14} && = 0 \\
(e_1, e_5, e_7): &\quad \alpha_{5,7}^{13} - \alpha_{5,8}^{14} - \alpha_{6,7}^{14} && = 0 \\
(e_2, e_3, e_7): &\quad \alpha_{2,11}^{14}\alpha_{3,7}^{11} - \alpha_{2,7}^{10}\alpha_{3,10}^{14} && = 0 \\
(e_2, e_4, e_6): &\quad \alpha_{2,11}^{14}\alpha_{4,6}^{11} - 2\alpha_{4,9}^{14} && = 0 \\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{14}\alpha_{4,5}^{10} + \alpha_{4,9}^{14} - \alpha_{5,8}^{14} && = 0
\end{aligned}
$$

Solution 1:

$$\alpha^{13}_{4,8} = 56/33$$
$$\alpha^{14}_{3,10} = -7/11$$
$$\alpha^{14}_{6,7} = -92/33$$
$$\alpha^{11}_{2,8} = 0$$
$$\alpha^{14}_{4,9} = 14/33$$
$$\alpha^{13}_{3,9} = -7/33$$
$$\alpha^{10}_{3,6} = 1/3$$
$$\alpha^{11}_{4,6} = -4/3$$
$$\alpha^{12}_{3,8} = 49/33$$
$$\alpha^{13}_{5,7} = -50/33$$
$$\alpha^{11}_{3,7} = 5/3$$
$$\alpha^{12}_{2,9} = -49/33$$
$$\alpha^{14}_{5,8} = 14/11$$
$$\alpha^{10}_{4,5} = -4/3$$
$$\alpha^{13}_{2,10} = -14/11$$
$$\alpha^{10}_{2,7} = 5/3$$
$$\alpha^{14}_{2,11} = -7/11$$
$$\alpha^{12}_{4,7} = 2/11$$
$$\alpha^{12}_{5,6} = -50/33$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{4,8} \to x_1$$
$$\alpha^{14}_{3,10} \to x_2$$
$$\alpha^{14}_{6,7} \to x_3$$
$$\alpha^{11}_{2,8} \to x_4$$
$$\alpha^{14}_{4,9} \to x_5$$
$$\alpha^{13}_{3,9} \to x_6$$
$$\alpha^{10}_{3,6} \to x_7$$
$$\alpha^{11}_{4,6} \to x_8$$

$$\alpha_{3,8}^{12} \to x_9$$

$$\alpha_{5,7}^{13} \to x_{10}$$

$$\alpha_{3,7}^{11} \to x_{11}$$

$$\alpha_{2,9}^{12} \to x_{12}$$

$$\alpha_{5,8}^{14} \to x_{13}$$

$$\alpha_{4,5}^{10} \to x_{14}$$

$$\alpha_{2,10}^{13} \to x_{15}$$

$$\alpha_{2,7}^{10} \to x_{16}$$

$$\alpha_{2,11}^{14} \to x_{17}$$

$$\alpha_{4,7}^{12} \to x_{18}$$

$$\alpha_{5,6}^{12} \to x_{19}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -x_{16}-x_7+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_{14}-x_7-1 &&=0\\
(e_1,e_2,e_7):&\quad -x_{11}+x_{16}-x_4 &&=0\\
(e_1,e_3,e_6):&\quad -x_{11}+x_7-x_8 &&=0\\
(e_1,e_4,e_5):&\quad x_{14}-x_8 &&=0\\
(e_2,e_3,e_4):&\quad -x_4 &&=0\\
(e_1,e_2,e_8):&\quad -x_{12}+x_4-x_9 &&=0\\
(e_1,e_3,e_7):&\quad x_{11}-x_{18}-x_9 &&=0\\
(e_1,e_4,e_6):&\quad -x_{18}-x_{19}+x_8 &&=0\\
(e_2,e_3,e_5):&\quad -x_{12}-x_9 &&=0\\
(e_1,e_2,e_9):&\quad x_{12}-x_{15}-x_6 &&=0\\
(e_1,e_3,e_8):&\quad -x_1-x_6+x_9 &&=0\\
(e_1,e_4,e_7):&\quad -x_1-x_{10}+x_{18} &&=0\\
(e_1,e_5,e_6):&\quad -x_{10}+x_{19} &&=0\\
(e_2,e_3,e_6):&\quad x_{15}x_7-2x_6 &&=0\\
(e_2,e_4,e_5):&\quad -x_1+x_{14}x_{15} &&=0\\
(e_1,e_2,e_{10}):&\quad x_{15}-x_{17}-x_2 &&=0\\
(e_1,e_3,e_9):&\quad -x_2-x_5+x_6 &&=0\\
(e_1,e_4,e_8):&\quad x_1-x_{13}-x_5 &&=0\\
(e_1,e_5,e_7):&\quad x_{10}-x_{13}-x_3 &&=0\\
(e_2,e_3,e_7):&\quad x_{11}x_{17}-x_{16}x_2 &&=0\\
(e_2,e_4,e_6):&\quad x_{17}x_8-2x_5 &&=0\\
(e_3,e_4,e_5):&\quad -x_{13}+x_{14}x_2+x_5 &&=0
\end{aligned}
$$

Groebner basis (19 variables, 19 linear, 0 nonlinear)

$$33x_1-56=0$$

$$11x_2+7=0$$

$$33x_3+92=0$$

$$x_4=0$$

$$33x_5-14=0$$

$$33x_6+7=0$$

$$3x_7-1=0$$

$$3x_8+4=0$$

$$33x_9 - 49 = 0$$
$$33x_{10} + 50 = 0$$
$$3x_{11} - 5 = 0$$
$$33x_{12} + 49 = 0$$
$$11x_{13} - 14 = 0$$
$$3x_{14} + 4 = 0$$
$$11x_{15} + 14 = 0$$
$$3x_{16} - 5 = 0$$
$$11x_{17} + 7 = 0$$
$$11x_{18} - 2 = 0$$
$$33x_{19} + 50 = 0$$

Solution 1:

$$x_1 = 56/33$$
$$x_2 = -7/11$$
$$x_3 = -92/33$$
$$x_4 = 0$$
$$x_5 = 14/33$$
$$x_6 = -7/33$$
$$x_7 = 1/3$$
$$x_8 = -4/3$$
$$x_9 = 49/33$$
$$x_10 = -50/33$$
$$x_11 = 5/3$$
$$x_12 = -49/33$$
$$x_13 = 14/11$$
$$x_14 = -4/3$$
$$x_15 = -14/11$$
$$x_16 = 5/3$$
$$x_17 = -7/11$$
$$x_18 = 2/11$$
$$x_19 = -50/33$$

$\mathfrak{m}_{9A}(3, 14)$

m9A314 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_3] = e_6 \qquad [e_2, e_4] = e_7$$
$$[e_2, e_5] = \alpha_{2,5}^{8} e_8 \qquad [e_2, e_6] = \alpha_{2,6}^{9} e_9$$
$$[e_2, e_7] = \alpha_{2,7}^{10} e_{10} \qquad [e_2, e_8] = \alpha_{2,8}^{11} e_{11}$$
$$[e_2, e_9] = \alpha_{2,9}^{12} e_{12} \qquad [e_2, e_{10}] = \alpha_{2,10}^{13} e_{13}$$
$$[e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} \qquad [e_3, e_4] = \alpha_{3,4}^{8} e_8$$
$$[e_3, e_5] = \alpha_{3,5}^{9} e_9 \qquad [e_3, e_6] = \alpha_{3,6}^{10} e_{10}$$
$$[e_3, e_7] = \alpha_{3,7}^{11} e_{11} \qquad [e_3, e_8] = \alpha_{3,8}^{12} e_{12}$$
$$[e_3, e_9] = \alpha_{3,9}^{13} e_{13} \qquad [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14}$$
$$[e_4, e_5] = \alpha_{4,5}^{10} e_{10} \qquad [e_4, e_6] = \alpha_{4,6}^{11} e_{11}$$
$$[e_4, e_7] = \alpha_{4,7}^{12} e_{12} \qquad [e_4, e_8] = \alpha_{4,8}^{13} e_{13}$$
$$[e_4, e_9] = \alpha_{4,9}^{14} e_{14} \qquad [e_5, e_6] = \alpha_{5,6}^{12} e_{12}$$
$$[e_5, e_7] = \alpha_{5,7}^{13} e_{13} \qquad [e_5, e_8] = \alpha_{5,8}^{14} e_{14}$$
$$[e_6, e_7] = \alpha_{6,7}^{14} e_{14}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha^8_{2,5} - \alpha^8_{3,4} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha^8_{2,5} - \alpha^9_{2,6} - \alpha^9_{3,5} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha^8_{3,4} - \alpha^9_{3,5} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha^9_{2,6} - \alpha^{10}_{2,7} - \alpha^{10}_{3,6} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha^9_{3,5} - \alpha^{10}_{3,6} - \alpha^{10}_{4,5} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha^{10}_{2,7} - \alpha^{11}_{2,8} - \alpha^{11}_{3,7} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha^{10}_{3,6} - \alpha^{11}_{3,7} - \alpha^{11}_{4,6} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha^{10}_{4,5} - \alpha^{11}_{4,6} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha^{11}_{2,8}\alpha^8_{3,4} - \alpha^{11}_{3,7} + \alpha^{11}_{4,6} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha^{11}_{2,8} - \alpha^{12}_{2,9} - \alpha^{12}_{3,8} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha^{11}_{3,7} - \alpha^{12}_{3,8} - \alpha^{12}_{4,7} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha^{11}_{4,6} - \alpha^{12}_{4,7} - \alpha^{12}_{5,6} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha^8_{2,5}\alpha^{12}_{3,8} + \alpha^{12}_{2,9}\alpha^9_{3,5} + \alpha^{12}_{5,6} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha^{13}_{2,10} + \alpha^{12}_{2,9} - \alpha^{13}_{3,9} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha^{12}_{3,8} - \alpha^{13}_{3,9} - \alpha^{13}_{4,8} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha^{12}_{4,7} - \alpha^{13}_{4,8} - \alpha^{13}_{5,7} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha^{12}_{5,6} - \alpha^{13}_{5,7} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha^{13}_{2,10}\alpha^{10}_{3,6} - \alpha^9_{2,6}\alpha^{13}_{3,9} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha^{13}_{2,10}\alpha^{10}_{4,5} - \alpha^8_{2,5}\alpha^{13}_{4,8} + \alpha^{13}_{5,7} &&= 0\\
(e_1, e_2, e_{10}): &\quad \alpha^{13}_{2,10} - \alpha^{14}_{2,11} - \alpha^{14}_{3,10} &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha^{14}_{3,10} + \alpha^{13}_{3,9} - \alpha^{14}_{4,9} &&= 0\\
(e_1, e_4, e_8): &\quad \alpha^{13}_{4,8} - \alpha^{14}_{4,9} - \alpha^{14}_{5,8} &&= 0\\
(e_1, e_5, e_7): &\quad \alpha^{13}_{5,7} - \alpha^{14}_{5,8} - \alpha^{14}_{6,7} &&= 0\\
(e_2, e_3, e_7): &\quad \alpha^{14}_{2,11}\alpha^{11}_{3,7} - \alpha^{10}_{2,7}\alpha^{14}_{3,10} - \alpha^{14}_{6,7} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha^{14}_{2,11}\alpha^{11}_{4,6} - \alpha^9_{2,6}\alpha^{14}_{4,9} + \alpha^{14}_{6,7} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha^{14}_{3,10}\alpha^{10}_{4,5} + \alpha^8_{3,4}\alpha^{14}_{5,8} - \alpha^9_{3,5}\alpha^{14}_{4,9} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{2,8} \to x_1$$

$$\alpha^{10}_{3,6} \to x_2$$

$$\alpha_{2,9}^{12} \to x_3$$

$$\alpha_{3,5}^{9} \to x_4$$

$$\alpha_{6,7}^{14} \to x_5$$

$$\alpha_{5,8}^{14} \to x_6$$

$$\alpha_{4,5}^{10} \to x_7$$

$$\alpha_{3,4}^{8} \to x_8$$

$$\alpha_{4,9}^{14} \to x_9$$

$$\alpha_{2,11}^{14} \to x_{10}$$

$$\alpha_{5,6}^{12} \to x_{11}$$

$$\alpha_{3,10}^{14} \to x_{12}$$

$$\alpha_{3,9}^{13} \to x_{13}$$

$$\alpha_{5,7}^{13} \to x_{14}$$

$$\alpha_{2,6}^{9} \to x_{15}$$

$$\alpha_{2,7}^{10} \to x_{16}$$

$$\alpha_{2,5}^{8} \to x_{17}$$

$$\alpha_{4,6}^{11} \to x_{18}$$

$$\alpha_{3,7}^{11} \to x_{19}$$

$$\alpha_{2,10}^{13} \to x_{20}$$

$$\alpha_{3,8}^{12} \to x_{21}$$

$$\alpha_{4,7}^{12} \to x_{22}$$

$$\alpha_{4,8}^{13} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{17} - x_8 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{15} + x_{17} - x_4 &&= 0\\
(e_1, e_3, e_4): &\quad -x_4 + x_8 &&= 0\\
(e_1, e_2, e_6): &\quad x_{15} - x_{16} - x_2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_2 + x_4 - x_7 &&= 0\\
(e_1, e_2, e_7): &\quad -x_1 + x_{16} - x_{19} &&= 0\\
(e_1, e_3, e_6): &\quad -x_{18} - x_{19} + x_2 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{18} + x_7 &&= 0\\
(e_2, e_3, e_4): &\quad x_1 x_8 + x_{18} - x_{19} &&= 0\\
(e_1, e_2, e_8): &\quad x_1 - x_{21} - x_3 &&= 0\\
(e_1, e_3, e_7): &\quad x_{19} - x_{21} - x_{22} &&= 0\\
(e_1, e_4, e_6): &\quad -x_{11} + x_{18} - x_{22} &&= 0\\
(e_2, e_3, e_5): &\quad x_{11} - x_{17} x_{21} + x_3 x_4 &&= 0\\
(e_1, e_2, e_9): &\quad -x_{13} - x_{20} + x_3 &&= 0\\
(e_1, e_3, e_8): &\quad -x_{13} + x_{21} - x_{23} &&= 0\\
(e_1, e_4, e_7): &\quad -x_{14} + x_{22} - x_{23} &&= 0\\
(e_1, e_5, e_6): &\quad x_{11} - x_{14} &&= 0\\
(e_2, e_3, e_6): &\quad -x_{13} x_{15} + x_2 x_{20} &&= 0\\
(e_2, e_4, e_5): &\quad x_{14} - x_{17} x_{23} + x_{20} x_7 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_{10} - x_{12} + x_{20} &&= 0\\
(e_1, e_3, e_9): &\quad -x_{12} + x_{13} - x_9 &&= 0\\
(e_1, e_4, e_8): &\quad x_{23} - x_6 - x_9 &&= 0\\
(e_1, e_5, e_7): &\quad x_{14} - x_5 - x_6 &&= 0\\
(e_2, e_3, e_7): &\quad x_{10} x_{19} - x_{12} x_{16} - x_5 &&= 0\\
(e_2, e_4, e_6): &\quad x_{10} x_{18} - x_{15} x_9 + x_5 &&= 0\\
(e_3, e_4, e_5): &\quad x_{12} x_7 - x_4 x_9 + x_6 x_8 &&= 0
\end{aligned}
$$

Groebner basis (23 variables, 19 linear, 7 nonlinear)

$$x_1 + 4x_{21} + 14x_{22} - 5x_{23} - 1 = 0$$

$$x_2 - x_{21} - 3x_{22} + x_{23} = 0$$

$$5x_{21} + 14x_{22} - 5x_{23} + x_3 - 1 = 0$$

$$-x_{21} - 5x_{22} + 2x_{23} + x_4 = 0$$

$$x_{12} - x_{21} - x_{22} + 3x_{23} + x_5 = 0$$

$$-x_{12} + x_{21} - 2x_{23} + x_6 = 0$$

$$-2x_{22} + x_{23} + x_7 = 0$$

$$-x_{21} - 5x_{22} + 2x_{23} + x_8 = 0$$

$$x_{12} - x_{21} + x_{23} + x_9 = 0$$

$$x_{10} + x_{12} + 6x_{21} + 14x_{22} - 6x_{23} - 1 = 0$$

$$x_{11} - x_{22} + x_{23} = 0$$

$$1176x_{12}x_{21}+516x_{12}x_{23}+6955x_{21}x_{23}-86016x_{22}^3+84480x_{22}^2x_{23}+12512x_{22}^2-24192x_{22}x_{23}^2+4387x_{22}x_{23}-1131x_{22}+1$$

$$588x_{12}x_{22}-288x_{12}x_{23}-355x_{21}x_{23}+7168x_{22}^3-7040x_{22}^2x_{23}-128x_{22}^2+2016x_{22}x_{23}^2-43x_{22}x_{23}+33x_{22}-144x_{23}^3-184x$$

$$60x_{12}x_{23}^2+3528x_{12}x_{23}-23643x_{21}x_{23}+71680x_{22}^3x_{23}+600768x_{22}^3-70400x_{22}^2x_{23}^2-563208x_{22}^2x_{23}-104598x_{22}^2+20160$$

$$x_{13} - x_{21} + x_{23} = 0$$

$$x_{14} - x_{22} + x_{23} = 0$$

$$x_{15} + 2x_{21} + 10x_{22} - 4x_{23} - 1 = 0$$

$$x_{16} + 3x_{21} + 13x_{22} - 5x_{23} - 1 = 0$$

$$x_{17} + x_{21} + 5x_{22} - 2x_{23} - 1 = 0$$

$$x_{18} - 2x_{22} + x_{23} = 0$$

$$x_{19} - x_{21} - x_{22} = 0$$

$$x_{20} + 6x_{21} + 14x_{22} - 6x_{23} - 1 = 0$$

$$24x_{21}^2 + 41x_{21}x_{23} - 56x_{22}^2 + 209x_{22}x_{23} + 15x_{22} - 76x_{23}^2 - 33x_{23} = 0$$

$$12x_{21}x_{22} - 7x_{21}x_{23} + 28x_{22}^2 - 31x_{22}x_{23} - 3x_{22} + 8x_{23}^2 + 3x_{23} = 0$$

$$20x_{21}x_{23}^2-45x_{21}x_{23}+1344x_{22}^3-1208x_{22}^2x_{23}-234x_{22}^2+380x_{22}x_{23}^2+96x_{22}x_{23}+27x_{22}-46x_{23}^3-42x_{23}^2-27x_{23} = 0$$

$$1792x_{22}^4-2656x_{22}^3x_{23}-312x_{22}^3+1384x_{22}^2x_{23}^2+450x_{22}^2x_{23}+36x_{22}^2-288x_{22}x_{23}^3-279x_{22}x_{23}^2-72x_{22}x_{23}+18x_{23}^4+66x_{23}^3$$

$\mathfrak{m}_{2A}(4,14)$

m2A414 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14} \\
[e_2,e_9] &= e_{13} & [e_2,e_{10}] &= 4e_{14} \\
[e_3,e_8] &= -e_{13} & [e_3,e_9] &= -3e_{14} \\
[e_4,e_7] &= e_{13} & [e_4,e_8] &= 2e_{14} \\
[e_5,e_6] &= -e_{13} & [e_5,e_7] &= -e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(4,14)$

m4A414 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14} \\
[e_2,e_7] &= e_{11} & [e_2,e_8] &= 3e_{12} \\
[e_2,e_9] &= \alpha_{2,9}^{13} e_{13} & [e_2,e_{10}] &= \alpha_{2,10}^{14} e_{14} \\
[e_3,e_6] &= -e_{11} & [e_3,e_7] &= -2e_{12} \\
[e_3,e_8] &= \alpha_{3,8}^{13} e_{13} & [e_3,e_9] &= \alpha_{3,9}^{14} e_{14} \\
[e_4,e_5] &= e_{11} & [e_4,e_6] &= e_{12} \\
[e_4,e_7] &= \alpha_{4,7}^{13} e_{13} & [e_4,e_8] &= \alpha_{4,8}^{14} e_{14} \\
[e_5,e_6] &= \alpha_{5,6}^{13} e_{13} & [e_5,e_7] &= \alpha_{5,7}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{13} - \alpha_{3,8}^{13} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{13} - \alpha_{4,7}^{13} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{13} - \alpha_{5,6}^{13} + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,8}^{14} \to x_1$$

$$\alpha_{3,8}^{13} \to x_2$$

$$\alpha_{2,10}^{14} \to x_3$$

$$\alpha_{5,7}^{14} \to x_4$$

$$\alpha_{4,7}^{13} \to x_5$$

$$\alpha_{2,9}^{13} \to x_6$$

$$\alpha_{5,6}^{13} \to x_7$$

$$\alpha_{3,9}^{14} \to x_8$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_2 - x_6 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_5 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_5 - x_7 + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -x_3 + x_6 - x_8 &&= 0\\
(e_1, e_3, e_8): &\quad -x_1 + x_2 - x_8 &&= 0\\
(e_1, e_4, e_7): &\quad -x_1 - x_4 + x_5 &&= 0\\
(e_1, e_5, e_6): &\quad -x_4 + x_7 &&= 0
\end{aligned}
$$

Groebner basis (8 variables, 7 linear, 0 nonlinear)

$$3x_1 + 2x_8 + 5 = 0$$

$$3x_2 - x_8 + 5 = 0$$
$$3x_3 + 4x_8 - 14 = 0$$
$$3x_4 - x_8 - 4 = 0$$
$$3x_5 + x_8 + 1 = 0$$
$$3x_6 + x_8 - 14 = 0$$
$$3x_7 - x_8 - 4 = 0$$

$\mathfrak{m}_{6A}(4, 14)$

m6A414 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_5] &= e_9 & [e_2, e_6] &= 2e_{10} \\
[e_2, e_7] &= \alpha_{2,7}^{11} e_{11} & [e_2, e_8] &= \alpha_{2,8}^{12} e_{12} \\
[e_2, e_9] &= \alpha_{2,9}^{13} e_{13} & [e_2, e_{10}] &= \alpha_{2,10}^{14} e_{14} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_3, e_9] &= \alpha_{3,9}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{11} e_{11} & [e_4, e_6] &= \alpha_{4,6}^{12} e_{12} \\
[e_4, e_7] &= \alpha_{4,7}^{13} e_{13} & [e_4, e_8] &= \alpha_{4,8}^{14} e_{14} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13} & [e_5, e_7] &= \alpha_{5,7}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,9}^{13} &= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,10}^{14} - \alpha_{3,9}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,7}^{12} \to x_1$$

$$\alpha_{4,8}^{14} \to x_2$$

$$\alpha_{3,8}^{13} \to x_3$$

$$\alpha_{2,10}^{14} \to x_4$$

$$\alpha_{5,7}^{14} \to x_5$$

$$\alpha_{2,8}^{12} \to x_6$$

$$\alpha_{4,6}^{12} \to x_7$$

$$\alpha_{3,6}^{11} \to x_8$$

$$\alpha_{4,7}^{13} \to x_9$$

$$\alpha_{2,9}^{13} \to x_{10}$$

$$\alpha_{5,6}^{13} \to x_{11}$$

$$\alpha_{4,5}^{11} \to x_{12}$$

$$\alpha_{3,9}^{14} \to x_{13}$$

$$\alpha_{2,7}^{11} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{14} - x_8 + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -x_{12} - x_8 - 1 &= 0 \\
(e_1, e_2, e_7): &\quad -x_1 + x_{14} - x_6 &= 0 \\
(e_1, e_3, e_6): &\quad -x_1 - x_7 + x_8 &= 0 \\
(e_1, e_4, e_5): &\quad x_{12} - x_7 &= 0 \\
(e_1, e_2, e_8): &\quad -x_{10} - x_3 + x_6 &= 0 \\
(e_1, e_3, e_7): &\quad x_1 - x_3 - x_9 &= 0 \\
(e_1, e_4, e_6): &\quad -x_{11} + x_7 - x_9 &= 0 \\
(e_2, e_3, e_4): &\quad -x_{10} &= 0 \\
(e_1, e_2, e_9): &\quad x_{10} - x_{13} - x_4 &= 0 \\
(e_1, e_3, e_8): &\quad -x_{13} - x_2 + x_3 &= 0 \\
(e_1, e_4, e_7): &\quad -x_2 - x_5 + x_9 &= 0 \\
(e_1, e_5, e_6): &\quad x_{11} - x_5 &= 0 \\
(e_2, e_3, e_5): &\quad -x_{13} - x_4 &= 0
\end{aligned}$$

Groebner basis (14 variables, 13 linear, 0 nonlinear)

$$x_1 + 2x_{14} - 5 = 0$$

$$11x_{14} + x_2 - 23 = 0$$

$$-3x_{14} + x_3 + 5 = 0$$

$$14x_{14} + x_4 - 28 = 0$$

$$-6x_{14} + x_5 + 13 = 0$$

$$-3x_{14} + x_6 + 5 = 0$$

$$-x_{14} + x_7 + 3 = 0$$

$$x_{14} + x_8 - 2 = 0$$

$$5x_{14} + x_9 - 10 = 0$$

$$x_{10} = 0$$

$$x_{11} - 6x_{14} + 13 = 0$$

$$x_{12} - x_{14} + 3 = 0$$

$$x_{13} - 14x_{14} + 28 = 0$$

$\mathfrak{m}_{8A}(4, 14)$

m8A414 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_3] &= e_7 & [e_2, e_4] &= e_8 \\
[e_2, e_5] &= \alpha_{2,5}^{9} e_9 & [e_2, e_6] &= \alpha_{2,6}^{10} e_{10} \\
[e_2, e_7] &= \alpha_{2,7}^{11} e_{11} & [e_2, e_8] &= \alpha_{2,8}^{12} e_{12} \\
[e_2, e_9] &= \alpha_{2,9}^{13} e_{13} & [e_2, e_{10}] &= \alpha_{2,10}^{14} e_{14} \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_3, e_9] &= \alpha_{3,9}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{11} e_{11} & [e_4, e_6] &= \alpha_{4,6}^{12} e_{12} \\
[e_4, e_7] &= \alpha_{4,7}^{13} e_{13} & [e_4, e_8] &= \alpha_{4,8}^{14} e_{14} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13} & [e_5, e_7] &= \alpha_{5,7}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0\\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0\\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &= 0\\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &= 0\\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &= 0\\
(e_2, e_3, e_4):&\quad \alpha_{2,9}^{13}\alpha_{3,4}^{9} - \alpha_{3,8}^{13} + \alpha_{4,7}^{13} &= 0\\
(e_1, e_2, e_9):&\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &= 0\\
(e_1, e_3, e_8):&\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &= 0\\
(e_1, e_4, e_7):&\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &= 0\\
(e_1, e_5, e_6):&\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &= 0\\
(e_2, e_3, e_5):&\quad \alpha_{2,10}^{14}\alpha_{3,5}^{10} - \alpha_{2,5}^{9}\alpha_{3,9}^{14} + \alpha_{5,7}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{3,7}^{12} \to x_1\\
\alpha_{4,8}^{14} \to x_2\\
\alpha_{3,8}^{13} \to x_3\\
\alpha_{2,6}^{10} \to x_4\\
\alpha_{2,10}^{14} \to x_5\\
\alpha_{5,7}^{14} \to x_6\\
\alpha_{2,8}^{12} \to x_7\\
\alpha_{4,6}^{12} \to x_8\\
\alpha_{3,5}^{10} \to x_9\\
\alpha_{3,6}^{11} \to x_{10}
\end{gathered}
$$

$$\alpha^{9}_{2,5} \to x_{11}$$
$$\alpha^{13}_{4,7} \to x_{12}$$
$$\alpha^{13}_{2,9} \to x_{13}$$
$$\alpha^{9}_{3,4} \to x_{14}$$
$$\alpha^{13}_{5,6} \to x_{15}$$
$$\alpha^{11}_{4,5} \to x_{16}$$
$$\alpha^{14}_{3,9} \to x_{17}$$
$$\alpha^{11}_{2,7} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4) :&\quad -x_{11} - x_{14} + 1 &&= 0\\
(e_1, e_2, e_5) :&\quad x_{11} - x_4 - x_9 &&= 0\\
(e_1, e_3, e_4) :&\quad x_{14} - x_9 &&= 0\\
(e_1, e_2, e_6) :&\quad -x_{10} - x_{18} + x_4 &&= 0\\
(e_1, e_3, e_5) :&\quad -x_{10} - x_{16} + x_9 &&= 0\\
(e_1, e_2, e_7) :&\quad -x_1 + x_{18} - x_7 &&= 0\\
(e_1, e_3, e_6) :&\quad -x_1 + x_{10} - x_8 &&= 0\\
(e_1, e_4, e_5) :&\quad x_{16} - x_8 &&= 0\\
(e_1, e_2, e_8) :&\quad -x_{13} - x_3 + x_7 &&= 0\\
(e_1, e_3, e_7) :&\quad x_1 - x_{12} - x_3 &&= 0\\
(e_1, e_4, e_6) :&\quad -x_{12} - x_{15} + x_8 &&= 0\\
(e_2, e_3, e_4) :&\quad x_{12} + x_{13}x_{14} - x_3 &&= 0\\
(e_1, e_2, e_9) :&\quad x_{13} - x_{17} - x_5 &&= 0\\
(e_1, e_3, e_8) :&\quad -x_{17} - x_2 + x_3 &&= 0\\
(e_1, e_4, e_7) :&\quad x_{12} - x_2 - x_6 &&= 0\\
(e_1, e_5, e_6) :&\quad x_{15} - x_6 &&= 0\\
(e_2, e_3, e_5) :&\quad -x_{11}x_{17} + x_5x_9 + x_6 &&= 0
\end{aligned}$$

Groebner basis (18 variables, 15 linear, 1 nonlinear)

$$3x_1 + 5x_{16} + x_{18} - 1 = 0$$
$$13x_{16} + 6x_{17} + 2x_{18} + 9x_2 - 2 = 0$$
$$13x_{16} - 3x_{17} + 2x_{18} + 9x_3 - 2 = 0$$
$$2x_{16} - 2x_{18} + 3x_4 - 1 = 0$$

$$-28x_{16} + 12x_{17} - 14x_{18} + 9x_5 + 5 = 0$$
$$-11x_{16} - 3x_{17} - x_{18} + 9x_6 + 1 = 0$$
$$-5x_{16} - 4x_{18} + 3x_7 + 1 = 0$$
$$-x_{16} + x_8 = 0$$
$$-x_{16} + x_{18} + 3x_9 - 1 = 0$$
$$3x_{10} + 2x_{16} + x_{18} - 1 = 0$$
$$3x_{11} + x_{16} - x_{18} - 2 = 0$$
$$9x_{12} + 2x_{16} + 3x_{17} + x_{18} - 1 = 0$$
$$9x_{13} - 28x_{16} + 3x_{17} - 14x_{18} + 5 = 0$$
$$3x_{14} - x_{16} + x_{18} - 1 = 0$$
$$9x_{15} - 11x_{16} - 3x_{17} - x_{18} + 1 = 0$$
$$28x_{16}^2 - 3x_{16}x_{17} - 14x_{16}x_{18} + 56x_{16} + 3x_{17}x_{18} - 21x_{17} - 14x_{18}^2 + 22x_{18} - 8 = 0$$

$\mathfrak{m}_{1A}(5, 14)$

m1A514 (this line included for string searching purposes)

Original brackets:

$$\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_9] = e_{14} & [e_3, e_8] = -e_{14} \\
[e_4, e_7] = e_{14} & [e_5, e_6] = -e_{14}
\end{array}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(5, 14)$

m3A514 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_7] &= e_{12} & [e_2, e_8] &= 3e_{13} \\
[e_2, e_9] &= \alpha_{2,9}^{14} e_{14} & [e_3, e_6] &= -e_{12} \\
[e_3, e_7] &= -2e_{13} & [e_3, e_8] &= \alpha_{3,8}^{14} e_{14} \\
[e_4, e_5] &= e_{12} & [e_4, e_6] &= e_{13} \\
[e_4, e_7] &= \alpha_{4,7}^{14} e_{14} & [e_5, e_6] &= \alpha_{5,6}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{14} - \alpha_{3,8}^{14} + 3 &= 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{14} - \alpha_{4,7}^{14} - 2 &= 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{14} - \alpha_{5,6}^{14} + 1 &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,9}^{14} &\to x_1 \\
\alpha_{5,6}^{14} &\to x_2 \\
\alpha_{3,8}^{14} &\to x_3 \\
\alpha_{4,7}^{14} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_3 + 3 &= 0 \\
(e_1, e_3, e_7): &\quad -x_3 - x_4 - 2 &= 0 \\
(e_1, e_4, e_6): &\quad -x_2 - x_4 + 1 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 - 5 = 0$$
$$x_2 + x_4 - 1 = 0$$
$$x_3 + x_4 + 2 = 0$$

$\mathfrak{m}_{5A}(5, 14)$

m5A514 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_5] &= e_{10} & [e_2, e_6] &= 2e_{11} \\
[e_2, e_7] &= \alpha_{2,7}^{12} e_{12} & [e_2, e_8] &= \alpha_{2,8}^{13} e_{13} \\
[e_2, e_9] &= \alpha_{2,9}^{14} e_{14} & [e_3, e_4] &= -e_{10} \\
[e_3, e_5] &= -e_{11} & [e_3, e_6] &= \alpha_{3,6}^{12} e_{12} \\
[e_3, e_7] &= \alpha_{3,7}^{13} e_{13} & [e_3, e_8] &= \alpha_{3,8}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{12} e_{12} & [e_4, e_6] &= \alpha_{4,6}^{13} e_{13} \\
[e_4, e_7] &= \alpha_{4,7}^{14} e_{14} & [e_5, e_6] &= \alpha_{5,6}^{14} e_{14}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &= 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{13} - \alpha_{2,9}^{14} - \alpha_{3,8}^{14} &= 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{13} - \alpha_{3,8}^{14} - \alpha_{4,7}^{14} &= 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{13} - \alpha_{4,7}^{14} - \alpha_{5,6}^{14} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{2,7} \to x_1$$
$$\alpha^{14}_{3,8} \to x_2$$
$$\alpha^{13}_{4,6} \to x_3$$
$$\alpha^{14}_{5,6} \to x_4$$
$$\alpha^{12}_{3,6} \to x_5$$
$$\alpha^{14}_{4,7} \to x_6$$
$$\alpha^{13}_{2,8} \to x_7$$
$$\alpha^{14}_{2,9} \to x_8$$
$$\alpha^{12}_{4,5} \to x_9$$
$$\alpha^{13}_{3,7} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_5 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_5 - x_9 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad x_1 - x_{10} - x_7 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_3 + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad -x_3 + x_9 &&= 0\\
(e_1, e_2, e_8): &\quad -x_2 + x_7 - x_8 &&= 0\\
(e_1, e_3, e_7): &\quad x_{10} - x_2 - x_6 &&= 0\\
(e_1, e_4, e_6): &\quad x_3 - x_4 - x_6 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$2x_1 + x_{10} - 5 = 0$$
$$3x_{10} + 2x_2 + 2x_8 - 5 = 0$$
$$x_{10} + 2x_3 + 1 = 0$$
$$3x_{10} + x_4 + x_8 - 2 = 0$$
$$-x_{10} + 2x_5 + 1 = 0$$
$$-5x_{10} + 2x_6 - 2x_8 + 5 = 0$$
$$3x_{10} + 2x_7 - 5 = 0$$
$$x_{10} + 2x_9 + 1 = 0$$

$\mathfrak{m}_{7A}(5,14)$

m7A514 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_1,e_{10}]=e_{11} && [e_1,e_{11}]=e_{12}\\
&[e_1,e_{12}]=e_{13} && [e_1,e_{13}]=e_{14}\\
&[e_2,e_3]=e_8 && [e_2,e_4]=e_9\\
&[e_2,e_5]=\alpha_{2,5}^{10}e_{10} && [e_2,e_6]=\alpha_{2,6}^{11}e_{11}\\
&[e_2,e_7]=\alpha_{2,7}^{12}e_{12} && [e_2,e_8]=\alpha_{2,8}^{13}e_{13}\\
&[e_2,e_9]=\alpha_{2,9}^{14}e_{14} && [e_3,e_4]=\alpha_{3,4}^{10}e_{10}\\
&[e_3,e_5]=\alpha_{3,5}^{11}e_{11} && [e_3,e_6]=\alpha_{3,6}^{12}e_{12}\\
&[e_3,e_7]=\alpha_{3,7}^{13}e_{13} && [e_3,e_8]=\alpha_{3,8}^{14}e_{14}\\
&[e_4,e_5]=\alpha_{4,5}^{12}e_{12} && [e_4,e_6]=\alpha_{4,6}^{13}e_{13}\\
&[e_4,e_7]=\alpha_{4,7}^{14}e_{14} && [e_5,e_6]=\alpha_{5,6}^{14}e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1,e_2,e_4): && -\alpha_{2,5}^{10}-\alpha_{3,4}^{10}+1 &&=0\\
&(e_1,e_2,e_5): && \alpha_{2,5}^{10}-\alpha_{2,6}^{11}-\alpha_{3,5}^{11} &&=0\\
&(e_1,e_3,e_4): && \alpha_{3,4}^{10}-\alpha_{3,5}^{11} &&=0\\
&(e_1,e_2,e_6): && \alpha_{2,6}^{11}-\alpha_{2,7}^{12}-\alpha_{3,6}^{12} &&=0\\
&(e_1,e_3,e_5): && \alpha_{3,5}^{11}-\alpha_{3,6}^{12}-\alpha_{4,5}^{12} &&=0\\
&(e_1,e_2,e_7): && \alpha_{2,7}^{12}-\alpha_{2,8}^{13}-\alpha_{3,7}^{13} &&=0\\
&(e_1,e_3,e_6): && \alpha_{3,6}^{12}-\alpha_{3,7}^{13}-\alpha_{4,6}^{13} &&=0\\
&(e_1,e_4,e_5): && \alpha_{4,5}^{12}-\alpha_{4,6}^{13} &&=0\\
&(e_1,e_2,e_8): && \alpha_{2,8}^{13}-\alpha_{2,9}^{14}-\alpha_{3,8}^{14} &&=0\\
&(e_1,e_3,e_7): && \alpha_{3,7}^{13}-\alpha_{3,8}^{14}-\alpha_{4,7}^{14} &&=0\\
&(e_1,e_4,e_6): && \alpha_{4,6}^{13}-\alpha_{4,7}^{14}-\alpha_{5,6}^{14} &&=0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha^{11}_{2,6} \to x_1$$
$$\alpha^{12}_{2,7} \to x_2$$
$$\alpha^{10}_{3,4} \to x_3$$
$$\alpha^{10}_{2,5} \to x_4$$
$$\alpha^{14}_{3,8} \to x_5$$
$$\alpha^{13}_{4,6} \to x_6$$
$$\alpha^{14}_{5,6} \to x_7$$
$$\alpha^{12}_{3,6} \to x_8$$
$$\alpha^{14}_{4,7} \to x_9$$
$$\alpha^{14}_{2,9} \to x_{10}$$
$$\alpha^{13}_{2,8} \to x_{11}$$
$$\alpha^{11}_{3,5} \to x_{12}$$
$$\alpha^{12}_{4,5} \to x_{13}$$
$$\alpha^{13}_{3,7} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_3 - x_4 + 1 &&= 0\\
(e_1,e_2,e_5):&\quad -x_1 - x_{12} + x_4 &&= 0\\
(e_1,e_3,e_4):&\quad -x_{12} + x_3 &&= 0\\
(e_1,e_2,e_6):&\quad x_1 - x_2 - x_8 &&= 0\\
(e_1,e_3,e_5):&\quad x_{12} - x_{13} - x_8 &&= 0\\
(e_1,e_2,e_7):&\quad -x_{11} - x_{14} + x_2 &&= 0\\
(e_1,e_3,e_6):&\quad -x_{14} - x_6 + x_8 &&= 0\\
(e_1,e_4,e_5):&\quad x_{13} - x_6 &&= 0\\
(e_1,e_2,e_8):&\quad -x_{10} + x_{11} - x_5 &&= 0\\
(e_1,e_3,e_7):&\quad x_{14} - x_5 - x_9 &&= 0\\
(e_1,e_4,e_6):&\quad x_6 - x_7 - x_9 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 0 nonlinear)

$$x_1 + 4x_{13} + 2x_{14} - 1 = 0$$

$$5x_{13} + 3x_{14} + x_2 - 1 = 0$$
$$-2x_{13} - x_{14} + x_3 = 0$$
$$2x_{13} + x_{14} + x_4 - 1 = 0$$
$$x_{10} + 5x_{13} + 4x_{14} + x_5 - 1 = 0$$
$$-x_{13} + x_6 = 0$$
$$x_{10} + 4x_{13} + 5x_{14} + x_7 - 1 = 0$$
$$-x_{13} - x_{14} + x_8 = 0$$
$$-x_{10} - 5x_{13} - 5x_{14} + x_9 + 1 = 0$$
$$x_{11} + 5x_{13} + 4x_{14} - 1 = 0$$
$$x_{12} - 2x_{13} - x_{14} = 0$$

$\mathfrak{m}_{2A}(6, 14)$

m2A614 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_7] &= e_{13} & [e_2, e_8] &= 3e_{14} \\
[e_3, e_6] &= -e_{13} & [e_3, e_7] &= -2e_{14} \\
[e_4, e_5] &= e_{13} & [e_4, e_6] &= e_{14}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(6, 14)$

m4A614 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_5] &= e_{11} & [e_2, e_6] &= 2e_{12} \\
[e_2, e_7] &= \alpha_{2,7}^{13} e_{13} & [e_2, e_8] &= \alpha_{2,8}^{14} e_{14} \\
[e_3, e_4] &= -e_{11} & [e_3, e_5] &= -e_{12} \\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_3, e_7] &= \alpha_{3,7}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{13} e_{13} & [e_4, e_6] &= \alpha_{4,6}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{13} - \alpha_{3,6}^{13} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{13} - \alpha_{4,5}^{13} - 1 &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{13} - \alpha_{2,8}^{14} - \alpha_{3,7}^{14} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{13} - \alpha_{3,7}^{14} - \alpha_{4,6}^{14} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{13} - \alpha_{4,6}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{13} \to x_1$$

$$\alpha_{2,7}^{13} \to x_2$$

$$\alpha_{4,6}^{14} \to x_3$$

$$\alpha_{3,7}^{14} \to x_4$$

$$\alpha_{3,6}^{13} \to x_5$$

$$\alpha_{2,8}^{14} \to x_6$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_5 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_1 - x_5 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad x_2 - x_4 - x_6 &&= 0\\
(e_1, e_3, e_6): &\quad -x_3 - x_4 + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad x_1 - x_3 &&= 0
\end{aligned}
$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$3x_1 - x_6 + 4 = 0$$

$$3x_2 - x_6 - 5 = 0$$

$$3x_3 - x_6 + 4 = 0$$

$$3x_4 + 2x_6 - 5 = 0$$

$$3x_5 + x_6 - 1 = 0$$

$\mathfrak{m}_{6A}(6, 14)$

m6A614 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_2, e_3] &= e_9 & [e_2, e_4] &= e_{10}\\
[e_2, e_5] &= \alpha_{2,5}^{11} e_{11} & [e_2, e_6] &= \alpha_{2,6}^{12} e_{12}\\
[e_2, e_7] &= \alpha_{2,7}^{13} e_{13} & [e_2, e_8] &= \alpha_{2,8}^{14} e_{14}\\
[e_3, e_4] &= \alpha_{3,4}^{11} e_{11} & [e_3, e_5] &= \alpha_{3,5}^{12} e_{12}\\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_3, e_7] &= \alpha_{3,7}^{14} e_{14}\\
[e_4, e_5] &= \alpha_{4,5}^{13} e_{13} & [e_4, e_6] &= \alpha_{4,6}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{12} - \alpha_{2,7}^{13} - \alpha_{3,6}^{13} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{12} - \alpha_{3,6}^{13} - \alpha_{4,5}^{13} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{13} - \alpha_{2,8}^{14} - \alpha_{3,7}^{14} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{13} - \alpha_{3,7}^{14} - \alpha_{4,6}^{14} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{13} - \alpha_{4,6}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{4,5}^{13} \to x_1\\
\alpha_{2,7}^{13} \to x_2\\
\alpha_{2,5}^{11} \to x_3\\
\alpha_{3,4}^{11} \to x_4\\
\alpha_{4,6}^{14} \to x_5\\
\alpha_{2,6}^{12} \to x_6\\
\alpha_{3,7}^{14} \to x_7\\
\alpha_{3,5}^{12} \to x_8\\
\alpha_{3,6}^{13} \to x_9\\
\alpha_{2,8}^{14} \to x_{10}
\end{gathered}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_4 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad x_3 - x_6 - x_8 &&= 0\\
(e_1, e_3, e_4): &\quad x_4 - x_8 &&= 0\\
(e_1, e_2, e_6): &\quad -x_2 + x_6 - x_9 &&= 0\\
(e_1, e_3, e_5): &\quad -x_1 + x_8 - x_9 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{10} + x_2 - x_7 &&= 0\\
(e_1, e_3, e_6): &\quad -x_5 - x_7 + x_9 &&= 0\\
(e_1, e_4, e_5): &\quad x_1 - x_5 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + x_{10} + 4x_9 - 1 = 0$$
$$-2x_{10} + x_2 - 5x_9 + 1 = 0$$
$$-x_{10} + x_3 - 3x_9 = 0$$
$$x_{10} + x_4 + 3x_9 - 1 = 0$$
$$x_{10} + x_5 + 4x_9 - 1 = 0$$
$$-2x_{10} + x_6 - 6x_9 + 1 = 0$$
$$-x_{10} + x_7 - 5x_9 + 1 = 0$$
$$x_{10} + x_8 + 3x_9 - 1 = 0$$

$\mathfrak{m}_{1A}(7, 14)$

m1A714 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_7] = e_{14} \qquad [e_3, e_6] = -e_{14}$$
$$[e_4, e_5] = e_{14}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(7, 14)$

m3A714 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_5] &= e_{12} & [e_2, e_6] &= 2e_{13} \\
[e_2, e_7] &= \alpha_{2,7}^{14} e_{14} & [e_3, e_4] &= -e_{12} \\
[e_3, e_5] &= -e_{13} & [e_3, e_6] &= \alpha_{3,6}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6) : \quad & -\alpha_{2,7}^{14} - \alpha_{3,6}^{14} + 2 && = 0 \\
(e_1, e_3, e_5) : \quad & -\alpha_{3,6}^{14} - \alpha_{4,5}^{14} - 1 && = 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\alpha_{3,6}^{14} \to x_1
$$

$$
\alpha_{4,5}^{14} \to x_2
$$

$$
\alpha_{2,7}^{14} \to x_3
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6) : \quad & -x_1 - x_3 + 2 && = 0 \\
(e_1, e_3, e_5) : \quad & -x_1 - x_2 - 1 && = 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$
x_1 + x_3 - 2 = 0
$$

$$
x_2 - x_3 + 3 = 0
$$

$\mathfrak{m}_{5A}(7,14)$

m5A714 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14}\\
[e_2,e_3] &= e_{10} & [e_2,e_4] &= e_{11}\\
[e_2,e_5] &= \alpha_{2,5}^{12} e_{12} & [e_2,e_6] &= \alpha_{2,6}^{13} e_{13}\\
[e_2,e_7] &= \alpha_{2,7}^{14} e_{14} & [e_3,e_4] &= \alpha_{3,4}^{12} e_{12}\\
[e_3,e_5] &= \alpha_{3,5}^{13} e_{13} & [e_3,e_6] &= \alpha_{3,6}^{14} e_{14}\\
[e_4,e_5] &= \alpha_{4,5}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{12}-\alpha_{3,4}^{12}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{12}-\alpha_{2,6}^{13}-\alpha_{3,5}^{13} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{12}-\alpha_{3,5}^{13} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{13}-\alpha_{2,7}^{14}-\alpha_{3,6}^{14} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{13}-\alpha_{3,6}^{14}-\alpha_{4,5}^{14} &&=0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,6}^{13} \to x_1$$

$$\alpha_{2,5}^{12} \to x_2$$

$$\alpha_{4,5}^{14} \to x_3$$

$$\alpha_{3,5}^{13} \to x_4$$

$$\alpha_{3,6}^{14} \to x_5$$

$$\alpha_{3,4}^{12} \to x_6$$

$$\alpha_{2,7}^{14} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_6 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 + x_2 - x_4 &&= 0\\
(e_1, e_3, e_4): &\quad -x_4 + x_6 &&= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_5 - x_7 &&= 0\\
(e_1, e_3, e_5): &\quad -x_3 + x_4 - x_5 &&= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 + 2x_6 - 1 = 0$$
$$x_2 + x_6 - 1 = 0$$
$$x_3 - 3x_6 - x_7 + 1 = 0$$
$$x_4 - x_6 = 0$$
$$x_5 + 2x_6 + x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(8, 14)$

m2A814 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_2, e_5] &= e_{13} & [e_2, e_6] &= 2e_{14}\\
[e_3, e_4] &= -e_{13} & [e_3, e_5] &= -e_{14}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(8, 14)$

m4A814 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_3] &= e_{11} & [e_2, e_4] &= e_{12} \\
[e_2, e_5] &= \alpha_{2,5}^{13} e_{13} & [e_2, e_6] &= \alpha_{2,6}^{14} e_{14} \\
[e_3, e_4] &= \alpha_{3,4}^{13} e_{13} & [e_3, e_5] &= \alpha_{3,5}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{13} - \alpha_{2,6}^{14} - \alpha_{3,5}^{14} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{13} - \alpha_{3,5}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,5}^{13} &\to x_1 \\
\alpha_{3,4}^{13} &\to x_2 \\
\alpha_{3,5}^{14} &\to x_3 \\
\alpha_{2,6}^{14} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_2 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad x_1 - x_3 - x_4 &= 0 \\
(e_1, e_3, e_4): &\quad x_2 - x_3 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
2x_1 - x_4 - 1 &= 0 \\
2x_2 + x_4 - 1 &= 0 \\
2x_3 + x_4 - 1 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(9,14)$

m1A914 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_1,e_{12}]&=e_{13} & [e_1,e_{13}]&=e_{14}\\
[e_2,e_5]&=e_{14} & [e_3,e_4]&=-e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(9,14)$

m3A914 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_1,e_{12}]&=e_{13} & [e_1,e_{13}]&=e_{14}\\
[e_2,e_3]&=e_{12} & [e_2,e_4]&=e_{13}\\
[e_2,e_5]&=\alpha_{2,5}^{14}e_{14} & [e_3,e_4]&=\alpha_{3,4}^{14}e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4):\quad -\alpha_{2,5}^{14}-\alpha_{3,4}^{14}+1 = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,5}^{14}\to x_1$$
$$\alpha_{3,4}^{14}\to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(10, 14)$

m2A1014 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_3] &= e_{13} & [e_2, e_4] &= e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(11, 14)$

m1A1114 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_3] &= e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2, 15)$

m1A215 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_{13}] = e_{15}$$
$$[e_3, e_{12}] = -e_{15} \qquad [e_4, e_{11}] = e_{15}$$
$$[e_5, e_{10}] = -e_{15} \qquad [e_6, e_9] = e_{15}$$
$$[e_7, e_8] = -e_{15}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2, 15)$

m3A215 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_{11}] = e_{13}$$
$$[e_2, e_{12}] = 5e_{14} \qquad [e_2, e_{13}] = 0$$
$$[e_3, e_{10}] = -e_{13} \qquad [e_3, e_{11}] = -4e_{14}$$
$$[e_3, e_{12}] = 5e_{15} \qquad [e_4, e_9] = e_{13}$$
$$[e_4, e_{10}] = 3e_{14} \qquad [e_4, e_{11}] = -9e_{15}$$
$$[e_5, e_8] = -e_{13} \qquad [e_5, e_9] = -2e_{14}$$
$$[e_5, e_{10}] = 12e_{15} \qquad [e_6, e_7] = e_{13}$$
$$[e_6, e_8] = e_{14} \qquad [e_6, e_9] = -14e_{15}$$
$$[e_7, e_8] = 15e_{15}$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14}\\
[e_1,e_{14}] &= e_{15} & [e_2,e_{11}] &= e_{13}\\
[e_2,e_{12}] &= 5e_{14} & [e_2,e_{13}] &= \alpha_{2,13}^{15}e_{15}\\
[e_3,e_{10}] &= -e_{13} & [e_3,e_{11}] &= -4e_{14}\\
[e_3,e_{12}] &= \alpha_{3,12}^{15}e_{15} & [e_4,e_9] &= e_{13}\\
[e_4,e_{10}] &= 3e_{14} & [e_4,e_{11}] &= \alpha_{4,11}^{15}e_{15}\\
[e_5,e_8] &= -e_{13} & [e_5,e_9] &= -2e_{14}\\
[e_5,e_{10}] &= \alpha_{5,10}^{15}e_{15} & [e_6,e_7] &= e_{13}\\
[e_6,e_8] &= e_{14} & [e_6,e_9] &= \alpha_{6,9}^{15}e_{15}\\
[e_7,e_8] &= \alpha_{7,8}^{15}e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{12}):&\quad -\alpha_{2,13}^{15}-\alpha_{3,12}^{15}+5 &&=0\\
(e_1,e_3,e_{11}):&\quad -\alpha_{3,12}^{15}-\alpha_{4,11}^{15}-4 &&=0\\
(e_1,e_4,e_{10}):&\quad -\alpha_{4,11}^{15}-\alpha_{5,10}^{15}+3 &&=0\\
(e_1,e_5,e_9):&\quad -\alpha_{5,10}^{15}-\alpha_{6,9}^{15}-2 &&=0\\
(e_1,e_6,e_8):&\quad -\alpha_{6,9}^{15}-\alpha_{7,8}^{15}+1 &&=0\\
(e_2,e_3,e_{10}):&\quad -\alpha_{2,13}^{15} &&=0\\
(e_2,e_4,e_9):&\quad \alpha_{2,13}^{15} &&=0\\
(e_2,e_5,e_8):&\quad -\alpha_{2,13}^{15} &&=0\\
(e_2,e_6,e_7):&\quad \alpha_{2,13}^{15} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha^{15}_{7,8} = 15$$
$$\alpha^{15}_{6,9} = -14$$
$$\alpha^{15}_{2,13} = 0$$
$$\alpha^{15}_{4,11} = -9$$
$$\alpha^{15}_{5,10} = 12$$
$$\alpha^{15}_{3,12} = 5$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{15}_{7,8} \to x_1$$
$$\alpha^{15}_{6,9} \to x_2$$
$$\alpha^{15}_{2,13} \to x_3$$
$$\alpha^{15}_{4,11} \to x_4$$
$$\alpha^{15}_{5,10} \to x_5$$
$$\alpha^{15}_{3,12} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}): &\quad -x_3 - x_6 + 5 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_4 - x_6 - 4 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_4 - x_5 + 3 &&= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_5 - 2 &&= 0\\
(e_1, e_6, e_8): &\quad -x_1 - x_2 + 1 &&= 0\\
(e_2, e_3, e_{10}): &\quad -x_3 &&= 0\\
(e_2, e_4, e_9): &\quad x_3 &&= 0\\
(e_2, e_5, e_8): &\quad -x_3 &&= 0\\
(e_2, e_6, e_7): &\quad x_3 &&= 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 - 15 = 0$$
$$x_2 + 14 = 0$$
$$x_3 = 0$$

$$x_4 + 9 = 0$$

$$x_5 - 12 = 0$$

$$x_6 - 5 = 0$$

Solution 1:

$$x_1 = 15$$

$$x_2 = -14$$

$$x_3 = 0$$

$$x_4 = -9$$

$$x_5 = 12$$

$$x_6 = 5$$

$\mathfrak{m}_{11A}(2, 15)$

m11A215 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7$$
$$[e_2, e_6] = e_8 \qquad [e_2, e_7] = e_9$$
$$[e_2, e_8] = e_{10} \qquad [e_2, e_9] = e_{11}$$
$$[e_2, e_{10}] = e_{12} \qquad [e_2, e_{11}] = e_{13}$$
$$[e_2, e_{12}] = e_{14} \qquad [e_2, e_{13}] = e_{15}$$
$$[e_3, e_4] = 0 \qquad [e_3, e_5] = 0$$
$$[e_3, e_6] = 0 \qquad [e_3, e_7] = 0$$
$$[e_3, e_8] = 0 \qquad [e_3, e_9] = 0$$
$$[e_3, e_{10}] = 0 \qquad [e_3, e_{11}] = 0$$
$$[e_3, e_{12}] = 0 \qquad [e_4, e_5] = 0$$
$$[e_4, e_6] = 0 \qquad [e_4, e_7] = 0$$
$$[e_4, e_8] = 0 \qquad [e_4, e_9] = 0$$
$$[e_4, e_{10}] = 0 \qquad [e_4, e_{11}] = 0$$
$$[e_5, e_6] = 0 \qquad [e_5, e_7] = 0$$
$$[e_5, e_8] = 0 \qquad [e_5, e_9] = 0$$
$$[e_5, e_{10}] = 0 \qquad [e_6, e_7] = 0$$
$$[e_6, e_8] = 0 \qquad [e_6, e_9] = 0$$
$$[e_7, e_8] = 0$$

Solution 2

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_1,e_{14}]=e_{15} \qquad [e_2,e_3]=e_5$$
$$[e_2,e_4]=e_6 \qquad [e_2,e_5]=\frac{9e_7}{10}$$
$$[e_2,e_6]=\frac{4e_8}{5} \qquad [e_2,e_7]=\frac{5e_9}{7}$$
$$[e_2,e_8]=\frac{9e_{10}}{14} \qquad [e_2,e_9]=\frac{7e_{11}}{12}$$
$$[e_2,e_{10}]=\frac{8e_{12}}{15} \qquad [e_2,e_{11}]=\frac{27e_{13}}{55}$$
$$[e_2,e_{12}]=\frac{5e_{14}}{11} \qquad [e_2,e_{13}]=\frac{11e_{15}}{26}$$
$$[e_3,e_4]=\frac{e_7}{10} \qquad [e_3,e_5]=\frac{e_8}{10}$$
$$[e_3,e_6]=\frac{3e_9}{35} \qquad [e_3,e_7]=\frac{e_{10}}{14}$$
$$[e_3,e_8]=\frac{5e_{11}}{84} \qquad [e_3,e_9]=\frac{e_{12}}{20}$$
$$[e_3,e_{10}]=\frac{7e_{13}}{165} \qquad [e_3,e_{11}]=\frac{2e_{14}}{55}$$
$$[e_3,e_{12}]=\frac{9e_{15}}{286} \qquad [e_4,e_5]=\frac{e_9}{70}$$
$$[e_4,e_6]=\frac{e_{10}}{70} \qquad [e_4,e_7]=\frac{e_{11}}{84}$$
$$[e_4,e_8]=\frac{e_{12}}{105} \qquad [e_4,e_9]=\frac{e_{13}}{132}$$
$$[e_4,e_{10}]=\frac{e_{14}}{165} \qquad [e_4,e_{11}]=\frac{7e_{15}}{1430}$$
$$[e_5,e_6]=\frac{e_{11}}{420} \qquad [e_5,e_7]=\frac{e_{12}}{420}$$
$$[e_5,e_8]=\frac{3e_{13}}{1540} \qquad [e_5,e_9]=\frac{e_{14}}{660}$$
$$[e_5,e_{10}]=\frac{e_{15}}{858} \qquad [e_6,e_7]=\frac{e_{13}}{2310}$$
$$[e_6,e_8]=\frac{e_{14}}{2310} \qquad [e_6,e_9]=\frac{e_{15}}{2860}$$
$$[e_7,e_8]=\frac{e_{15}}{12012}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_2, e_{12}] = \alpha_{2,12}^{14} e_{14} & [e_2, e_{13}] = \alpha_{2,13}^{15} e_{15} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] = \alpha_{3,8}^{11} e_{11} & [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_3, e_{11}] = \alpha_{3,11}^{14} e_{14} \\
[e_3, e_{12}] = \alpha_{3,12}^{15} e_{15} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_8] = \alpha_{4,8}^{12} e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_4, e_{10}] = \alpha_{4,10}^{14} e_{14} & [e_4, e_{11}] = \alpha_{4,11}^{15} e_{15} \\
[e_5, e_6] = \alpha_{5,6}^{11} e_{11} & [e_5, e_7] = \alpha_{5,7}^{12} e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_5, e_9] = \alpha_{5,9}^{14} e_{14} \\
[e_5, e_{10}] = \alpha_{5,10}^{15} e_{15} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13} \\
[e_6, e_8] = \alpha_{6,8}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{15} e_{15} \\
[e_7, e_8] = \alpha_{7,8}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{7}_{2,5}-\alpha^{7}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{7}_{2,5}-\alpha^{8}_{2,6}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{7}_{3,4}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{8}_{2,6}-\alpha^{9}_{2,7}-\alpha^{9}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{8}_{3,5}-\alpha^{9}_{3,6}-\alpha^{9}_{4,5} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{9}_{2,7}\alpha^{7}_{3,4}-\alpha^{9}_{3,6}+\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{9}_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{9}_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{9}_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha^{8}_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^{9}_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^{9}_{4,5}+\alpha^{11}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{12}_{2,10}+\alpha^{11}_{2,9}-\alpha^{12}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{11}_{3,8}-\alpha^{12}_{3,9}-\alpha^{12}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{11}_{4,7}-\alpha^{12}_{4,8}-\alpha^{12}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{11}_{5,6}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_3,e_7):&\quad \alpha^{12}_{2,10}\alpha^{10}_{3,7}-\alpha^{9}_{2,7}\alpha^{12}_{3,9}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_4,e_6):&\quad \alpha^{12}_{2,10}\alpha^{10}_{4,6}-\alpha^{8}_{2,6}\alpha^{12}_{4,8} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^{7}_{3,4}\alpha^{12}_{5,7}-\alpha^{8}_{3,5}\alpha^{12}_{4,8}+\alpha^{12}_{3,9}\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha^{12}_{2,10}-\alpha^{13}_{2,11}-\alpha^{13}_{3,10} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{13}_{3,10}+\alpha^{12}_{3,9}-\alpha^{13}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad \alpha^{12}_{4,8}-\alpha^{13}_{4,9}-\alpha^{13}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad \alpha^{12}_{5,7}-\alpha^{13}_{5,8}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_3,e_8):&\quad \alpha^{13}_{2,11}\alpha^{11}_{3,8}-\alpha^{10}_{2,8}\alpha^{13}_{3,10}-\alpha^{13}_{5,8} &&=0\\
(e_2,e_4,e_7):&\quad \alpha^{13}_{2,11}\alpha^{11}_{4,7}-\alpha^{9}_{2,7}\alpha^{13}_{4,9}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_5,e_6):&\quad \alpha^{13}_{2,11}\alpha^{11}_{5,6}+\alpha^{7}_{2,5}\alpha^{13}_{6,7}-\alpha^{8}_{2,6}\alpha^{13}_{5,8} &&=0\\
(e_3,e_4,e_6):&\quad \alpha^{13}_{3,10}\alpha^{10}_{4,6}+\alpha^{7}_{3,4}\alpha^{13}_{6,7}-\alpha^{9}_{3,6}\alpha^{13}_{4,9} &&=0\\
(e_1,e_2,e_{11}):&\quad \alpha^{13}_{2,11}-\alpha^{14}_{2,12}-\alpha^{14}_{3,11} &&=0\\
(e_1,e_3,e_{10}):&\quad \alpha^{13}_{3,10}-\alpha^{14}_{3,11}-\alpha^{14}_{4,10} &&=0\\
(e_1,e_4,e_9):&\quad -\alpha^{14}_{4,10}+\alpha^{13}_{4,9}-\alpha^{14}_{5,9} &&=0\\
(e_1,e_5,e_8):&\quad \alpha^{13}_{5,8}-\alpha^{14}_{5,9}-\alpha^{14}_{6,8} &&=0\\
(e_1,e_6,e_7):&\quad \alpha^{13}_{6,7}-\alpha^{14}_{6,8} &&=0\\
(e_2,e_3,e_9):&\quad \alpha^{14}_{2,12}\alpha^{12}_{3,9}-\alpha^{11}_{2,9}\alpha^{14}_{3,11}-\alpha^{14}_{5,9} &&=0\\
(e_2,e_4,e_8):&\quad \alpha^{14}_{2,12}\alpha^{12}_{4,8}-\alpha^{10}_{2,8}\alpha^{14}_{4,10}-\alpha^{14}_{6,8} &&=0\\
(e_2,e_5,e_7):&\quad \alpha^{14}_{2,12}\alpha^{12}_{5,7}-\alpha^{9}_{2,7}\alpha^{14}_{5,9} &&=0\\
(e_3,e_4,e_7):&\quad \alpha^{14}_{3,11}\alpha^{11}_{4,7}-\alpha^{10}_{3,7}\alpha^{14}_{4,10} &&=0\\
(e_3,e_5,e_6):&\quad \alpha^{14}_{3,11}\alpha^{11}_{5,6}+\alpha^{8}_{3,5}\alpha^{14}_{6,8}-\alpha^{9}_{3,6}\alpha^{14}_{5,9} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha^{14}_{4,10} = 0$$
$$\alpha^{13}_{3,10} = 0$$
$$\alpha^{13}_{4,9} = 0$$
$$\alpha^{9}_{3,6} = 0$$
$$\alpha^{13}_{2,11} = 1$$
$$\alpha^{7}_{3,4} = 0$$
$$\alpha^{15}_{3,12} = 0$$
$$\alpha^{11}_{2,9} = 1$$
$$\alpha^{14}_{6,8} = 0$$
$$\alpha^{9}_{2,7} = 1$$
$$\alpha^{8}_{2,6} = 1$$
$$\alpha^{7}_{2,5} = 1$$
$$\alpha^{15}_{6,9} = 0$$
$$\alpha^{12}_{5,7} = 0$$
$$\alpha^{13}_{6,7} = 0$$
$$\alpha^{8}_{3,5} = 0$$
$$\alpha^{15}_{5,10} = 0$$
$$\alpha^{10}_{4,6} = 0$$
$$\alpha^{14}_{3,11} = 0$$
$$\alpha^{14}_{5,9} = 0$$
$$\alpha^{15}_{7,8} = 0$$
$$\alpha^{12}_{3,9} = 0$$
$$\alpha^{10}_{3,7} = 0$$
$$\alpha^{15}_{2,13} = 1$$
$$\alpha^{12}_{2,10} = 1$$
$$\alpha^{10}_{2,8} = 1$$
$$\alpha^{11}_{4,7} = 0$$
$$\alpha^{11}_{5,6} = 0$$
$$\alpha^{14}_{2,12} = 1$$
$$\alpha^{13}_{5,8} = 0$$
$$\alpha^{15}_{4,11} = 0$$
$$\alpha^{9}_{4,5} = 0$$
$$\alpha^{12}_{4,8} = 0$$
$$\alpha^{11}_{3,8} = 0$$

Solution 2:

$$\alpha^{14}_{4,10} = 1/165$$
$$\alpha^{13}_{3,10} = 7/165$$
$$\alpha^{13}_{4,9} = 1/132$$
$$\alpha^{9}_{3,6} = 3/35$$
$$\alpha^{13}_{2,11} = 27/55$$
$$\alpha^{7}_{3,4} = 1/10$$
$$\alpha^{15}_{3,12} = 9/286$$
$$\alpha^{11}_{2,9} = 7/12$$
$$\alpha^{14}_{6,8} = 1/2310$$
$$\alpha^{9}_{2,7} = 5/7$$
$$\alpha^{8}_{2,6} = 4/5$$
$$\alpha^{7}_{2,5} = 9/10$$
$$\alpha^{15}_{6,9} = 1/2860$$
$$\alpha^{12}_{5,7} = 1/420$$
$$\alpha^{13}_{6,7} = 1/2310$$
$$\alpha^{8}_{3,5} = 1/10$$
$$\alpha^{15}_{5,10} = 1/858$$
$$\alpha^{10}_{4,6} = 1/70$$
$$\alpha^{14}_{3,11} = 2/55$$
$$\alpha^{14}_{5,9} = 1/660$$
$$\alpha^{15}_{7,8} = 1/12012$$
$$\alpha^{12}_{3,9} = 1/20$$
$$\alpha^{10}_{3,7} = 1/14$$
$$\alpha^{15}_{2,13} = 11/26$$
$$\alpha^{12}_{2,10} = 8/15$$
$$\alpha^{10}_{2,8} = 9/14$$
$$\alpha^{11}_{4,7} = 1/84$$
$$\alpha^{11}_{5,6} = 1/420$$
$$\alpha^{14}_{2,12} = 5/11$$
$$\alpha^{13}_{5,8} = 3/1540$$
$$\alpha^{15}_{4,11} = 7/1430$$
$$\alpha^{9}_{4,5} = 1/70$$
$$\alpha^{12}_{4,8} = 1/105$$
$$\alpha^{11}_{3,8} = 5/84$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,10}^{14} \to x_1$$

$$\alpha_{3,10}^{13} \to x_2$$

$$\alpha_{4,9}^{13} \to x_3$$

$$\alpha_{3,6}^{9} \to x_4$$

$$\alpha_{2,11}^{13} \to x_5$$

$$\alpha_{3,4}^{7} \to x_6$$

$$\alpha_{3,12}^{15} \to x_7$$

$$\alpha_{2,9}^{11} \to x_8$$

$$\alpha_{6,8}^{14} \to x_9$$

$$\alpha_{2,7}^{9} \to x_{10}$$

$$\alpha_{2,6}^{8} \to x_{11}$$

$$\alpha_{2,5}^{7} \to x_{12}$$

$$\alpha_{6,9}^{15} \to x_{13}$$

$$\alpha_{5,7}^{12} \to x_{14}$$

$$\alpha_{6,7}^{13} \to x_{15}$$

$$\alpha_{3,5}^{8} \to x_{16}$$

$$\alpha_{5,10}^{15} \to x_{17}$$

$$\alpha_{4,6}^{10} \to x_{18}$$

$$\alpha_{3,11}^{14} \to x_{19}$$

$$\alpha_{5,9}^{14} \to x_{20}$$

$$\alpha_{7,8}^{15} \to x_{21}$$

$$\alpha_{3,9}^{12} \to x_{22}$$

$$\alpha_{3,7}^{10} \to x_{23}$$

$$\alpha_{2,13}^{15} \to x_{24}$$

$$\alpha_{2,10}^{12} \to x_{25}$$

$$\alpha_{2,8}^{10} \to x_{26}$$

$$\alpha_{4,7}^{11} \to x_{27}$$

$$\alpha_{5,6}^{11} \to x_{28}$$

$$\alpha_{2,12}^{14} \to x_{29}$$

$$\alpha_{5,8}^{13} \to x_{30}$$

$$\alpha_{4,11}^{15} \to x_{31}$$

$$\alpha_{4,5}^{9} \to x_{32}$$

$$\alpha_{4,8}^{12} \to x_{33}$$

$$\alpha_{3,8}^{11} \to x_{34}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{12}-x_6+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{11}+x_{12}-x_{16} &&=0\\
(e_1,e_3,e_4):&\quad -x_{16}+x_6 &&=0\\
(e_1,e_2,e_6):&\quad -x_{10}+x_{11}-x_4 &&=0\\
(e_1,e_3,e_5):&\quad x_{16}-x_{32}-x_4 &&=0\\
(e_2,e_3,e_4):&\quad x_{10}x_6+x_{32}-x_4 &&=0\\
(e_1,e_2,e_7):&\quad x_{10}-x_{23}-x_{26} &&=0\\
(e_1,e_3,e_6):&\quad -x_{18}-x_{23}+x_4 &&=0\\
(e_1,e_4,e_5):&\quad -x_{18}+x_{32} &&=0\\
(e_2,e_3,e_5):&\quad -x_{12}x_{23}+x_{16}x_{26} &&=0\\
(e_1,e_2,e_8):&\quad x_{26}-x_{34}-x_8 &&=0\\
(e_1,e_3,e_7):&\quad x_{23}-x_{27}-x_{34} &&=0\\
(e_1,e_4,e_6):&\quad x_{18}-x_{27}-x_{28} &&=0\\
(e_2,e_3,e_6):&\quad -x_{11}x_{34}-x_{28}+x_4x_8 &&=0\\
(e_2,e_4,e_5):&\quad -x_{12}x_{27}+x_{28}+x_{32}x_8 &&=0\\
(e_1,e_2,e_9):&\quad -x_{22}-x_{25}+x_8 &&=0\\
(e_1,e_3,e_8):&\quad -x_{22}-x_{33}+x_{34} &&=0\\
(e_1,e_4,e_7):&\quad -x_{14}+x_{27}-x_{33} &&=0\\
(e_1,e_5,e_6):&\quad -x_{14}+x_{28} &&=0\\
(e_2,e_3,e_7):&\quad -x_{10}x_{22}-x_{14}+x_{23}x_{25} &&=0\\
(e_2,e_4,e_6):&\quad -x_{11}x_{33}+x_{18}x_{25} &&=0\\
(e_3,e_4,e_5):&\quad x_{14}x_6-x_{16}x_{33}+x_{22}x_{32} &&=0\\
(e_1,e_2,e_{10}):&\quad -x_2+x_{25}-x_5 &&=0\\
(e_1,e_3,e_9):&\quad -x_2+x_{22}-x_3 &&=0\\
(e_1,e_4,e_8):&\quad -x_3-x_{30}+x_{33} &&=0\\
(e_1,e_5,e_7):&\quad x_{14}-x_{15}-x_{30} &&=0\\
(e_2,e_3,e_8):&\quad -x_2x_{26}-x_{30}+x_{34}x_5 &&=0\\
(e_2,e_4,e_7):&\quad -x_{10}x_3-x_{15}+x_{27}x_5 &&=0\\
(e_2,e_5,e_6):&\quad -x_{11}x_{30}+x_{12}x_{15}+x_{28}x_5 &&=0\\
(e_3,e_4,e_6):&\quad x_{15}x_6+x_{18}x_2-x_3x_4 &&=0\\
(e_1,e_2,e_{11}):&\quad -x_{19}-x_{29}+x_5 &&=0\\
(e_1,e_3,e_{10}):&\quad -x_1-x_{19}+x_2 &&=0\\
(e_1,e_4,e_9):&\quad -x_1-x_{20}+x_3 &&=0\\
(e_1,e_5,e_8):&\quad -x_{20}+x_{30}-x_9 &&=0\\
(e_1,e_6,e_7):&\quad x_{15}-x_9 &&=0\\
(e_2,e_3,e_9):&\quad -x_{19}x_8-x_{20}+x_{22}x_{29} &&=0\\
(e_2,e_4,e_8):&\quad -x_1x_{26}+x_{29}x_{33}-x_9 &&=0\\
(e_2,e_5,e_7):&\quad -x_{10}x_{20}+x_{14}x_{29} &&=0\\
(e_3,e_4,e_7):&\quad -x_1x_{23}+x_{19}x_{27} &&=0\\
(e_3,e_5,e_6):&\quad x_{16}x_9+x_{19}x_{28}-x_{20}x_4 &&=0\\
(e_1,e_2,e_{12}):&\quad -x_{24}+x_{29}-x_7 &&=0\\
(e_1,e_3,e_{11}):&\quad x_{19}-x_{31}-x_7 &&=0\\
(e_1,e_4,e_{10}):&\quad x_1-x_{17}-x_{31} &&=0\\
(e_1,e_5,e_9):&\quad -x_{13}-x_{17}+x_{20} &&=0
\end{aligned}
$$

Groebner basis (34 variables, 0 linear, 34 nonlinear)

$$1375000x_1 - 761129964x_{34}^5 - 19078125x_{34}^4 - 1031250x_{34}^3 - 2062500x_{34}^2 = 0$$

$$343750x_2+1429436988x_{34}^5+31453125x_{34}^4+3609375x_{34}^3+1031250x_{34}^2-343750x_{34} = 0$$

$$275000x_3 - 390368916x_{34}^5 - 8971875x_{34}^4 - 1031250x_{34}^3 - 412500x_{34}^2 = 0$$

$$-337363992x_{34}^5 - 6843750x_{34}^4 - 703125x_{34}^3 - 93750x_{34}^2 - 31250x_{34} + 31250x_4 = 0$$

$$14633280144x_{34}^5+292875000x_{34}^4+29390625x_{34}^3+3093750x_{34}^2+2406250x_{34}+343750x_5-343750 = 0$$

$$-2134974996x_{34}^5-43171875x_{34}^4-4406250x_{34}^3-562500x_{34}^2-125000x_{34}+125000x_6 = 0$$

$$2118820032x_{34}^5+48262500x_{34}^4+4021875x_{34}^3+2681250x_{34}^2-446875x_{34}+446875x_7 = 0$$

$$247333716x_{34}^5+5011875x_{34}^4+513750x_{34}^3+67500x_{34}^2+25000x_{34}+5000x_8-5000 = 0$$

$$-155836044x_{34}^5 - 2578125x_{34}^4 + 343750x_9 = 0$$

$$12500x_{10}+561940596x_{34}^5+11371875x_{34}^4+1162500x_{34}^3+150000x_{34}^2+37500x_{34}-12500 = 0$$

$$62500x_{11}+2134974996x_{34}^5+43171875x_{34}^4+4406250x_{34}^3+562500x_{34}^2+125000x_{34}-62500 = 0$$

$$125000x_{12}+2134974996x_{34}^5+43171875x_{34}^4+4406250x_{34}^3+562500x_{34}^2+125000x_{34}-125000 = 0$$

$$4468750x_{13} - 1527997212x_{34}^5 - 33515625x_{34}^4 = 0$$

$$62500x_{14} - 110791044x_{34}^5 - 2109375x_{34}^4 - 187500x_{34}^3 = 0$$

$$343750x_{15} - 155836044x_{34}^5 - 2578125x_{34}^4 = 0$$

$$125000x_{16}-2134974996x_{34}^5-43171875x_{34}^4-4406250x_{34}^3-562500x_{34}^2-125000x_{34} = 0$$

$$1787500x_{17} - 936730116x_{34}^5 - 20109375x_{34}^4 - 5362500x_{34}^3 = 0$$

$$125000x_{18} - 785519028x_{34}^5 - 15796875x_{34}^4 - 1593750x_{34}^3 - 187500x_{34}^2 = 0$$

$$1375000x_{19}+6478877916x_{34}^5+144890625x_{34}^4+15468750x_{34}^3+6187500x_{34}^2-1375000x_{34} = 0$$

$$687500x_{20} - 595357308x_{34}^5 - 12890625x_{34}^4 - 2062500x_{34}^3 = 0$$

$$446875x_{21} - 49787136x_{34}^5 = 0$$

$$125000x_{22}+342354852x_{34}^5+7359375x_{34}^4+843750x_{34}^3+187500x_{34}^2-125000x_{34} = 0$$

$$25000x_{23} - 112787388x_{34}^5 - 2315625x_{34}^4 - 243750x_{34}^3 - 37500x_{34}^2 - 25000x_{34} = 0$$

$$65000x_{24}+2152554012x_{34}^5+41510625x_{34}^4+4241250x_{34}^3-97500x_{34}^2+585000x_{34}-65000 = 0$$

$$31250x_{25}+1460247012x_{34}^5+29484375x_{34}^4+3000000x_{34}^3+375000x_{34}^2+187500x_{34}-31250 = 0$$

$$5000x_{26}+247333716x_{34}^5+5011875x_{34}^4+513750x_{34}^3+67500x_{34}^2+20000x_{34}-5000 = 0$$

$$25000x_{27} - 112787388x_{34}^5 - 2315625x_{34}^4 - 243750x_{34}^3 - 37500x_{34}^2 = 0$$

$$62500x_{28} - 110791044x_{34}^5 - 2109375x_{34}^4 - 187500x_{34}^3 = 0$$

$$275000x_{29}+10410848532x_{34}^5+205321875x_{34}^4+20418750x_{34}^3+1237500x_{34}^2+2200000x_{34}-275000 = 0$$

$$687500x_{30} - 907029396x_{34}^5 - 18046875x_{34}^4 - 2062500x_{34}^3 = 0$$

$$17875000x_{31} - 527388372x_{34}^5 - 46921875x_{34}^4 + 40218750x_{34}^3 - 26812500x_{34}^2 = 0$$
$$125000x_{32} - 785519028x_{34}^5 - 15796875x_{34}^4 - 1593750x_{34}^3 - 187500x_{34}^2 = 0$$
$$125000x_{33} - 342354852x_{34}^5 - 7359375x_{34}^4 - 843750x_{34}^3 - 187500x_{34}^2 = 0$$
$$84x_{34}^6 - 5x_{34}^5 = 0$$

Solution 1:

$$x_1 = 0$$
$$x_2 = 0$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 1$$
$$x_6 = 0$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = 0$$
$$x_10 = 1$$
$$x_11 = 1$$
$$x_12 = 1$$
$$x_13 = 0$$
$$x_14 = 0$$
$$x_15 = 0$$
$$x_16 = 0$$
$$x_17 = 0$$
$$x_18 = 0$$
$$x_19 = 0$$
$$x_20 = 0$$
$$x_21 = 0$$
$$x_22 = 0$$
$$x_23 = 0$$
$$x_24 = 1$$
$$x_25 = 1$$
$$x_26 = 1$$
$$x_27 = 0$$

$$x_28 = 0$$

$$x_29 = 1$$

$$x_30 = 0$$

$$x_31 = 0$$

$$x_32 = 0$$

$$x_33 = 0$$

$$x_34 = 0$$

Solution 2:

$$x_1 = 1/165$$

$$x_2 = 7/165$$

$$x_3 = 1/132$$

$$x_4 = 3/35$$

$$x_5 = 27/55$$

$$x_6 = 1/10$$

$$x_7 = 9/286$$

$$x_8 = 7/12$$

$$x_9 = 1/2310$$

$$x_10 = 5/7$$

$$x_11 = 4/5$$

$$x_12 = 9/10$$

$$x_13 = 1/2860$$

$$x_14 = 1/420$$

$$x_15 = 1/2310$$

$$x_16 = 1/10$$

$$x_17 = 1/858$$

$$x_18 = 1/70$$

$$x_19 = 2/55$$

$$x_20 = 1/660$$

$$x_21 = 1/12012$$

$$x_22 = 1/20$$

$$x_23 = 1/14$$

$$x_24 = 11/26$$

$$x_25 = 8/15$$
$$x_26 = 9/14$$
$$x_27 = 1/84$$
$$x_28 = 1/420$$
$$x_29 = 5/11$$
$$x_30 = 3/1540$$
$$x_31 = 7/1430$$
$$x_32 = 1/70$$
$$x_33 = 1/105$$
$$x_34 = 5/84$$

$\mathfrak{m}_{2A}(3, 15)$

m2A315 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_{11}] &= e_{14} \\
[e_2, e_{12}] &= 5e_{15} & [e_3, e_{10}] &= -e_{14} \\
[e_3, e_{11}] &= -4e_{15} & [e_4, e_9] &= e_{14} \\
[e_4, e_{10}] &= 3e_{15} & [e_5, e_8] &= -e_{14} \\
[e_5, e_9] &= -2e_{15} & [e_6, e_7] &= e_{14} \\
[e_6, e_8] &= e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3,15)$

m4A315 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_1,e_{14}]=e_{15} \qquad [e_2,e_9]=e_{12}$$
$$[e_2,e_{10}]=4e_{13} \qquad [e_2,e_{11}]=6e_{14}$$
$$[e_2,e_{12}]=0 \qquad [e_3,e_8]=-e_{12}$$
$$[e_3,e_9]=-3e_{13} \qquad [e_3,e_{10}]=-2e_{14}$$
$$[e_3,e_{11}]=6e_{15} \qquad [e_4,e_7]=e_{12}$$
$$[e_4,e_8]=2e_{13} \qquad [e_4,e_9]=-e_{14}$$
$$[e_4,e_{10}]=-8e_{15} \qquad [e_5,e_6]=-e_{12}$$
$$[e_5,e_7]=-e_{13} \qquad [e_5,e_8]=3e_{14}$$
$$[e_5,e_9]=7e_{15} \qquad [e_6,e_7]=-4e_{14}$$
$$[e_6,e_8]=-4e_{15}$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_1, e_{14}] = e_{15} && [e_2, e_9] = e_{12} \\
&[e_2, e_{10}] = 4e_{13} && [e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} \\
&[e_2, e_{12}] = \alpha_{2,12}^{15} e_{15} && [e_3, e_8] = -e_{12} \\
&[e_3, e_9] = -3e_{13} && [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14} \\
&[e_3, e_{11}] = \alpha_{3,11}^{15} e_{15} && [e_4, e_7] = e_{12} \\
&[e_4, e_8] = 2e_{13} && [e_4, e_9] = \alpha_{4,9}^{14} e_{14} \\
&[e_4, e_{10}] = \alpha_{4,10}^{15} e_{15} && [e_5, e_6] = -e_{12} \\
&[e_5, e_7] = -e_{13} && [e_5, e_8] = \alpha_{5,8}^{14} e_{14} \\
&[e_5, e_9] = \alpha_{5,9}^{15} e_{15} && [e_6, e_7] = \alpha_{6,7}^{14} e_{14} \\
&[e_6, e_8] = \alpha_{6,8}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}):&\quad -\alpha_{2,11}^{14} - \alpha_{3,10}^{14} + 4 &&= 0 \\
(e_1, e_3, e_9):&\quad -\alpha_{3,10}^{14} - \alpha_{4,9}^{14} - 3 &&= 0 \\
(e_1, e_4, e_8):&\quad -\alpha_{4,9}^{14} - \alpha_{5,8}^{14} + 2 &&= 0 \\
(e_1, e_5, e_7):&\quad -\alpha_{5,8}^{14} - \alpha_{6,7}^{14} - 1 &&= 0 \\
(e_1, e_2, e_{11}):&\quad \alpha_{2,11}^{14} - \alpha_{2,12}^{15} - \alpha_{3,11}^{15} &&= 0 \\
(e_1, e_3, e_{10}):&\quad \alpha_{3,10}^{14} - \alpha_{3,11}^{15} - \alpha_{4,10}^{15} &&= 0 \\
(e_1, e_4, e_9):&\quad -\alpha_{4,10}^{15} + \alpha_{4,9}^{14} - \alpha_{5,9}^{15} &&= 0 \\
(e_1, e_5, e_8):&\quad \alpha_{5,8}^{14} - \alpha_{5,9}^{15} - \alpha_{6,8}^{15} &&= 0 \\
(e_1, e_6, e_7):&\quad \alpha_{6,7}^{14} - \alpha_{6,8}^{15} &&= 0 \\
(e_2, e_3, e_8):&\quad -\alpha_{2,12}^{15} &&= 0 \\
(e_2, e_4, e_7):&\quad \alpha_{2,12}^{15} &&= 0 \\
(e_2, e_5, e_6):&\quad -\alpha_{2,12}^{15} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{15}_{4,10} = -8$$
$$\alpha^{15}_{5,9} = 7$$
$$\alpha^{14}_{3,10} = -2$$
$$\alpha^{14}_{6,7} = -4$$
$$\alpha^{15}_{2,12} = 0$$
$$\alpha^{14}_{5,8} = 3$$
$$\alpha^{15}_{6,8} = -4$$
$$\alpha^{14}_{4,9} = -1$$
$$\alpha^{14}_{2,11} = 6$$
$$\alpha^{15}_{3,11} = 6$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{15}_{4,10} \to x_1$$
$$\alpha^{15}_{5,9} \to x_2$$
$$\alpha^{14}_{3,10} \to x_3$$
$$\alpha^{14}_{6,7} \to x_4$$
$$\alpha^{15}_{2,12} \to x_5$$
$$\alpha^{14}_{5,8} \to x_6$$
$$\alpha^{15}_{6,8} \to x_7$$
$$\alpha^{14}_{4,9} \to x_8$$
$$\alpha^{14}_{2,11} \to x_9$$
$$\alpha^{15}_{3,11} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_{10}): &\quad -x_3 - x_9 + 4 && = 0\\
(e_1,e_3,e_9): &\quad -x_3 - x_8 - 3 && = 0\\
(e_1,e_4,e_8): &\quad -x_6 - x_8 + 2 && = 0\\
(e_1,e_5,e_7): &\quad -x_4 - x_6 - 1 && = 0\\
(e_1,e_2,e_{11}): &\quad -x_{10} - x_5 + x_9 && = 0\\
(e_1,e_3,e_{10}): &\quad -x_1 - x_{10} + x_3 && = 0\\
(e_1,e_4,e_9): &\quad -x_1 - x_2 + x_8 && = 0\\
(e_1,e_5,e_8): &\quad -x_2 + x_6 - x_7 && = 0\\
(e_1,e_6,e_7): &\quad x_4 - x_7 && = 0\\
(e_2,e_3,e_8): &\quad -x_5 && = 0\\
(e_2,e_4,e_7): &\quad x_5 && = 0\\
(e_2,e_5,e_6): &\quad -x_5 && = 0
\end{aligned}
$$

Groebner basis (10 variables, 10 linear, 0 nonlinear)

$$
\begin{gathered}
x_1 + 8 = 0\\
x_2 - 7 = 0\\
x_3 + 2 = 0\\
x_4 + 4 = 0\\
x_5 = 0\\
x_6 - 3 = 0\\
x_7 + 4 = 0\\
x_8 + 1 = 0\\
x_9 - 6 = 0\\
x_{10} - 6 = 0
\end{gathered}
$$

Solution 1:

$$
\begin{gathered}
x_1 = -8\\
x_2 = 7\\
x_3 = -2\\
x_4 = -4\\
x_5 = 0\\
x_6 = 3\\
x_7 = -4\\
x_8 = -1\\
x_9 = 6\\
x_10 = 6
\end{gathered}
$$

$\mathfrak{m}_{6A}(3,15)$

m6A315 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_1,e_{14}]=e_{15} \qquad [e_2,e_7]=e_{10}$$
$$[e_2,e_8]=3e_{11} \qquad [e_2,e_9]=\alpha_{2,9}^{12}e_{12}$$
$$[e_2,e_{10}]=\alpha_{2,10}^{13}e_{13} \qquad [e_2,e_{11}]=\alpha_{2,11}^{14}e_{14}$$
$$[e_2,e_{12}]=\alpha_{2,12}^{15}e_{15} \qquad [e_3,e_6]=-e_{10}$$
$$[e_3,e_7]=-2e_{11} \qquad [e_3,e_8]=\alpha_{3,8}^{12}e_{12}$$
$$[e_3,e_9]=\alpha_{3,9}^{13}e_{13} \qquad [e_3,e_{10}]=\alpha_{3,10}^{14}e_{14}$$
$$[e_3,e_{11}]=\alpha_{3,11}^{15}e_{15} \qquad [e_4,e_5]=e_{10}$$
$$[e_4,e_6]=e_{11} \qquad [e_4,e_7]=\alpha_{4,7}^{12}e_{12}$$
$$[e_4,e_8]=\alpha_{4,8}^{13}e_{13} \qquad [e_4,e_9]=\alpha_{4,9}^{14}e_{14}$$
$$[e_4,e_{10}]=\alpha_{4,10}^{15}e_{15} \qquad [e_5,e_6]=\alpha_{5,6}^{12}e_{12}$$
$$[e_5,e_7]=\alpha_{5,7}^{13}e_{13} \qquad [e_5,e_8]=\alpha_{5,8}^{14}e_{14}$$
$$[e_5,e_9]=\alpha_{5,9}^{15}e_{15} \qquad [e_6,e_7]=\alpha_{6,7}^{14}e_{14}$$
$$[e_6,e_8]=\alpha_{6,8}^{15}e_{15}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,10}^{13} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13} &&= 0\\
(e_1, e_2, e_{10}): &\quad \alpha_{2,10}^{13} - \alpha_{2,11}^{14} - \alpha_{3,10}^{14} &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{14} + \alpha_{3,9}^{13} - \alpha_{4,9}^{14} &&= 0\\
(e_1, e_4, e_8): &\quad \alpha_{4,8}^{13} - \alpha_{4,9}^{14} - \alpha_{5,8}^{14} &&= 0\\
(e_1, e_5, e_7): &\quad \alpha_{5,7}^{13} - \alpha_{5,8}^{14} - \alpha_{6,7}^{14} &&= 0\\
(e_2, e_3, e_7): &\quad -2\alpha_{2,11}^{14} - \alpha_{3,10}^{14} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,11}^{14} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{14} &&= 0\\
(e_1, e_2, e_{11}): &\quad \alpha_{2,11}^{14} - \alpha_{2,12}^{15} - \alpha_{3,11}^{15} &&= 0\\
(e_1, e_3, e_{10}): &\quad \alpha_{3,10}^{14} - \alpha_{3,11}^{15} - \alpha_{4,10}^{15} &&= 0\\
(e_1, e_4, e_9): &\quad -\alpha_{4,10}^{15} + \alpha_{4,9}^{14} - \alpha_{5,9}^{15} &&= 0\\
(e_1, e_5, e_8): &\quad \alpha_{5,8}^{14} - \alpha_{5,9}^{15} - \alpha_{6,8}^{15} &&= 0\\
(e_1, e_6, e_7): &\quad \alpha_{6,7}^{14} - \alpha_{6,8}^{15} &&= 0\\
(e_2, e_3, e_8): &\quad \alpha_{2,12}^{15}\alpha_{3,8}^{12} - 3\alpha_{3,11}^{15} &&= 0\\
(e_2, e_4, e_7): &\quad \alpha_{2,12}^{15}\alpha_{4,7}^{12} - \alpha_{4,10}^{15} &&= 0\\
(e_2, e_5, e_6): &\quad \alpha_{2,12}^{15}\alpha_{5,6}^{12} &&= 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,11}^{15} + \alpha_{4,10}^{15} &&= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,8}^{13} \to x_1$$

$$\alpha_{4,10}^{15} \to x_2$$

$$\alpha_{5,9}^{15} \to x_3$$

$$\alpha_{3,10}^{14} \to x_4$$

$$\alpha_{6,7}^{14} \to x_5$$

$$\alpha_{4,9}^{14} \to x_6$$

$$\alpha_{2,12}^{15} \to x_7$$

$$\alpha_{3,9}^{13} \to x_8$$

$$\alpha_{3,8}^{12} \to x_9$$

$$\alpha_{5,8}^{14} \to x_{10}$$

$$\alpha_{5,7}^{13} \to x_{11}$$

$$\alpha_{6,8}^{15} \to x_{12}$$

$$\alpha_{2,10}^{13} \to x_{13}$$

$$\alpha_{2,9}^{12} \to x_{14}$$

$$\alpha_{2,11}^{14} \to x_{15}$$

$$\alpha_{4,7}^{12} \to x_{16}$$

$$\alpha_{5,6}^{12} \to x_{17}$$

$$\alpha_{3,11}^{15} \to x_{18}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -x_{14}-x_9+3 &&=0\\
(e_1,e_3,e_7):&\quad -x_{16}-x_9-2 &&=0\\
(e_1,e_4,e_6):&\quad -x_{16}-x_{17}+1 &&=0\\
(e_1,e_2,e_9):&\quad -x_{13}+x_{14}-x_8 &&=0\\
(e_1,e_3,e_8):&\quad -x_1-x_8+x_9 &&=0\\
(e_1,e_4,e_7):&\quad -x_1-x_{11}+x_{16} &&=0\\
(e_1,e_5,e_6):&\quad -x_{11}+x_{17} &&=0\\
(e_2,e_3,e_6):&\quad -x_{13} &&=0\\
(e_2,e_4,e_5):&\quad x_{13} &&=0\\
(e_1,e_2,e_{10}):&\quad x_{13}-x_{15}-x_4 &&=0\\
(e_1,e_3,e_9):&\quad -x_4-x_6+x_8 &&=0\\
(e_1,e_4,e_8):&\quad x_1-x_{10}-x_6 &&=0\\
(e_1,e_5,e_7):&\quad -x_{10}+x_{11}-x_5 &&=0\\
(e_2,e_3,e_7):&\quad -2x_{15}-x_4 &&=0\\
(e_2,e_4,e_6):&\quad x_{15} &&=0\\
(e_3,e_4,e_5):&\quad x_4 &&=0\\
(e_1,e_2,e_{11}):&\quad x_{15}-x_{18}-x_7 &&=0\\
(e_1,e_3,e_{10}):&\quad -x_{18}-x_2+x_4 &&=0\\
(e_1,e_4,e_9):&\quad -x_2-x_3+x_6 &&=0\\
(e_1,e_5,e_8):&\quad x_{10}-x_{12}-x_3 &&=0\\
(e_1,e_6,e_7):&\quad -x_{12}+x_5 &&=0\\
(e_2,e_3,e_8):&\quad -3x_{18}+x_7x_9 &&=0\\
(e_2,e_4,e_7):&\quad x_{16}x_7-x_2 &&=0\\
(e_2,e_5,e_6):&\quad x_{17}x_7 &&=0\\
(e_3,e_4,e_6):&\quad x_{18}+x_2 &&=0
\end{aligned}
$$

Groebner basis (18 variables, 1 linear, 0 nonlinear)

$$1=0$$

$\mathfrak{m}_{8A}(3,15)$

m8A315 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= 2e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_2, e_9] &= \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] &= \alpha_{2,10}^{13} e_{13} & [e_2, e_{11}] &= \alpha_{2,11}^{14} e_{14} \\
[e_2, e_{12}] &= \alpha_{2,12}^{15} e_{15} & [e_3, e_4] &= -e_8 \\
[e_3, e_5] &= -e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] &= \alpha_{3,9}^{13} e_{13} & [e_3, e_{10}] &= \alpha_{3,10}^{14} e_{14} \\
[e_3, e_{11}] &= \alpha_{3,11}^{15} e_{15} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11} & [e_4, e_7] &= \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] &= \alpha_{4,8}^{13} e_{13} & [e_4, e_9] &= \alpha_{4,9}^{14} e_{14} \\
[e_4, e_{10}] &= \alpha_{4,10}^{15} e_{15} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] &= \alpha_{5,7}^{13} e_{13} & [e_5, e_8] &= \alpha_{5,8}^{14} e_{14} \\
[e_5, e_9] &= \alpha_{5,9}^{15} e_{15} & [e_6, e_7] &= \alpha_{6,7}^{14} e_{14} \\
[e_6, e_8] &= \alpha_{6,8}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4):&\quad -\alpha_{2,8}^{11} &&= 0\\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0\\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0\\
(e_2, e_3, e_5):&\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_2, e_9):&\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8):&\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7):&\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6):&\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6):&\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - 2\alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_5):&\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_2, e_{10}):&\quad \alpha_{2,10}^{13} - \alpha_{2,11}^{14} - \alpha_{3,10}^{14} &&= 0\\
(e_1, e_3, e_9):&\quad -\alpha_{3,10}^{14} + \alpha_{3,9}^{13} - \alpha_{4,9}^{14} &&= 0\\
(e_1, e_4, e_8):&\quad \alpha_{4,8}^{13} - \alpha_{4,9}^{14} - \alpha_{5,8}^{14} &&= 0\\
(e_1, e_5, e_7):&\quad \alpha_{5,7}^{13} - \alpha_{5,8}^{14} - \alpha_{6,7}^{14} &&= 0\\
(e_2, e_3, e_7):&\quad \alpha_{2,11}^{14}\alpha_{3,7}^{11} - \alpha_{2,7}^{10}\alpha_{3,10}^{14} &&= 0\\
(e_2, e_4, e_6):&\quad \alpha_{2,11}^{14}\alpha_{4,6}^{11} - 2\alpha_{4,9}^{14} &&= 0\\
(e_3, e_4, e_5):&\quad \alpha_{3,10}^{14}\alpha_{4,5}^{10} + \alpha_{4,9}^{14} - \alpha_{5,8}^{14} &&= 0\\
(e_1, e_2, e_{11}):&\quad \alpha_{2,11}^{14} - \alpha_{2,12}^{15} - \alpha_{3,11}^{15} &&= 0\\
(e_1, e_3, e_{10}):&\quad \alpha_{3,10}^{14} - \alpha_{3,11}^{15} - \alpha_{4,10}^{15} &&= 0\\
(e_1, e_4, e_9):&\quad -\alpha_{4,10}^{15} + \alpha_{4,9}^{14} - \alpha_{5,9}^{15} &&= 0\\
(e_1, e_5, e_8):&\quad \alpha_{5,8}^{14} - \alpha_{5,9}^{15} - \alpha_{6,8}^{15} &&= 0\\
(e_1, e_6, e_7):&\quad \alpha_{6,7}^{14} - \alpha_{6,8}^{15} &&= 0\\
(e_2, e_3, e_8):&\quad \alpha_{2,12}^{15}\alpha_{3,8}^{12} - \alpha_{2,8}^{11}\alpha_{3,11}^{15} &&= 0\\
(e_2, e_4, e_7):&\quad \alpha_{2,12}^{15}\alpha_{4,7}^{12} - \alpha_{2,7}^{10}\alpha_{4,10}^{15} &&= 0\\
(e_2, e_5, e_6):&\quad \alpha_{2,12}^{15}\alpha_{5,6}^{12} - 2\alpha_{5,9}^{15} + \alpha_{6,8}^{15} &&= 0\\
(e_3, e_4, e_6):&\quad \alpha_{3,11}^{15}\alpha_{4,6}^{11} - \alpha_{3,6}^{10}\alpha_{4,10}^{15} - \alpha_{6,8}^{15} &&= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,8}^{11} \to x_1$$

$$\alpha_{3,6}^{10} \to x_2$$

$$\alpha_{2,9}^{12} \to x_3$$

$$\alpha_{5,9}^{15} \to x_4$$

$$\alpha_{4,10}^{15} \to x_5$$

$$\alpha_{6,7}^{14} \to x_6$$

$$\alpha_{5,8}^{14} \to x_7$$

$$\alpha_{4,5}^{10} \to x_8$$

$$\alpha_{4,9}^{14} \to x_9$$

$$\alpha_{2,11}^{14} \to x_{10}$$

$$\alpha_{5,6}^{12} \to x_{11}$$

$$\alpha_{3,10}^{14} \to x_{12}$$

$$\alpha_{2,12}^{15} \to x_{13}$$

$$\alpha_{3,9}^{13} \to x_{14}$$

$$\alpha_{5,7}^{13} \to x_{15}$$

$$\alpha_{2,7}^{10} \to x_{16}$$

$$\alpha_{4,6}^{11} \to x_{17}$$

$$\alpha_{3,7}^{11} \to x_{18}$$

$$\alpha_{6,8}^{15} \to x_{19}$$

$$\alpha_{2,10}^{13} \to x_{20}$$

$$\alpha_{3,8}^{12} \to x_{21}$$

$$\alpha_{4,7}^{12} \to x_{22}$$

$$\alpha_{4,8}^{13} \to x_{23}$$

$$\alpha_{3,11}^{15} \to x_{24}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{16} - x_2 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_2 - x_8 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad -x_1 + x_{16} - x_{18} &&= 0\\
(e_1, e_3, e_6): &\quad -x_{17} - x_{18} + x_2 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{17} + x_8 &&= 0\\
(e_2, e_3, e_4): &\quad -x_1 &&= 0\\
(e_1, e_2, e_8): &\quad x_1 - x_{21} - x_3 &&= 0\\
(e_1, e_3, e_7): &\quad x_{18} - x_{21} - x_{22} &&= 0\\
(e_1, e_4, e_6): &\quad -x_{11} + x_{17} - x_{22} &&= 0\\
(e_2, e_3, e_5): &\quad -x_{21} - x_3 &&= 0\\
(e_1, e_2, e_9): &\quad -x_{14} - x_{20} + x_3 &&= 0\\
(e_1, e_3, e_8): &\quad -x_{14} + x_{21} - x_{23} &&= 0\\
(e_1, e_4, e_7): &\quad -x_{15} + x_{22} - x_{23} &&= 0\\
(e_1, e_5, e_6): &\quad x_{11} - x_{15} &&= 0\\
(e_2, e_3, e_6): &\quad -2x_{14} + x_2 x_{20} &&= 0\\
(e_2, e_4, e_5): &\quad x_{20} x_8 - x_{23} &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_{10} - x_{12} + x_{20} &&= 0\\
(e_1, e_3, e_9): &\quad -x_{12} + x_{14} - x_9 &&= 0\\
(e_1, e_4, e_8): &\quad x_{23} - x_7 - x_9 &&= 0\\
(e_1, e_5, e_7): &\quad x_{15} - x_6 - x_7 &&= 0\\
(e_2, e_3, e_7): &\quad x_{10} x_{18} - x_{12} x_{16} &&= 0\\
(e_2, e_4, e_6): &\quad x_{10} x_{17} - 2x_9 &&= 0\\
(e_3, e_4, e_5): &\quad x_{12} x_8 - x_7 + x_9 &&= 0\\
(e_1, e_2, e_{11}): &\quad x_{10} - x_{13} - x_{24} &&= 0\\
(e_1, e_3, e_{10}): &\quad x_{12} - x_{24} - x_5 &&= 0\\
(e_1, e_4, e_9): &\quad -x_4 - x_5 + x_9 &&= 0\\
(e_1, e_5, e_8): &\quad -x_{19} - x_4 + x_7 &&= 0\\
(e_1, e_6, e_7): &\quad -x_{19} + x_6 &&= 0\\
(e_2, e_3, e_8): &\quad -x_1 x_{24} + x_{13} x_{21} &&= 0\\
(e_2, e_4, e_7): &\quad x_{13} x_{22} - x_{16} x_5 &&= 0\\
(e_2, e_5, e_6): &\quad x_{11} x_{13} + x_{19} - 2x_4 &&= 0\\
(e_3, e_4, e_6): &\quad x_{17} x_{24} - x_{19} - x_2 x_5 &&= 0
\end{aligned}$$

Groebner basis (24 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{10A}(3, 15)$

m10A315 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{2e_8}{5}$$
$$[e_2, e_6] = -\frac{e_9}{5} \qquad [e_2, e_7] = -\frac{e_{10}}{2}$$
$$[e_2, e_8] = -\frac{e_{11}}{2} \qquad [e_2, e_9] = -\frac{e_{12}}{5}$$
$$[e_2, e_{10}] = \frac{2e_{13}}{5} \qquad [e_2, e_{11}] = e_{14}$$
$$[e_2, e_{12}] = e_{15} \qquad [e_3, e_4] = \frac{3e_8}{5}$$
$$[e_3, e_5] = \frac{3e_9}{5} \qquad [e_3, e_6] = \frac{3e_{10}}{10}$$
$$[e_3, e_7] = 0 \qquad [e_3, e_8] = -\frac{3e_{12}}{10}$$
$$[e_3, e_9] = -\frac{3e_{13}}{5} \qquad [e_3, e_{10}] = -\frac{3e_{14}}{5}$$
$$[e_3, e_{11}] = 0 \qquad [e_4, e_5] = \frac{3e_{10}}{10}$$
$$[e_4, e_6] = \frac{3e_{11}}{10} \qquad [e_4, e_7] = \frac{3e_{12}}{10}$$
$$[e_4, e_8] = \frac{3e_{13}}{10} \qquad [e_4, e_9] = 0$$
$$[e_4, e_{10}] = -\frac{3e_{15}}{5} \qquad [e_5, e_6] = 0$$
$$[e_5, e_7] = 0 \qquad [e_5, e_8] = \frac{3e_{14}}{10}$$
$$[e_5, e_9] = \frac{3e_{15}}{5} \qquad [e_6, e_7] = -\frac{3e_{14}}{10}$$
$$[e_6, e_8] = -\frac{3e_{15}}{10}$$

Solution 2

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_3] = e_6 \\
[e_2, e_4] = e_7 & [e_2, e_5] = \frac{6e_8}{7} \\
[e_2, e_6] = \frac{5e_9}{7} & [e_2, e_7] = \frac{25e_{10}}{42} \\
[e_2, e_8] = \frac{e_{11}}{2} & [e_2, e_9] = \frac{14e_{12}}{33} \\
[e_2, e_{10}] = \frac{4e_{13}}{11} & [e_2, e_{11}] = \frac{45e_{14}}{143} \\
[e_2, e_{12}] = \frac{25e_{15}}{91} & [e_3, e_4] = \frac{e_8}{7} \\
[e_3, e_5] = \frac{e_9}{7} & [e_3, e_6] = \frac{5e_{10}}{42} \\
[e_3, e_7] = \frac{2e_{11}}{21} & [e_3, e_8] = \frac{5e_{12}}{66} \\
[e_3, e_9] = \frac{2e_{13}}{33} & [e_3, e_{10}] = \frac{7e_{14}}{143} \\
[e_3, e_{11}] = \frac{40e_{15}}{1001} & [e_4, e_5] = \frac{e_{10}}{42} \\
[e_4, e_6] = \frac{e_{11}}{42} & [e_4, e_7] = \frac{3e_{12}}{154} \\
[e_4, e_8] = \frac{e_{13}}{66} & [e_4, e_9] = \frac{5e_{14}}{429} \\
[e_4, e_{10}] = \frac{9e_{15}}{1001} & [e_5, e_6] = \frac{e_{12}}{231} \\
[e_5, e_7] = \frac{e_{13}}{231} & [e_5, e_8] = \frac{e_{14}}{286} \\
[e_5, e_9] = \frac{8e_{15}}{3003} & [e_6, e_7] = \frac{5e_{14}}{6006} \\
[e_6, e_8] = \frac{5e_{15}}{6006} &
\end{array}
$$

Solution 3

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = e_8$$
$$[e_2, e_6] = e_9 \qquad [e_2, e_7] = e_{10}$$
$$[e_2, e_8] = e_{11} \qquad [e_2, e_9] = e_{12}$$
$$[e_2, e_{10}] = e_{13} \qquad [e_2, e_{11}] = e_{14}$$
$$[e_2, e_{12}] = e_{15} \qquad [e_3, e_4] = 0$$
$$[e_3, e_5] = 0 \qquad [e_3, e_6] = 0$$
$$[e_3, e_7] = 0 \qquad [e_3, e_8] = 0$$
$$[e_3, e_9] = 0 \qquad [e_3, e_{10}] = 0$$
$$[e_3, e_{11}] = 0 \qquad [e_4, e_5] = 0$$
$$[e_4, e_6] = 0 \qquad [e_4, e_7] = 0$$
$$[e_4, e_8] = 0 \qquad [e_4, e_9] = 0$$
$$[e_4, e_{10}] = 0 \qquad [e_5, e_6] = 0$$
$$[e_5, e_7] = 0 \qquad [e_5, e_8] = 0$$
$$[e_5, e_9] = 0 \qquad [e_6, e_7] = 0$$
$$[e_6, e_8] = 0$$

Solution 4

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_1,e_{14}]=e_{15} \qquad [e_2,e_3]=e_6$$
$$[e_2,e_4]=e_7 \qquad [e_2,e_5]=e_8\left(\frac{3\sqrt{11}}{7}+\frac{13}{7}\right)$$
$$[e_2,e_6]=e_9\left(\frac{19}{7}+\frac{6\sqrt{11}}{7}\right) \qquad [e_2,e_7]=e_{10}\left(1+\frac{3\sqrt{11}}{7}\right)$$
$$[e_2,e_8]=e_{11}\left(-\frac{23}{7}-\frac{6\sqrt{11}}{7}\right) \qquad [e_2,e_9]=e_{12}\left(-\frac{31}{5}-\frac{9\sqrt{11}}{5}\right)$$
$$[e_2,e_{10}]=e_{13}\left(-\frac{6\sqrt{11}}{5}-\frac{19}{5}\right) \qquad [e_2,e_{11}]=e_{14}\left(-\frac{3\sqrt{11}}{5}-\frac{7}{5}\right)$$
$$[e_2,e_{12}]=e_{15}\left(-\frac{30\sqrt{11}}{7}-\frac{95}{7}\right) \qquad [e_3,e_4]=e_8\left(-\frac{3\sqrt{11}}{7}-\frac{6}{7}\right)$$
$$[e_3,e_5]=e_9\left(-\frac{3\sqrt{11}}{7}-\frac{6}{7}\right) \qquad [e_3,e_6]=e_{10}\left(\frac{3\sqrt{11}}{7}+\frac{12}{7}\right)$$
$$[e_3,e_7]=e_{11}\left(\frac{9\sqrt{11}}{7}+\frac{30}{7}\right) \qquad [e_3,e_8]=e_{12}\left(\frac{102}{35}+\frac{33\sqrt{11}}{35}\right)$$
$$[e_3,e_9]=e_{13}\left(-\frac{12}{5}-\frac{3\sqrt{11}}{5}\right) \qquad [e_3,e_{10}]=e_{14}\left(-\frac{12}{5}-\frac{3\sqrt{11}}{5}\right)$$
$$[e_3,e_{11}]=e_{15}\left(\frac{426}{35}+\frac{129\sqrt{11}}{35}\right) \qquad [e_4,e_5]=e_{10}\left(-\frac{6\sqrt{11}}{7}-\frac{18}{7}\right)$$
$$[e_4,e_6]=e_{11}\left(-\frac{6\sqrt{11}}{7}-\frac{18}{7}\right) \qquad [e_4,e_7]=e_{12}\left(\frac{12\sqrt{11}}{35}+\frac{48}{35}\right)$$
$$[e_4,e_8]=e_{13}\left(\frac{54\sqrt{11}}{35}+\frac{186}{35}\right) \qquad [e_4,e_9]=0$$
$$[e_4,e_{10}]=e_{15}\left(-\frac{102}{7}-\frac{30\sqrt{11}}{7}\right) \qquad [e_5,e_6]=e_{12}\left(-\frac{6\sqrt{11}}{5}-\frac{138}{35}\right)$$
$$[e_5,e_7]=e_{13}\left(-\frac{6\sqrt{11}}{5}-\frac{138}{35}\right) \qquad [e_5,e_8]=e_{14}\left(\frac{54\sqrt{11}}{35}+\frac{186}{35}\right)$$
$$[e_5,e_9]=e_{15}\left(\frac{30\sqrt{11}}{7}+\frac{102}{7}\right) \qquad [e_6,e_7]=e_{14}\left(-\frac{324}{35}-\frac{96\sqrt{11}}{35}\right)$$
$$[e_6,e_8]=e_{15}\left(-\frac{324}{35}-\frac{96\sqrt{11}}{35}\right)$$

Solution 5

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = e_8 \left(\frac{13}{7} - \frac{3\sqrt{11}}{7}\right)$$
$$[e_2, e_6] = e_9 \left(\frac{19}{7} - \frac{6\sqrt{11}}{7}\right) \qquad [e_2, e_7] = e_{10} \left(1 - \frac{3\sqrt{11}}{7}\right)$$
$$[e_2, e_8] = e_{11} \left(-\frac{23}{7} + \frac{6\sqrt{11}}{7}\right) \qquad [e_2, e_9] = e_{12} \left(-\frac{31}{5} + \frac{9\sqrt{11}}{5}\right)$$
$$[e_2, e_{10}] = e_{13} \left(-\frac{19}{5} + \frac{6\sqrt{11}}{5}\right) \qquad [e_2, e_{11}] = e_{14} \left(-\frac{7}{5} + \frac{3\sqrt{11}}{5}\right)$$
$$[e_2, e_{12}] = e_{15} \left(-\frac{95}{7} + \frac{30\sqrt{11}}{7}\right) \qquad [e_3, e_4] = e_8 \left(-\frac{6}{7} + \frac{3\sqrt{11}}{7}\right)$$
$$[e_3, e_5] = e_9 \left(-\frac{6}{7} + \frac{3\sqrt{11}}{7}\right) \qquad [e_3, e_6] = e_{10} \left(\frac{12}{7} - \frac{3\sqrt{11}}{7}\right)$$
$$[e_3, e_7] = e_{11} \left(\frac{30}{7} - \frac{9\sqrt{11}}{7}\right) \qquad [e_3, e_8] = e_{12} \left(\frac{102}{35} - \frac{33\sqrt{11}}{35}\right)$$
$$[e_3, e_9] = e_{13} \left(-\frac{12}{5} + \frac{3\sqrt{11}}{5}\right) \qquad [e_3, e_{10}] = e_{14} \left(-\frac{12}{5} + \frac{3\sqrt{11}}{5}\right)$$
$$[e_3, e_{11}] = e_{15} \left(\frac{426}{35} - \frac{129\sqrt{11}}{35}\right) \qquad [e_4, e_5] = e_{10} \left(-\frac{18}{7} + \frac{6\sqrt{11}}{7}\right)$$
$$[e_4, e_6] = e_{11} \left(-\frac{18}{7} + \frac{6\sqrt{11}}{7}\right) \qquad [e_4, e_7] = e_{12} \left(\frac{48}{35} - \frac{12\sqrt{11}}{35}\right)$$
$$[e_4, e_8] = e_{13} \left(\frac{186}{35} - \frac{54\sqrt{11}}{35}\right) \qquad [e_4, e_9] = 0$$
$$[e_4, e_{10}] = e_{15} \left(-\frac{102}{7} + \frac{30\sqrt{11}}{7}\right) \qquad [e_5, e_6] = e_{12} \left(-\frac{138}{35} + \frac{6\sqrt{11}}{5}\right)$$
$$[e_5, e_7] = e_{13} \left(-\frac{138}{35} + \frac{6\sqrt{11}}{5}\right) \qquad [e_5, e_8] = e_{14} \left(\frac{186}{35} - \frac{54\sqrt{11}}{35}\right)$$
$$[e_5, e_9] = e_{15} \left(\frac{102}{7} - \frac{30\sqrt{11}}{7}\right) \qquad [e_6, e_7] = e_{14} \left(-\frac{324}{35} + \frac{96\sqrt{11}}{35}\right)$$
$$[e_6, e_8] = e_{15} \left(-\frac{324}{35} + \frac{96\sqrt{11}}{35}\right)$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_3] = e_6 \\
[e_2, e_4] = e_7 & [e_2, e_5] = \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] = \alpha_{2,6}^{9} e_9 & [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] = \alpha_{2,8}^{11} e_{11} & [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} & [e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} \\
[e_2, e_{12}] = \alpha_{2,12}^{15} e_{15} & [e_3, e_4] = \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] = \alpha_{3,5}^{9} e_9 & [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] = \alpha_{3,7}^{11} e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14} \\
[e_3, e_{11}] = \alpha_{3,11}^{15} e_{15} & [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] = \alpha_{4,6}^{11} e_{11} & [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] = \alpha_{4,8}^{13} e_{13} & [e_4, e_9] = \alpha_{4,9}^{14} e_{14} \\
[e_4, e_{10}] = \alpha_{4,10}^{15} e_{15} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} & [e_5, e_8] = \alpha_{5,8}^{14} e_{14} \\
[e_5, e_9] = \alpha_{5,9}^{15} e_{15} & [e_6, e_7] = \alpha_{6,7}^{14} e_{14} \\
[e_6, e_8] = \alpha_{6,8}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{8}_{2,5}-\alpha^{8}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{8}_{2,5}-\alpha^{9}_{2,6}-\alpha^{9}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{8}_{3,4}-\alpha^{9}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{9}_{2,6}-\alpha^{10}_{2,7}-\alpha^{10}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{9}_{3,5}-\alpha^{10}_{3,6}-\alpha^{10}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{10}_{2,7}-\alpha^{11}_{2,8}-\alpha^{11}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{10}_{3,6}-\alpha^{11}_{3,7}-\alpha^{11}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{10}_{4,5}-\alpha^{11}_{4,6} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{11}_{2,8}\alpha^{8}_{3,4}-\alpha^{11}_{3,7}+\alpha^{11}_{4,6} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{11}_{2,8}-\alpha^{12}_{2,9}-\alpha^{12}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{11}_{3,7}-\alpha^{12}_{3,8}-\alpha^{12}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{11}_{4,6}-\alpha^{12}_{4,7}-\alpha^{12}_{5,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{8}_{2,5}\alpha^{12}_{3,8}+\alpha^{12}_{2,9}\alpha^{9}_{3,5}+\alpha^{12}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{13}_{2,10}+\alpha^{12}_{2,9}-\alpha^{13}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{12}_{3,8}-\alpha^{13}_{3,9}-\alpha^{13}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{12}_{4,7}-\alpha^{13}_{4,8}-\alpha^{13}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{12}_{5,6}-\alpha^{13}_{5,7} &&=0\\
(e_2,e_3,e_6):&\quad \alpha^{13}_{2,10}\alpha^{10}_{3,6}-\alpha^{9}_{2,6}\alpha^{13}_{3,9} &&=0\\
(e_2,e_4,e_5):&\quad \alpha^{13}_{2,10}\alpha^{10}_{4,5}-\alpha^{8}_{2,5}\alpha^{13}_{4,8}+\alpha^{13}_{5,7} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha^{13}_{2,10}-\alpha^{14}_{2,11}-\alpha^{14}_{3,10} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{14}_{3,10}+\alpha^{13}_{3,9}-\alpha^{14}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad \alpha^{13}_{4,8}-\alpha^{14}_{4,9}-\alpha^{14}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad \alpha^{13}_{5,7}-\alpha^{14}_{5,8}-\alpha^{14}_{6,7} &&=0\\
(e_2,e_3,e_7):&\quad \alpha^{14}_{2,11}\alpha^{11}_{3,7}-\alpha^{10}_{2,7}\alpha^{14}_{3,10}-\alpha^{14}_{6,7} &&=0\\
(e_2,e_4,e_6):&\quad \alpha^{14}_{2,11}\alpha^{11}_{4,6}-\alpha^{9}_{2,6}\alpha^{14}_{4,9}+\alpha^{14}_{6,7} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^{14}_{3,10}\alpha^{10}_{4,5}+\alpha^{8}_{3,4}\alpha^{14}_{5,8}-\alpha^{9}_{3,5}\alpha^{14}_{4,9} &&=0\\
(e_1,e_2,e_{11}):&\quad \alpha^{14}_{2,11}-\alpha^{15}_{2,12}-\alpha^{15}_{3,11} &&=0\\
(e_1,e_3,e_{10}):&\quad \alpha^{14}_{3,10}-\alpha^{15}_{3,11}-\alpha^{15}_{4,10} &&=0\\
(e_1,e_4,e_9):&\quad -\alpha^{15}_{4,10}+\alpha^{14}_{4,9}-\alpha^{15}_{5,9} &&=0\\
(e_1,e_5,e_8):&\quad \alpha^{14}_{5,8}-\alpha^{15}_{5,9}-\alpha^{15}_{6,8} &&=0\\
(e_1,e_6,e_7):&\quad \alpha^{14}_{6,7}-\alpha^{15}_{6,8} &&=0\\
(e_2,e_3,e_8):&\quad \alpha^{15}_{2,12}\alpha^{12}_{3,8}-\alpha^{11}_{2,8}\alpha^{15}_{3,11}-\alpha^{15}_{6,8} &&=0\\
(e_2,e_4,e_7):&\quad \alpha^{15}_{2,12}\alpha^{12}_{4,7}-\alpha^{10}_{2,7}\alpha^{15}_{4,10} &&=0\\
(e_2,e_5,e_6):&\quad \alpha^{15}_{2,12}\alpha^{12}_{5,6}+\alpha^{8}_{2,5}\alpha^{15}_{6,8}-\alpha^{9}_{2,6}\alpha^{15}_{5,9} &&=0\\
(e_3,e_4,e_6):&\quad \alpha^{15}_{3,11}\alpha^{11}_{4,6}+\alpha^{8}_{3,4}\alpha^{15}_{6,8}-\alpha^{10}_{3,6}\alpha^{15}_{4,10} &&=0
\end{aligned}$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,8}^{11} &= -1/2 \\
\alpha_{3,6}^{10} &= 3/10 \\
\alpha_{2,9}^{12} &= -1/5 \\
\alpha_{5,9}^{15} &= 3/5 \\
\alpha_{3,5}^{9} &= 3/5 \\
\alpha_{4,10}^{15} &= -3/5 \\
\alpha_{6,7}^{14} &= -3/10 \\
\alpha_{5,8}^{14} &= 3/10 \\
\alpha_{4,5}^{10} &= 3/10 \\
\alpha_{3,4}^{8} &= 3/5 \\
\alpha_{4,9}^{14} &= 0 \\
\alpha_{2,11}^{14} &= 1 \\
\alpha_{5,6}^{12} &= 0 \\
\alpha_{3,10}^{14} &= -3/5 \\
\alpha_{2,12}^{15} &= 1 \\
\alpha_{3,9}^{13} &= -3/5 \\
\alpha_{5,7}^{13} &= 0 \\
\alpha_{2,6}^{9} &= -1/5 \\
\alpha_{2,7}^{10} &= -1/2 \\
\alpha_{2,5}^{8} &= 2/5 \\
\alpha_{4,6}^{11} &= 3/10 \\
\alpha_{3,7}^{11} &= 0 \\
\alpha_{6,8}^{15} &= -3/10 \\
\alpha_{2,10}^{13} &= 2/5 \\
\alpha_{3,8}^{12} &= -3/10 \\
\alpha_{4,7}^{12} &= 3/10 \\
\alpha_{4,8}^{13} &= 3/10 \\
\alpha_{3,11}^{15} &= 0
\end{aligned}
$$

Solution 2:

$$\alpha^{11}_{2,8} = 1/2$$
$$\alpha^{10}_{3,6} = 5/42$$
$$\alpha^{12}_{2,9} = 14/33$$
$$\alpha^{15}_{5,9} = 8/3003$$
$$\alpha^{9}_{3,5} = 1/7$$
$$\alpha^{15}_{4,10} = 9/1001$$
$$\alpha^{14}_{6,7} = 5/6006$$
$$\alpha^{14}_{5,8} = 1/286$$
$$\alpha^{10}_{4,5} = 1/42$$
$$\alpha^{8}_{3,4} = 1/7$$
$$\alpha^{14}_{4,9} = 5/429$$
$$\alpha^{14}_{2,11} = 45/143$$
$$\alpha^{12}_{5,6} = 1/231$$
$$\alpha^{14}_{3,10} = 7/143$$
$$\alpha^{15}_{2,12} = 25/91$$
$$\alpha^{13}_{3,9} = 2/33$$
$$\alpha^{13}_{5,7} = 1/231$$
$$\alpha^{9}_{2,6} = 5/7$$
$$\alpha^{10}_{2,7} = 25/42$$
$$\alpha^{8}_{2,5} = 6/7$$
$$\alpha^{11}_{4,6} = 1/42$$
$$\alpha^{11}_{3,7} = 2/21$$
$$\alpha^{15}_{6,8} = 5/6006$$
$$\alpha^{13}_{2,10} = 4/11$$
$$\alpha^{12}_{3,8} = 5/66$$
$$\alpha^{12}_{4,7} = 3/154$$
$$\alpha^{13}_{4,8} = 1/66$$
$$\alpha^{15}_{3,11} = 40/1001$$

Solution 3:

$$\alpha_{2,8}^{11} = 1$$
$$\alpha_{3,6}^{10} = 0$$
$$\alpha_{2,9}^{12} = 1$$
$$\alpha_{5,9}^{15} = 0$$
$$\alpha_{3,5}^{9} = 0$$
$$\alpha_{4,10}^{15} = 0$$
$$\alpha_{6,7}^{14} = 0$$
$$\alpha_{5,8}^{14} = 0$$
$$\alpha_{4,5}^{10} = 0$$
$$\alpha_{3,4}^{8} = 0$$
$$\alpha_{4,9}^{14} = 0$$
$$\alpha_{2,11}^{14} = 1$$
$$\alpha_{5,6}^{12} = 0$$
$$\alpha_{3,10}^{14} = 0$$
$$\alpha_{2,12}^{15} = 1$$
$$\alpha_{3,9}^{13} = 0$$
$$\alpha_{5,7}^{13} = 0$$
$$\alpha_{2,6}^{9} = 1$$
$$\alpha_{2,7}^{10} = 1$$
$$\alpha_{2,5}^{8} = 1$$
$$\alpha_{4,6}^{11} = 0$$
$$\alpha_{3,7}^{11} = 0$$
$$\alpha_{6,8}^{15} = 0$$
$$\alpha_{2,10}^{13} = 1$$
$$\alpha_{3,8}^{12} = 0$$
$$\alpha_{4,7}^{12} = 0$$
$$\alpha_{4,8}^{13} = 0$$
$$\alpha_{3,11}^{15} = 0$$

Solution 4:

$$\alpha^{11}_{2,8} = -23/7 - 6 * sqrt(11)/7$$
$$\alpha^{10}_{3,6} = 3 * sqrt(11)/7 + 12/7$$
$$\alpha^{12}_{2,9} = -31/5 - 9 * sqrt(11)/5$$
$$\alpha^{15}_{5,9} = 30 * sqrt(11)/7 + 102/7$$
$$\alpha^{9}_{3,5} = -3 * sqrt(11)/7 - 6/7$$
$$\alpha^{15}_{4,10} = -102/7 - 30 * sqrt(11)/7$$
$$\alpha^{14}_{6,7} = -324/35 - 96 * sqrt(11)/35$$
$$\alpha^{14}_{5,8} = 54 * sqrt(11)/35 + 186/35$$
$$\alpha^{10}_{4,5} = -6 * sqrt(11)/7 - 18/7$$
$$\alpha^{8}_{3,4} = -3 * sqrt(11)/7 - 6/7$$
$$\alpha^{14}_{4,9} = 0$$
$$\alpha^{14}_{2,11} = -3 * sqrt(11)/5 - 7/5$$
$$\alpha^{12}_{5,6} = -6 * sqrt(11)/5 - 138/35$$
$$\alpha^{14}_{3,10} = -12/5 - 3 * sqrt(11)/5$$
$$\alpha^{15}_{2,12} = -30 * sqrt(11)/7 - 95/7$$
$$\alpha^{13}_{3,9} = -12/5 - 3 * sqrt(11)/5$$
$$\alpha^{13}_{5,7} = -6 * sqrt(11)/5 - 138/35$$
$$\alpha^{9}_{2,6} = 19/7 + 6 * sqrt(11)/7$$
$$\alpha^{10}_{2,7} = 1 + 3 * sqrt(11)/7$$
$$\alpha^{8}_{2,5} = 3 * sqrt(11)/7 + 13/7$$
$$\alpha^{11}_{4,6} = -6 * sqrt(11)/7 - 18/7$$
$$\alpha^{11}_{3,7} = 9 * sqrt(11)/7 + 30/7$$
$$\alpha^{15}_{6,8} = -324/35 - 96 * sqrt(11)/35$$
$$\alpha^{13}_{2,10} = -6 * sqrt(11)/5 - 19/5$$
$$\alpha^{12}_{3,8} = 102/35 + 33 * sqrt(11)/35$$
$$\alpha^{12}_{4,7} = 12 * sqrt(11)/35 + 48/35$$
$$\alpha^{13}_{4,8} = 54 * sqrt(11)/35 + 186/35$$
$$\alpha^{15}_{3,11} = 426/35 + 129 * sqrt(11)/35$$

Solution 5:

$$\alpha^{11}_{2,8} = -23/7 + 6 * sqrt(11)/7$$
$$\alpha^{10}_{3,6} = 12/7 - 3 * sqrt(11)/7$$
$$\alpha^{12}_{2,9} = -31/5 + 9 * sqrt(11)/5$$
$$\alpha^{15}_{5,9} = 102/7 - 30 * sqrt(11)/7$$
$$\alpha^{9}_{3,5} = -6/7 + 3 * sqrt(11)/7$$
$$\alpha^{15}_{4,10} = -102/7 + 30 * sqrt(11)/7$$
$$\alpha^{14}_{6,7} = -324/35 + 96 * sqrt(11)/35$$
$$\alpha^{14}_{5,8} = 186/35 - 54 * sqrt(11)/35$$
$$\alpha^{10}_{4,5} = -18/7 + 6 * sqrt(11)/7$$
$$\alpha^{8}_{3,4} = -6/7 + 3 * sqrt(11)/7$$
$$\alpha^{14}_{4,9} = 0$$
$$\alpha^{14}_{2,11} = -7/5 + 3 * sqrt(11)/5$$
$$\alpha^{12}_{5,6} = -138/35 + 6 * sqrt(11)/5$$
$$\alpha^{14}_{3,10} = -12/5 + 3 * sqrt(11)/5$$
$$\alpha^{15}_{2,12} = -95/7 + 30 * sqrt(11)/7$$
$$\alpha^{13}_{3,9} = -12/5 + 3 * sqrt(11)/5$$
$$\alpha^{13}_{5,7} = -138/35 + 6 * sqrt(11)/5$$
$$\alpha^{9}_{2,6} = 19/7 - 6 * sqrt(11)/7$$
$$\alpha^{10}_{2,7} = 1 - 3 * sqrt(11)/7$$
$$\alpha^{8}_{2,5} = 13/7 - 3 * sqrt(11)/7$$
$$\alpha^{11}_{4,6} = -18/7 + 6 * sqrt(11)/7$$
$$\alpha^{11}_{3,7} = 30/7 - 9 * sqrt(11)/7$$
$$\alpha^{15}_{6,8} = -324/35 + 96 * sqrt(11)/35$$
$$\alpha^{13}_{2,10} = -19/5 + 6 * sqrt(11)/5$$
$$\alpha^{12}_{3,8} = 102/35 - 33 * sqrt(11)/35$$
$$\alpha^{12}_{4,7} = 48/35 - 12 * sqrt(11)/35$$
$$\alpha^{13}_{4,8} = 186/35 - 54 * sqrt(11)/35$$
$$\alpha^{15}_{3,11} = 426/35 - 129 * sqrt(11)/35$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{2,8} \to x_1$$

$$\alpha^{10}_{3,6} \to x_2$$
$$\alpha^{12}_{2,9} \to x_3$$
$$\alpha^{15}_{5,9} \to x_4$$
$$\alpha^{9}_{3,5} \to x_5$$
$$\alpha^{15}_{4,10} \to x_6$$
$$\alpha^{14}_{6,7} \to x_7$$
$$\alpha^{14}_{5,8} \to x_8$$
$$\alpha^{10}_{4,5} \to x_9$$
$$\alpha^{8}_{3,4} \to x_{10}$$
$$\alpha^{14}_{4,9} \to x_{11}$$
$$\alpha^{14}_{2,11} \to x_{12}$$
$$\alpha^{12}_{5,6} \to x_{13}$$
$$\alpha^{14}_{3,10} \to x_{14}$$
$$\alpha^{15}_{2,12} \to x_{15}$$
$$\alpha^{13}_{3,9} \to x_{16}$$
$$\alpha^{13}_{5,7} \to x_{17}$$
$$\alpha^{9}_{2,6} \to x_{18}$$
$$\alpha^{10}_{2,7} \to x_{19}$$
$$\alpha^{8}_{2,5} \to x_{20}$$
$$\alpha^{11}_{4,6} \to x_{21}$$
$$\alpha^{11}_{3,7} \to x_{22}$$
$$\alpha^{15}_{6,8} \to x_{23}$$
$$\alpha^{13}_{2,10} \to x_{24}$$
$$\alpha^{12}_{3,8} \to x_{25}$$
$$\alpha^{12}_{4,7} \to x_{26}$$
$$\alpha^{13}_{4,8} \to x_{27}$$
$$\alpha^{15}_{3,11} \to x_{28}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{10}-x_{20}+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{18}+x_{20}-x_5 &&=0\\
(e_1,e_3,e_4):&\quad x_{10}-x_5 &&=0\\
(e_1,e_2,e_6):&\quad x_{18}-x_{19}-x_2 &&=0\\
(e_1,e_3,e_5):&\quad -x_2+x_5-x_9 &&=0\\
(e_1,e_2,e_7):&\quad -x_1+x_{19}-x_{22} &&=0\\
(e_1,e_3,e_6):&\quad x_2-x_{21}-x_{22} &&=0\\
(e_1,e_4,e_5):&\quad -x_{21}+x_9 &&=0\\
(e_2,e_3,e_4):&\quad x_1x_{10}+x_{21}-x_{22} &&=0\\
(e_1,e_2,e_8):&\quad x_1-x_{25}-x_3 &&=0\\
(e_1,e_3,e_7):&\quad x_{22}-x_{25}-x_{26} &&=0\\
(e_1,e_4,e_6):&\quad -x_{13}+x_{21}-x_{26} &&=0\\
(e_2,e_3,e_5):&\quad x_{13}-x_{20}x_{25}+x_3x_5 &&=0\\
(e_1,e_2,e_9):&\quad -x_{16}-x_{24}+x_3 &&=0\\
(e_1,e_3,e_8):&\quad -x_{16}+x_{25}-x_{27} &&=0\\
(e_1,e_4,e_7):&\quad -x_{17}+x_{26}-x_{27} &&=0\\
(e_1,e_5,e_6):&\quad x_{13}-x_{17} &&=0\\
(e_2,e_3,e_6):&\quad -x_{16}x_{18}+x_2x_{24} &&=0\\
(e_2,e_4,e_5):&\quad x_{17}-x_{20}x_{27}+x_{24}x_9 &&=0\\
(e_1,e_2,e_{10}):&\quad -x_{12}-x_{14}+x_{24} &&=0\\
(e_1,e_3,e_9):&\quad -x_{11}-x_{14}+x_{16} &&=0\\
(e_1,e_4,e_8):&\quad -x_{11}+x_{27}-x_8 &&=0\\
(e_1,e_5,e_7):&\quad x_{17}-x_7-x_8 &&=0\\
(e_2,e_3,e_7):&\quad x_{12}x_{22}-x_{14}x_{19}-x_7 &&=0\\
(e_2,e_4,e_6):&\quad -x_{11}x_{18}+x_{12}x_{21}+x_7 &&=0\\
(e_3,e_4,e_5):&\quad x_{10}x_8-x_{11}x_5+x_{14}x_9 &&=0\\
(e_1,e_2,e_{11}):&\quad x_{12}-x_{15}-x_{28} &&=0\\
(e_1,e_3,e_{10}):&\quad x_{14}-x_{28}-x_6 &&=0\\
(e_1,e_4,e_9):&\quad x_{11}-x_4-x_6 &&=0\\
(e_1,e_5,e_8):&\quad -x_{23}-x_4+x_8 &&=0\\
(e_1,e_6,e_7):&\quad -x_{23}+x_7 &&=0\\
(e_2,e_3,e_8):&\quad -x_1x_{28}+x_{15}x_{25}-x_{23} &&=0\\
(e_2,e_4,e_7):&\quad x_{15}x_{26}-x_{19}x_6 &&=0\\
(e_2,e_5,e_6):&\quad x_{13}x_{15}-x_{18}x_4+x_{20}x_{23} &&=0\\
(e_3,e_4,e_6):&\quad x_{10}x_{23}-x_2x_6+x_{21}x_{28} &&=0
\end{aligned}
$$

Groebner basis (28 variables, 0 linear, 29 nonlinear)

$20324812999680000000x_1+101624064999840000000x_{27}-1704840018080248531173947255x_{28}^8+4125177816316$

$1058584010400000000x_2-1058584010400000000x_{27}+239859666971964902576864l5x_{28}^8-58102409930065547$

$812992519987200000000x_{27}-237129391506308402504608054 85x_{28}^8+5746374986442453211306515 62092x_{28}^7+93$

$-8129925199872000000000x_{27}-5978127848506145335639485 85x_{28}^8+1452653536969579775000133 40412x_{28}^7+1401$

$-203248129996800000000 0x_{27}+1616241681608653461656048755x_{28}^8-3892395021724686289006326 6836x_{28}^7-12$

$2032481299968000000000x_{27}+2413400723520356901635268965x_{28}^8-5868809846526325729505045 5948x_{28}^7-4589$

$4234336041600000000x_{27}+2398596669719649025768641 5x_{28}^8-5810240993006554772923435 88x_{28}^7-1006737911$

$-675242956800000000x_{27}-6105440275041997127743505x_{28}^8+1486495683788954346844990 36x_{28}^7+7221911958$

$-1016240649998400000000x_{27}-6864111213222096030818470 85x_{28}^8+1685436331561606293000171 7612x_{28}^7-265$

$1016240649998400000000x_{10}-2032481299968000000000x_{27}+1616241681608653461656048755x_{28}^8-38923950217$

$675242956800000000x_{11}+6105440275041997127743505x_{28}^8-1486495683788954346844990 36x_{28}^7-722191195897$

$967848238080000000x_{12}-1676394265119043285354889 35x_{28}^8+4067301964701647029289940 932x_{28}^7+545342640$

$2032481299968000000000x_{13}-6864111213222096030818470 85x_{28}^8+1685436331561606293000171 7612x_{28}^7-26585$

$1016240649998400000000x_{14}+2032481299968000000000x_{27}+2413400723520356901635268965x_{28}^8-58688098465$

$967848238080000000x_{15}-1676394265119043285354889 35x_{28}^8+4067301964701647029289940 932x_{28}^7+545342640$

$6774937666560000000x_{16}+13549875333120000000x_{27}+2221512989942784979573777645x_{28}^8-540399056708$

$2032481299968000000000x_{17}-6864111213222096030818470 85x_{28}^8+1685436331561606293000171 7612x_{28}^7-26585$

$508120324992000000000x_{18}+2032481299968000000000x_{27}-1616241681608653461656048755x_{28}^8+389239502172$

$1016240649998400000000x_{19}+508120324992000000000x_{27}-5535136166148169988049993 35x_{28}^8+13362621396735$

$1016240649998400000000x_{20}+2032481299968000000000x_{27}-1616241681608653461656048755x_{28}^8+38923950217$

$1016240649998400000000x_{21}-1016240649998400000000x_{27}-6864111213222096030818470 85x_{28}^8+168543633156$

$20324812999680000000x_{22}+5978127848506145335639485 85x_{28}^8-1452653536969579775000133 40412x_{28}^7-140122$

$4234336041600000000x_{23}+4234336041600000000x_{27}+2398596669719649025768641 5x_{28}^8-58102409930065547$

$508120324992000000000x_{24}+1016240649998400000000x_{27}-7594369530114798797295534605x_{28}^8+184189303914$

$6774937666560000000x_{25}+6774937666560000000x_{27}+2221512989942784979573777645x_{28}^8-5403990567085$

$2032481299968000000000x_{26}-2032481299968000000000x_{27}-6864111213222096030818470 85x_{28}^8+168543633156$

$47424563665920000000 0x_{27}^2-14227369099776000000000x_{27}+3259154052236660331785816785x_{28}^8-811261088$

$112915627776000000 00x_{27}x_{28}+2941865979813930657372885x_{28}^8-7183385948991632078167137 2x_{28}^7+159536873$

$$35035x_{28}^9-854252x_{28}^8-10965x_{28}^7+1800x_{28}^6=0$$

Solution 1:

$$x_1 = -1/2$$
$$x_2 = 3/10$$
$$x_3 = -1/5$$
$$x_4 = 3/5$$
$$x_5 = 3/5$$
$$x_6 = -3/5$$
$$x_7 = -3/10$$
$$x_8 = 3/10$$
$$x_9 = 3/10$$
$$x_1 0 = 3/5$$
$$x_1 1 = 0$$
$$x_1 2 = 1$$
$$x_1 3 = 0$$
$$x_1 4 = -3/5$$
$$x_1 5 = 1$$
$$x_1 6 = -3/5$$
$$x_1 7 = 0$$
$$x_1 8 = -1/5$$
$$x_1 9 = -1/2$$
$$x_2 0 = 2/5$$
$$x_2 1 = 3/10$$
$$x_2 2 = 0$$
$$x_2 3 = -3/10$$
$$x_2 4 = 2/5$$
$$x_2 5 = -3/10$$
$$x_2 6 = 3/10$$
$$x_2 7 = 3/10$$
$$x_2 8 = 0$$

Solution 2:

$$x_1 = 1/2$$
$$x_2 = 5/42$$

$$x_3 = 14/33$$
$$x_4 = 8/3003$$
$$x_5 = 1/7$$
$$x_6 = 9/1001$$
$$x_7 = 5/6006$$
$$x_8 = 1/286$$
$$x_9 = 1/42$$
$$x_10 = 1/7$$
$$x_11 = 5/429$$
$$x_12 = 45/143$$
$$x_13 = 1/231$$
$$x_14 = 7/143$$
$$x_15 = 25/91$$
$$x_16 = 2/33$$
$$x_17 = 1/231$$
$$x_18 = 5/7$$
$$x_19 = 25/42$$
$$x_20 = 6/7$$
$$x_21 = 1/42$$
$$x_22 = 2/21$$
$$x_23 = 5/6006$$
$$x_24 = 4/11$$
$$x_25 = 5/66$$
$$x_26 = 3/154$$
$$x_27 = 1/66$$
$$x_28 = 40/1001$$

Solution 3:

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_4 = 0$$
$$x_5 = 0$$

$$x_6 = 0$$
$$x_7 = 0$$
$$x_8 = 0$$
$$x_9 = 0$$
$$x_10 = 0$$
$$x_11 = 0$$
$$x_12 = 1$$
$$x_13 = 0$$
$$x_14 = 0$$
$$x_15 = 1$$
$$x_16 = 0$$
$$x_17 = 0$$
$$x_18 = 1$$
$$x_19 = 1$$
$$x_20 = 1$$
$$x_21 = 0$$
$$x_22 = 0$$
$$x_23 = 0$$
$$x_24 = 1$$
$$x_25 = 0$$
$$x_26 = 0$$
$$x_27 = 0$$
$$x_28 = 0$$

Solution 4:

$$x_1 = -23/7 - 6 * sqrt(11)/7$$
$$x_2 = 3 * sqrt(11)/7 + 12/7$$
$$x_3 = -31/5 - 9 * sqrt(11)/5$$
$$x_4 = 30 * sqrt(11)/7 + 102/7$$
$$x_5 = -3 * sqrt(11)/7 - 6/7$$
$$x_6 = -102/7 - 30 * sqrt(11)/7$$
$$x_7 = -324/35 - 96 * sqrt(11)/35$$
$$x_8 = 54 * sqrt(11)/35 + 186/35$$

$$x_9 = -6 * sqrt(11)/7 - 18/7$$
$$x_10 = -3 * sqrt(11)/7 - 6/7$$
$$x_11 = 0$$
$$x_12 = -3 * sqrt(11)/5 - 7/5$$
$$x_13 = -6 * sqrt(11)/5 - 138/35$$
$$x_14 = -12/5 - 3 * sqrt(11)/5$$
$$x_15 = -30 * sqrt(11)/7 - 95/7$$
$$x_16 = -12/5 - 3 * sqrt(11)/5$$
$$x_17 = -6 * sqrt(11)/5 - 138/35$$
$$x_18 = 19/7 + 6 * sqrt(11)/7$$
$$x_19 = 1 + 3 * sqrt(11)/7$$
$$x_20 = 3 * sqrt(11)/7 + 13/7$$
$$x_21 = -6 * sqrt(11)/7 - 18/7$$
$$x_22 = 9 * sqrt(11)/7 + 30/7$$
$$x_23 = -324/35 - 96 * sqrt(11)/35$$
$$x_24 = -6 * sqrt(11)/5 - 19/5$$
$$x_25 = 102/35 + 33 * sqrt(11)/35$$
$$x_26 = 12 * sqrt(11)/35 + 48/35$$
$$x_27 = 54 * sqrt(11)/35 + 186/35$$
$$x_28 = 426/35 + 129 * sqrt(11)/35$$

Solution 5:

$$x_1 = -23/7 + 6 * sqrt(11)/7$$
$$x_2 = 12/7 - 3 * sqrt(11)/7$$
$$x_3 = -31/5 + 9 * sqrt(11)/5$$
$$x_4 = 102/7 - 30 * sqrt(11)/7$$
$$x_5 = -6/7 + 3 * sqrt(11)/7$$
$$x_6 = -102/7 + 30 * sqrt(11)/7$$
$$x_7 = -324/35 + 96 * sqrt(11)/35$$
$$x_8 = 186/35 - 54 * sqrt(11)/35$$
$$x_9 = -18/7 + 6 * sqrt(11)/7$$
$$x_10 = -6/7 + 3 * sqrt(11)/7$$
$$x_11 = 0$$

$$x_12 = -7/5 + 3 * sqrt(11)/5$$

$$x_13 = -138/35 + 6 * sqrt(11)/5$$

$$x_14 = -12/5 + 3 * sqrt(11)/5$$

$$x_15 = -95/7 + 30 * sqrt(11)/7$$

$$x_16 = -12/5 + 3 * sqrt(11)/5$$

$$x_17 = -138/35 + 6 * sqrt(11)/5$$

$$x_18 = 19/7 - 6 * sqrt(11)/7$$

$$x_19 = 1 - 3 * sqrt(11)/7$$

$$x_20 = 13/7 - 3 * sqrt(11)/7$$

$$x_21 = -18/7 + 6 * sqrt(11)/7$$

$$x_22 = 30/7 - 9 * sqrt(11)/7$$

$$x_23 = -324/35 + 96 * sqrt(11)/35$$

$$x_24 = -19/5 + 6 * sqrt(11)/5$$

$$x_25 = 102/35 - 33 * sqrt(11)/35$$

$$x_26 = 48/35 - 12 * sqrt(11)/35$$

$$x_27 = 186/35 - 54 * sqrt(11)/35$$

$$x_28 = 426/35 - 129 * sqrt(11)/35$$

$\mathfrak{m}_{1A}(4, 15)$

m1A415 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_1, e_{14}] &= e_{15} & [e_2, e_{11}] &= e_{15} \\ [e_3, e_{10}] &= -e_{15} & [e_4, e_9] &= e_{15} \\ [e_5, e_8] &= -e_{15} & [e_6, e_7] &= e_{15} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4,15)$

m3A415 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14} \\
[e_1,e_{14}] &= e_{15} & [e_2,e_9] &= e_{13} \\
[e_2,e_{10}] &= 4e_{14} & [e_2,e_{11}] &= \alpha_{2,11}^{15}e_{15} \\
[e_3,e_8] &= -e_{13} & [e_3,e_9] &= -3e_{14} \\
[e_3,e_{10}] &= \alpha_{3,10}^{15}e_{15} & [e_4,e_7] &= e_{13} \\
[e_4,e_8] &= 2e_{14} & [e_4,e_9] &= \alpha_{4,9}^{15}e_{15} \\
[e_5,e_6] &= -e_{13} & [e_5,e_7] &= -e_{14} \\
[e_5,e_8] &= \alpha_{5,8}^{15}e_{15} & [e_6,e_7] &= \alpha_{6,7}^{15}e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}): &\quad -\alpha_{2,11}^{15} - \alpha_{3,10}^{15} + 4 &&= 0 \\
(e_1,e_3,e_9): &\quad -\alpha_{3,10}^{15} - \alpha_{4,9}^{15} - 3 &&= 0 \\
(e_1,e_4,e_8): &\quad -\alpha_{4,9}^{15} - \alpha_{5,8}^{15} + 2 &&= 0 \\
(e_1,e_5,e_7): &\quad -\alpha_{5,8}^{15} - \alpha_{6,7}^{15} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{4,9}^{15} &\to x_1 \\
\alpha_{3,10}^{15} &\to x_2 \\
\alpha_{2,11}^{15} &\to x_3 \\
\alpha_{6,7}^{15} &\to x_4 \\
\alpha_{5,8}^{15} &\to x_5
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_{10}):& \quad -x_2 - x_3 + 4 &&= 0\\
(e_1, e_3, e_9):& \quad -x_1 - x_2 - 3 &&= 0\\
(e_1, e_4, e_8):& \quad -x_1 - x_5 + 2 &&= 0\\
(e_1, e_5, e_7):& \quad -x_4 - x_5 - 1 &&= 0
\end{aligned}
$$

Groebner basis (5 variables, 4 linear, 0 nonlinear)

$$x_1 + x_5 - 2 = 0$$

$$x_2 - x_5 + 5 = 0$$

$$x_3 + x_5 - 9 = 0$$

$$x_4 + x_5 + 1 = 0$$

$\mathfrak{m}_{5A}(4, 15)$

m5A415 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_1, e_{14}] &= e_{15} & [e_2, e_7] &= e_{11}\\
[e_2, e_8] &= 3e_{12} & [e_2, e_9] &= \alpha_{2,9}^{13} e_{13}\\
[e_2, e_{10}] &= \alpha_{2,10}^{14} e_{14} & [e_2, e_{11}] &= \alpha_{2,11}^{15} e_{15}\\
[e_3, e_6] &= -e_{11} & [e_3, e_7] &= -2e_{12}\\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_3, e_9] &= \alpha_{3,9}^{14} e_{14}\\
[e_3, e_{10}] &= \alpha_{3,10}^{15} e_{15} & [e_4, e_5] &= e_{11}\\
[e_4, e_6] &= e_{12} & [e_4, e_7] &= \alpha_{4,7}^{13} e_{13}\\
[e_4, e_8] &= \alpha_{4,8}^{14} e_{14} & [e_4, e_9] &= \alpha_{4,9}^{15} e_{15}\\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13} & [e_5, e_7] &= \alpha_{5,7}^{14} e_{14}\\
[e_5, e_8] &= \alpha_{5,8}^{15} e_{15} & [e_6, e_7] &= \alpha_{6,7}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha^{13}_{2,9} - \alpha^{13}_{3,8} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha^{13}_{3,8} - \alpha^{13}_{4,7} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha^{13}_{4,7} - \alpha^{13}_{5,6} + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha^{14}_{2,10} + \alpha^{13}_{2,9} - \alpha^{14}_{3,9} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha^{13}_{3,8} - \alpha^{14}_{3,9} - \alpha^{14}_{4,8} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha^{13}_{4,7} - \alpha^{14}_{4,8} - \alpha^{14}_{5,7} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha^{13}_{5,6} - \alpha^{14}_{5,7} &&= 0\\
(e_1, e_2, e_{10}): &\quad \alpha^{14}_{2,10} - \alpha^{15}_{2,11} - \alpha^{15}_{3,10} &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha^{15}_{3,10} + \alpha^{14}_{3,9} - \alpha^{15}_{4,9} &&= 0\\
(e_1, e_4, e_8): &\quad \alpha^{14}_{4,8} - \alpha^{15}_{4,9} - \alpha^{15}_{5,8} &&= 0\\
(e_1, e_5, e_7): &\quad \alpha^{14}_{5,7} - \alpha^{15}_{5,8} - \alpha^{15}_{6,7} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha^{15}_{2,11} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha^{15}_{2,11} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{14}_{4,8} \to x_1$$

$$\alpha^{13}_{3,8} \to x_2$$

$$\alpha^{15}_{5,8} \to x_3$$

$$\alpha^{14}_{2,10} \to x_4$$

$$\alpha^{14}_{5,7} \to x_5$$

$$\alpha^{15}_{4,9} \to x_6$$

$$\alpha^{15}_{3,10} \to x_7$$

$$\alpha^{13}_{4,7} \to x_8$$

$$\alpha^{13}_{2,9} \to x_9$$

$$\alpha^{15}_{2,11} \to x_{10}$$

$$\alpha^{13}_{5,6} \to x_{11}$$

$$\alpha^{15}_{6,7} \to x_{12}$$

$$\alpha^{14}_{3,9} \to x_{13}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8):&\quad -x_2 - x_9 + 3 &&= 0\\
(e_1, e_3, e_7):&\quad -x_2 - x_8 - 2 &&= 0\\
(e_1, e_4, e_6):&\quad -x_{11} - x_8 + 1 &&= 0\\
(e_1, e_2, e_9):&\quad -x_{13} - x_4 + x_9 &&= 0\\
(e_1, e_3, e_8):&\quad -x_1 - x_{13} + x_2 &&= 0\\
(e_1, e_4, e_7):&\quad -x_1 - x_5 + x_8 &&= 0\\
(e_1, e_5, e_6):&\quad x_{11} - x_5 &&= 0\\
(e_1, e_2, e_{10}):&\quad -x_{10} + x_4 - x_7 &&= 0\\
(e_1, e_3, e_9):&\quad x_{13} - x_6 - x_7 &&= 0\\
(e_1, e_4, e_8):&\quad x_1 - x_3 - x_6 &&= 0\\
(e_1, e_5, e_7):&\quad -x_{12} - x_3 + x_5 &&= 0\\
(e_2, e_3, e_6):&\quad -x_{10} &&= 0\\
(e_2, e_4, e_5):&\quad x_{10} &&= 0
\end{aligned}
$$

Groebner basis (13 variables, 12 linear, 0 nonlinear)

$$3x_1 + 2x_{13} + 5 = 0$$
$$-x_{13} + 3x_2 + 5 = 0$$
$$3x_{13} + x_3 - 3 = 0$$
$$4x_{13} + 3x_4 - 14 = 0$$
$$-x_{13} + 3x_5 - 4 = 0$$
$$-7x_{13} + 3x_6 + 14 = 0$$
$$4x_{13} + 3x_7 - 14 = 0$$
$$x_{13} + 3x_8 + 1 = 0$$
$$x_{13} + 3x_9 - 14 = 0$$
$$x_{10} = 0$$
$$3x_{11} - x_{13} - 4 = 0$$
$$3x_{12} - 10x_{13} + 5 = 0$$

$\mathfrak{m}_{7A}(4,15)$

m7A415 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_1,e_{14}]=e_{15} \qquad [e_2,e_5]=e_9$$
$$[e_2,e_6]=2e_{10} \qquad [e_2,e_7]=\alpha_{2,7}^{11}e_{11}$$
$$[e_2,e_8]=\alpha_{2,8}^{12}e_{12} \qquad [e_2,e_9]=\alpha_{2,9}^{13}e_{13}$$
$$[e_2,e_{10}]=\alpha_{2,10}^{14}e_{14} \qquad [e_2,e_{11}]=\alpha_{2,11}^{15}e_{15}$$
$$[e_3,e_4]=-e_9 \qquad [e_3,e_5]=-e_{10}$$
$$[e_3,e_6]=\alpha_{3,6}^{11}e_{11} \qquad [e_3,e_7]=\alpha_{3,7}^{12}e_{12}$$
$$[e_3,e_8]=\alpha_{3,8}^{13}e_{13} \qquad [e_3,e_9]=\alpha_{3,9}^{14}e_{14}$$
$$[e_3,e_{10}]=\alpha_{3,10}^{15}e_{15} \qquad [e_4,e_5]=\alpha_{4,5}^{11}e_{11}$$
$$[e_4,e_6]=\alpha_{4,6}^{12}e_{12} \qquad [e_4,e_7]=\alpha_{4,7}^{13}e_{13}$$
$$[e_4,e_8]=\alpha_{4,8}^{14}e_{14} \qquad [e_4,e_9]=\alpha_{4,9}^{15}e_{15}$$
$$[e_5,e_6]=\alpha_{5,6}^{13}e_{13} \qquad [e_5,e_7]=\alpha_{5,7}^{14}e_{14}$$
$$[e_5,e_8]=\alpha_{5,8}^{15}e_{15} \qquad [e_6,e_7]=\alpha_{6,7}^{15}e_{15}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,9}^{13} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,10}^{14} - \alpha_{3,9}^{14} &&= 0\\
(e_1, e_2, e_{10}): &\quad \alpha_{2,10}^{14} - \alpha_{2,11}^{15} - \alpha_{3,10}^{15} &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{15} + \alpha_{3,9}^{14} - \alpha_{4,9}^{15} &&= 0\\
(e_1, e_4, e_8): &\quad \alpha_{4,8}^{14} - \alpha_{4,9}^{15} - \alpha_{5,8}^{15} &&= 0\\
(e_1, e_5, e_7): &\quad \alpha_{5,7}^{14} - \alpha_{5,8}^{15} - \alpha_{6,7}^{15} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,11}^{15}\alpha_{3,6}^{11} - 2\alpha_{3,10}^{15} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,11}^{15}\alpha_{4,5}^{11} - \alpha_{4,9}^{15} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,7}^{12} \to x_1$$

$$\alpha_{4,8}^{14} \to x_2$$

$$\alpha_{3,8}^{13} \to x_3$$

$$\alpha_{5,8}^{15} \to x_4$$

$$\alpha_{2,10}^{14} \to x_5$$

$$\alpha_{5,7}^{14} \to x_6$$

$$\alpha_{4,9}^{15} \to x_7$$

$$\alpha_{2,8}^{12} \to x_8$$
$$\alpha_{4,6}^{12} \to x_9$$
$$\alpha_{3,10}^{15} \to x_{10}$$
$$\alpha_{3,6}^{11} \to x_{11}$$
$$\alpha_{6,7}^{15} \to x_{12}$$
$$\alpha_{4,7}^{13} \to x_{13}$$
$$\alpha_{2,9}^{13} \to x_{14}$$
$$\alpha_{2,11}^{15} \to x_{15}$$
$$\alpha_{5,6}^{13} \to x_{16}$$
$$\alpha_{4,5}^{11} \to x_{17}$$
$$\alpha_{3,9}^{14} \to x_{18}$$
$$\alpha_{2,7}^{11} \to x_{19}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{11} - x_{19} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{11} - x_{17} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad -x_1 + x_{19} - x_8 &&= 0\\
(e_1, e_3, e_6): &\quad -x_1 + x_{11} - x_9 &&= 0\\
(e_1, e_4, e_5): &\quad x_{17} - x_9 &&= 0\\
(e_1, e_2, e_8): &\quad -x_{14} - x_3 + x_8 &&= 0\\
(e_1, e_3, e_7): &\quad x_1 - x_{13} - x_3 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{13} - x_{16} + x_9 &&= 0\\
(e_2, e_3, e_4): &\quad -x_{14} &&= 0\\
(e_1, e_2, e_9): &\quad x_{14} - x_{18} - x_5 &&= 0\\
(e_1, e_3, e_8): &\quad -x_{18} - x_2 + x_3 &&= 0\\
(e_1, e_4, e_7): &\quad x_{13} - x_2 - x_6 &&= 0\\
(e_1, e_5, e_6): &\quad x_{16} - x_6 &&= 0\\
(e_2, e_3, e_5): &\quad -x_{18} - x_5 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_{10} - x_{15} + x_5 &&= 0\\
(e_1, e_3, e_9): &\quad -x_{10} + x_{18} - x_7 &&= 0\\
(e_1, e_4, e_8): &\quad x_2 - x_4 - x_7 &&= 0\\
(e_1, e_5, e_7): &\quad -x_{12} - x_4 + x_6 &&= 0\\
(e_2, e_3, e_6): &\quad -2x_{10} + x_{11}x_{15} &&= 0\\
(e_2, e_4, e_5): &\quad x_{15}x_{17} - x_7 &&= 0
\end{aligned}$$

Groebner basis (19 variables, 17 linear, 1 nonlinear)

$$x_1 + 2x_{19} - 5 = 0$$
$$11x_{19} + x_2 - 23 = 0$$
$$-3x_{19} + x_3 + 5 = 0$$
$$x_{15} + 39x_{19} + x_4 - 79 = 0$$
$$14x_{19} + x_5 - 28 = 0$$
$$-6x_{19} + x_6 + 13 = 0$$
$$-x_{15} - 28x_{19} + x_7 + 56 = 0$$
$$-3x_{19} + x_8 + 5 = 0$$
$$-x_{19} + x_9 + 3 = 0$$
$$x_{10} + x_{15} + 14x_{19} - 28 = 0$$
$$x_{11} + x_{19} - 2 = 0$$
$$x_{12} - x_{15} - 45x_{19} + 92 = 0$$
$$x_{13} + 5x_{19} - 10 = 0$$
$$x_{14} = 0$$
$$x_{15}x_{19} - 4x_{15} - 28x_{19} + 56 = 0$$
$$x_{16} - 6x_{19} + 13 = 0$$
$$x_{17} - x_{19} + 3 = 0$$
$$x_{18} - 14x_{19} + 28 = 0$$

$\mathfrak{m}_{9A}(4, 15)$

m9A415 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_1, e_{14}] = e_{15} && [e_2, e_3] = e_7 \\
&[e_2, e_4] = e_8 && [e_2, e_5] = \alpha_{2,5}^{9} e_9 \\
&[e_2, e_6] = \alpha_{2,6}^{10} e_{10} && [e_2, e_7] = \alpha_{2,7}^{11} e_{11} \\
&[e_2, e_8] = \alpha_{2,8}^{12} e_{12} && [e_2, e_9] = \alpha_{2,9}^{13} e_{13} \\
&[e_2, e_{10}] = \alpha_{2,10}^{14} e_{14} && [e_2, e_{11}] = \alpha_{2,11}^{15} e_{15} \\
&[e_3, e_4] = \alpha_{3,4}^{9} e_9 && [e_3, e_5] = \alpha_{3,5}^{10} e_{10} \\
&[e_3, e_6] = \alpha_{3,6}^{11} e_{11} && [e_3, e_7] = \alpha_{3,7}^{12} e_{12} \\
&[e_3, e_8] = \alpha_{3,8}^{13} e_{13} && [e_3, e_9] = \alpha_{3,9}^{14} e_{14} \\
&[e_3, e_{10}] = \alpha_{3,10}^{15} e_{15} && [e_4, e_5] = \alpha_{4,5}^{11} e_{11} \\
&[e_4, e_6] = \alpha_{4,6}^{12} e_{12} && [e_4, e_7] = \alpha_{4,7}^{13} e_{13} \\
&[e_4, e_8] = \alpha_{4,8}^{14} e_{14} && [e_4, e_9] = \alpha_{4,9}^{15} e_{15} \\
&[e_5, e_6] = \alpha_{5,6}^{13} e_{13} && [e_5, e_7] = \alpha_{5,7}^{14} e_{14} \\
&[e_5, e_8] = \alpha_{5,8}^{15} e_{15} && [e_6, e_7] = \alpha_{6,7}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha_{2,5}^{9}-\alpha_{3,4}^{9}+1 &&=0\\
(e_1,e_2,e_5): &\quad \alpha_{2,5}^{9}-\alpha_{2,6}^{10}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_3,e_4): &\quad \alpha_{3,4}^{9}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_2,e_6): &\quad \alpha_{2,6}^{10}-\alpha_{2,7}^{11}-\alpha_{3,6}^{11} &&=0\\
(e_1,e_3,e_5): &\quad \alpha_{3,5}^{10}-\alpha_{3,6}^{11}-\alpha_{4,5}^{11} &&=0\\
(e_1,e_2,e_7): &\quad \alpha_{2,7}^{11}-\alpha_{2,8}^{12}-\alpha_{3,7}^{12} &&=0\\
(e_1,e_3,e_6): &\quad \alpha_{3,6}^{11}-\alpha_{3,7}^{12}-\alpha_{4,6}^{12} &&=0\\
(e_1,e_4,e_5): &\quad \alpha_{4,5}^{11}-\alpha_{4,6}^{12} &&=0\\
(e_1,e_2,e_8): &\quad \alpha_{2,8}^{12}-\alpha_{2,9}^{13}-\alpha_{3,8}^{13} &&=0\\
(e_1,e_3,e_7): &\quad \alpha_{3,7}^{12}-\alpha_{3,8}^{13}-\alpha_{4,7}^{13} &&=0\\
(e_1,e_4,e_6): &\quad \alpha_{4,6}^{12}-\alpha_{4,7}^{13}-\alpha_{5,6}^{13} &&=0\\
(e_2,e_3,e_4): &\quad \alpha_{2,9}^{13}\alpha_{3,4}^{9}-\alpha_{3,8}^{13}+\alpha_{4,7}^{13} &&=0\\
(e_1,e_2,e_9): &\quad -\alpha_{2,10}^{14}+\alpha_{2,9}^{13}-\alpha_{3,9}^{14} &&=0\\
(e_1,e_3,e_8): &\quad \alpha_{3,8}^{13}-\alpha_{3,9}^{14}-\alpha_{4,8}^{14} &&=0\\
(e_1,e_4,e_7): &\quad \alpha_{4,7}^{13}-\alpha_{4,8}^{14}-\alpha_{5,7}^{14} &&=0\\
(e_1,e_5,e_6): &\quad \alpha_{5,6}^{13}-\alpha_{5,7}^{14} &&=0\\
(e_2,e_3,e_5): &\quad \alpha_{2,10}^{14}\alpha_{3,5}^{10}-\alpha_{2,5}^{9}\alpha_{3,9}^{14}+\alpha_{5,7}^{14} &&=0\\
(e_1,e_2,e_{10}): &\quad \alpha_{2,10}^{14}-\alpha_{2,11}^{15}-\alpha_{3,10}^{15} &&=0\\
(e_1,e_3,e_9): &\quad -\alpha_{3,10}^{15}+\alpha_{3,9}^{14}-\alpha_{4,9}^{15} &&=0\\
(e_1,e_4,e_8): &\quad \alpha_{4,8}^{14}-\alpha_{4,9}^{15}-\alpha_{5,8}^{15} &&=0\\
(e_1,e_5,e_7): &\quad \alpha_{5,7}^{14}-\alpha_{5,8}^{15}-\alpha_{6,7}^{15} &&=0\\
(e_2,e_3,e_6): &\quad \alpha_{2,11}^{15}\alpha_{3,6}^{11}-\alpha_{2,6}^{10}\alpha_{3,10}^{15}+\alpha_{6,7}^{15} &&=0\\
(e_2,e_4,e_5): &\quad \alpha_{2,11}^{15}\alpha_{4,5}^{11}-\alpha_{2,5}^{9}\alpha_{4,9}^{15}+\alpha_{5,8}^{15} &&=0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{5,7}^{14} &\to x_1\\
\alpha_{2,11}^{15} &\to x_2\\
\alpha_{6,7}^{15} &\to x_3\\
\alpha_{3,9}^{14} &\to x_4\\
\alpha_{2,7}^{11} &\to x_5
\end{aligned}
$$

$$\alpha^{12}_{3,7} \to x_6$$
$$\alpha^{14}_{4,8} \to x_7$$
$$\alpha^{15}_{4,9} \to x_8$$
$$\alpha^{12}_{2,8} \to x_9$$
$$\alpha^{12}_{4,6} \to x_{10}$$
$$\alpha^{10}_{3,5} \to x_{11}$$
$$\alpha^{9}_{2,5} \to x_{12}$$
$$\alpha^{13}_{4,7} \to x_{13}$$
$$\alpha^{13}_{2,9} \to x_{14}$$
$$\alpha^{11}_{4,5} \to x_{15}$$
$$\alpha^{13}_{3,8} \to x_{16}$$
$$\alpha^{10}_{2,6} \to x_{17}$$
$$\alpha^{14}_{2,10} \to x_{18}$$
$$\alpha^{15}_{3,10} \to x_{19}$$
$$\alpha^{11}_{3,6} \to x_{20}$$
$$\alpha^{9}_{3,4} \to x_{21}$$
$$\alpha^{13}_{5,6} \to x_{22}$$
$$\alpha^{15}_{5,8} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -x_{12} - x_{21} + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_{11} + x_{12} - x_{17} &&= 0\\
(e_1, e_3, e_4):&\quad -x_{11} + x_{21} &&= 0\\
(e_1, e_2, e_6):&\quad x_{17} - x_{20} - x_5 &&= 0\\
(e_1, e_3, e_5):&\quad x_{11} - x_{15} - x_{20} &&= 0\\
(e_1, e_2, e_7):&\quad x_5 - x_6 - x_9 &&= 0\\
(e_1, e_3, e_6):&\quad -x_{10} + x_{20} - x_6 &&= 0\\
(e_1, e_4, e_5):&\quad -x_{10} + x_{15} &&= 0\\
(e_1, e_2, e_8):&\quad -x_{14} - x_{16} + x_9 &&= 0\\
(e_1, e_3, e_7):&\quad -x_{13} - x_{16} + x_6 &&= 0\\
(e_1, e_4, e_6):&\quad x_{10} - x_{13} - x_{22} &&= 0\\
(e_2, e_3, e_4):&\quad x_{13} + x_{14}x_{21} - x_{16} &&= 0\\
(e_1, e_2, e_9):&\quad x_{14} - x_{18} - x_4 &&= 0\\
(e_1, e_3, e_8):&\quad x_{16} - x_4 - x_7 &&= 0\\
(e_1, e_4, e_7):&\quad -x_1 + x_{13} - x_7 &&= 0\\
(e_1, e_5, e_6):&\quad -x_1 + x_{22} &&= 0\\
(e_2, e_3, e_5):&\quad x_1 + x_{11}x_{18} - x_{12}x_4 &&= 0\\
(e_1, e_2, e_{10}):&\quad x_{18} - x_{19} - x_2 &&= 0\\
(e_1, e_3, e_9):&\quad -x_{19} + x_4 - x_8 &&= 0\\
(e_1, e_4, e_8):&\quad -x_{23} + x_7 - x_8 &&= 0\\
(e_1, e_5, e_7):&\quad x_1 - x_{23} - x_3 &&= 0\\
(e_2, e_3, e_6):&\quad -x_{17}x_{19} + x_2x_{20} + x_3 &&= 0\\
(e_2, e_4, e_5):&\quad -x_{12}x_8 + x_{15}x_2 + x_{23} &&= 0
\end{aligned}
$$

Groebner basis (23 variables, 19 linear, 4 nonlinear)

$$x_1 - x_{22} = 0$$

$$x_2 + 15x_{20} - 8x_{21} + 9x_{22} + x_{23} - 1 = 0$$

$$-x_{22} + x_{23} + x_3 = 0$$

$$-4x_{20} + 3x_{21} - 3x_{22} + x_4 = 0$$

$$x_{20} + 2x_{21} + x_5 - 1 = 0$$

$$-2x_{20} + x_{21} + x_6 = 0$$

$$x_{20} - x_{21} + 2x_{22} + x_7 = 0$$

$$x_{20} - x_{21} + 2x_{22} + x_{23} + x_8 = 0$$

$$3x_{20} + x_{21} + x_9 - 1 = 0$$

$$x_{10} + x_{20} - x_{21} = 0$$

$$x_{11} - x_{21} = 0$$

$$x_{12} + x_{21} - 1 = 0$$

$$x_{13} + x_{20} - x_{21} + x_{22} = 0$$

$$x_{14} + 6x_{20} - x_{21} + x_{22} - 1 = 0$$

$$x_{15} + x_{20} - x_{21} = 0$$

$$x_{16} - 3x_{20} + 2x_{21} - x_{22} = 0$$

$$x_{17} + 2x_{21} - 1 = 0$$

$$x_{18} + 10x_{20} - 4x_{21} + 4x_{22} - 1 = 0$$

$$x_{19} - 5x_{20} + 4x_{21} - 5x_{22} - x_{23} = 0$$

$$600x_{20}^2+40x_{20}x_{23}+40x_{20}+585x_{21}^3-684x_{21}^2x_{22}-198x_{21}^2x_{23}-550x_{21}^2-117x_{21}x_{22}^2-18x_{21}x_{22}x_{23}+236x_{21}x_{22}+64x_{21}$$

$$6x_{20}x_{21} + 4x_{20} - x_{21}^2 + x_{21}x_{22} - 4x_{21} + 2x_{22} = 0$$

$$120x_{20}x_{22}+200x_{20}-195x_{21}^3+228x_{21}^2x_{22}+66x_{21}^2x_{23}+250x_{21}^2+39x_{21}x_{22}^2+6x_{21}x_{22}x_{23}-172x_{21}x_{22}-48x_{21}x_{23}-200x$$

$$195x_{21}^4-228x_{21}^3x_{22}-66x_{21}^3x_{23}-120x_{21}^3-39x_{21}^2x_{22}^2-6x_{21}^2x_{22}x_{23}+4x_{21}^2x_{23}-84x_{21}x_{22}^2-16x_{21}x_{22}x_{23}+16x_{21}x_{22}+80x$$

$\mathfrak{m}_{2A}(5, 15)$

m2A515 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_9] = e_{14}$$
$$[e_2, e_{10}] = 4e_{15} \qquad [e_3, e_8] = -e_{14}$$
$$[e_3, e_9] = -3e_{15} \qquad [e_4, e_7] = e_{14}$$
$$[e_4, e_8] = 2e_{15} \qquad [e_5, e_6] = -e_{14}$$
$$[e_5, e_7] = -e_{15}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(5, 15)$

m4A515 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_7] &= e_{12} \\
[e_2, e_8] &= 3e_{13} & [e_2, e_9] &= \alpha_{2,9}^{14} e_{14} \\
[e_2, e_{10}] &= \alpha_{2,10}^{15} e_{15} & [e_3, e_6] &= -e_{12} \\
[e_3, e_7] &= -2e_{13} & [e_3, e_8] &= \alpha_{3,8}^{14} e_{14} \\
[e_3, e_9] &= \alpha_{3,9}^{15} e_{15} & [e_4, e_5] &= e_{12} \\
[e_4, e_6] &= e_{13} & [e_4, e_7] &= \alpha_{4,7}^{14} e_{14} \\
[e_4, e_8] &= \alpha_{4,8}^{15} e_{15} & [e_5, e_6] &= \alpha_{5,6}^{14} e_{14} \\
[e_5, e_7] &= \alpha_{5,7}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8):&\quad -\alpha_{2,9}^{14} - \alpha_{3,8}^{14} + 3 &= 0 \\
(e_1, e_3, e_7):&\quad -\alpha_{3,8}^{14} - \alpha_{4,7}^{14} - 2 &= 0 \\
(e_1, e_4, e_6):&\quad -\alpha_{4,7}^{14} - \alpha_{5,6}^{14} + 1 &= 0 \\
(e_1, e_2, e_9):&\quad -\alpha_{2,10}^{15} + \alpha_{2,9}^{14} - \alpha_{3,9}^{15} &= 0 \\
(e_1, e_3, e_8):&\quad \alpha_{3,8}^{14} - \alpha_{3,9}^{15} - \alpha_{4,8}^{15} &= 0 \\
(e_1, e_4, e_7):&\quad \alpha_{4,7}^{14} - \alpha_{4,8}^{15} - \alpha_{5,7}^{15} &= 0 \\
(e_1, e_5, e_6):&\quad \alpha_{5,6}^{14} - \alpha_{5,7}^{15} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{5,7}^{15} &\to x_1 \\
\alpha_{3,8}^{14} &\to x_2 \\
\alpha_{3,9}^{15} &\to x_3 \\
\alpha_{2,10}^{15} &\to x_4
\end{aligned}
$$

$$\alpha_{4,7}^{14} \to x_5$$

$$\alpha_{2,9}^{14} \to x_6$$

$$\alpha_{5,6}^{14} \to x_7$$

$$\alpha_{4,8}^{15} \to x_8$$

Jacobi Tests

$$\begin{array}{lll}
(e_1, e_2, e_8): & -x_2 - x_6 + 3 & = 0 \\
(e_1, e_3, e_7): & -x_2 - x_5 - 2 & = 0 \\
(e_1, e_4, e_6): & -x_5 - x_7 + 1 & = 0 \\
(e_1, e_2, e_9): & -x_3 - x_4 + x_6 & = 0 \\
(e_1, e_3, e_8): & x_2 - x_3 - x_8 & = 0 \\
(e_1, e_4, e_7): & -x_1 + x_5 - x_8 & = 0 \\
(e_1, e_5, e_6): & -x_1 + x_7 & = 0
\end{array}$$

Groebner basis (8 variables, 7 linear, 0 nonlinear)

$$2x_1 + x_8 - 1 = 0$$

$$2x_2 + x_8 + 5 = 0$$

$$2x_3 + 3x_8 + 5 = 0$$

$$x_4 - 2x_8 - 8 = 0$$

$$2x_5 - x_8 - 1 = 0$$

$$2x_6 - x_8 - 11 = 0$$

$$2x_7 + x_8 - 1 = 0$$

$\mathfrak{m}_{6A}(5, 15)$

m6A515 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_1,e_{13}]=e_{14} \\
[e_1,e_{14}]=e_{15} & [e_2,e_5]=e_{10} \\
[e_2,e_6]=2e_{11} & [e_2,e_7]=\alpha_{2,7}^{12}e_{12} \\
[e_2,e_8]=\alpha_{2,8}^{13}e_{13} & [e_2,e_9]=\alpha_{2,9}^{14}e_{14} \\
[e_2,e_{10}]=\alpha_{2,10}^{15}e_{15} & [e_3,e_4]=-e_{10} \\
[e_3,e_5]=-e_{11} & [e_3,e_6]=\alpha_{3,6}^{12}e_{12} \\
[e_3,e_7]=\alpha_{3,7}^{13}e_{13} & [e_3,e_8]=\alpha_{3,8}^{14}e_{14} \\
[e_3,e_9]=\alpha_{3,9}^{15}e_{15} & [e_4,e_5]=\alpha_{4,5}^{12}e_{12} \\
[e_4,e_6]=\alpha_{4,6}^{13}e_{13} & [e_4,e_7]=\alpha_{4,7}^{14}e_{14} \\
[e_4,e_8]=\alpha_{4,8}^{15}e_{15} & [e_5,e_6]=\alpha_{5,6}^{14}e_{14} \\
[e_5,e_7]=\alpha_{5,7}^{15}e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1,e_2,e_6): & -\alpha_{2,7}^{12}-\alpha_{3,6}^{12}+2 & =0 \\
(e_1,e_3,e_5): & -\alpha_{3,6}^{12}-\alpha_{4,5}^{12}-1 & =0 \\
(e_1,e_2,e_7): & \alpha_{2,7}^{12}-\alpha_{2,8}^{13}-\alpha_{3,7}^{13} & =0 \\
(e_1,e_3,e_6): & \alpha_{3,6}^{12}-\alpha_{3,7}^{13}-\alpha_{4,6}^{13} & =0 \\
(e_1,e_4,e_5): & \alpha_{4,5}^{12}-\alpha_{4,6}^{13} & =0 \\
(e_1,e_2,e_8): & \alpha_{2,8}^{13}-\alpha_{2,9}^{14}-\alpha_{3,8}^{14} & =0 \\
(e_1,e_3,e_7): & \alpha_{3,7}^{13}-\alpha_{3,8}^{14}-\alpha_{4,7}^{14} & =0 \\
(e_1,e_4,e_6): & \alpha_{4,6}^{13}-\alpha_{4,7}^{14}-\alpha_{5,6}^{14} & =0 \\
(e_1,e_2,e_9): & -\alpha_{2,10}^{15}+\alpha_{2,9}^{14}-\alpha_{3,9}^{15} & =0 \\
(e_1,e_3,e_8): & \alpha_{3,8}^{14}-\alpha_{3,9}^{15}-\alpha_{4,8}^{15} & =0 \\
(e_1,e_4,e_7): & \alpha_{4,7}^{14}-\alpha_{4,8}^{15}-\alpha_{5,7}^{15} & =0 \\
(e_1,e_5,e_6): & \alpha_{5,6}^{14}-\alpha_{5,7}^{15} & =0 \\
(e_2,e_3,e_4): & -\alpha_{2,10}^{15} & =0
\end{array}
$$

Infinite number of solutions.
How the solution(s) were or were not found:

Change variables

$$\alpha^{15}_{5,7} \to x_1$$
$$\alpha^{12}_{2,7} \to x_2$$
$$\alpha^{14}_{3,8} \to x_3$$
$$\alpha^{15}_{3,9} \to x_4$$
$$\alpha^{15}_{2,10} \to x_5$$
$$\alpha^{13}_{4,6} \to x_6$$
$$\alpha^{14}_{5,6} \to x_7$$
$$\alpha^{12}_{3,6} \to x_8$$
$$\alpha^{14}_{4,7} \to x_9$$
$$\alpha^{13}_{2,8} \to x_{10}$$
$$\alpha^{14}_{2,9} \to x_{11}$$
$$\alpha^{12}_{4,5} \to x_{12}$$
$$\alpha^{13}_{3,7} \to x_{13}$$
$$\alpha^{15}_{4,8} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_2 - x_8 + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -x_{12} - x_8 - 1 &&= 0\\
(e_1, e_2, e_7):&\quad -x_{10} - x_{13} + x_2 &&= 0\\
(e_1, e_3, e_6):&\quad -x_{13} - x_6 + x_8 &&= 0\\
(e_1, e_4, e_5):&\quad x_{12} - x_6 &&= 0\\
(e_1, e_2, e_8):&\quad x_{10} - x_{11} - x_3 &&= 0\\
(e_1, e_3, e_7):&\quad x_{13} - x_3 - x_9 &&= 0\\
(e_1, e_4, e_6):&\quad x_6 - x_7 - x_9 &&= 0\\
(e_1, e_2, e_9):&\quad x_{11} - x_4 - x_5 &&= 0\\
(e_1, e_3, e_8):&\quad -x_{14} + x_3 - x_4 &&= 0\\
(e_1, e_4, e_7):&\quad -x_1 - x_{14} + x_9 &&= 0\\
(e_1, e_5, e_6):&\quad -x_1 + x_7 &&= 0\\
(e_2, e_3, e_4):&\quad -x_5 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 13 linear, 0 nonlinear)

$$8x_1 + 5x_{14} + 3 = 0$$
$$x_{14} + 4x_2 - 9 = 0$$
$$-x_{14} + 8x_3 - 7 = 0$$
$$7x_{14} + 8x_4 - 7 = 0$$
$$x_5 = 0$$
$$x_{14} + 4x_6 + 3 = 0$$
$$5x_{14} + 8x_7 + 3 = 0$$
$$-x_{14} + 4x_8 + 1 = 0$$
$$-3x_{14} + 8x_9 + 3 = 0$$
$$4x_{10} + 3x_{14} - 7 = 0$$
$$8x_{11} + 7x_{14} - 7 = 0$$
$$4x_{12} + x_{14} + 3 = 0$$
$$2x_{13} - x_{14} - 1 = 0$$

$\mathfrak{m}_{8A}(5, 15)$

m8A515 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 & &[e_1, e_3] = e_4\\
&[e_1, e_4] = e_5 & &[e_1, e_5] = e_6\\
&[e_1, e_6] = e_7 & &[e_1, e_7] = e_8\\
&[e_1, e_8] = e_9 & &[e_1, e_9] = e_{10}\\
&[e_1, e_{10}] = e_{11} & &[e_1, e_{11}] = e_{12}\\
&[e_1, e_{12}] = e_{13} & &[e_1, e_{13}] = e_{14}\\
&[e_1, e_{14}] = e_{15} & &[e_2, e_3] = e_8\\
&[e_2, e_4] = e_9 & &[e_2, e_5] = \alpha_{2,5}^{10} e_{10}\\
&[e_2, e_6] = \alpha_{2,6}^{11} e_{11} & &[e_2, e_7] = \alpha_{2,7}^{12} e_{12}\\
&[e_2, e_8] = \alpha_{2,8}^{13} e_{13} & &[e_2, e_9] = \alpha_{2,9}^{14} e_{14}\\
&[e_2, e_{10}] = \alpha_{2,10}^{15} e_{15} & &[e_3, e_4] = \alpha_{3,4}^{10} e_{10}\\
&[e_3, e_5] = \alpha_{3,5}^{11} e_{11} & &[e_3, e_6] = \alpha_{3,6}^{12} e_{12}\\
&[e_3, e_7] = \alpha_{3,7}^{13} e_{13} & &[e_3, e_8] = \alpha_{3,8}^{14} e_{14}\\
&[e_3, e_9] = \alpha_{3,9}^{15} e_{15} & &[e_4, e_5] = \alpha_{4,5}^{12} e_{12}\\
&[e_4, e_6] = \alpha_{4,6}^{13} e_{13} & &[e_4, e_7] = \alpha_{4,7}^{14} e_{14}\\
&[e_4, e_8] = \alpha_{4,8}^{15} e_{15} & &[e_5, e_6] = \alpha_{5,6}^{14} e_{14}\\
&[e_5, e_7] = \alpha_{5,7}^{15} e_{15} & &
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{13} - \alpha_{2,9}^{14} - \alpha_{3,8}^{14} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{13} - \alpha_{3,8}^{14} - \alpha_{4,7}^{14} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{13} - \alpha_{4,7}^{14} - \alpha_{5,6}^{14} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{15} + \alpha_{2,9}^{14} - \alpha_{3,9}^{15} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{14} - \alpha_{3,9}^{15} - \alpha_{4,8}^{15} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{14} - \alpha_{4,8}^{15} - \alpha_{5,7}^{15} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{14} - \alpha_{5,7}^{15} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,10}^{15}\alpha_{3,4}^{10} - \alpha_{3,9}^{15} + \alpha_{4,8}^{15} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{5,7}^{15} \to x_1$$

$$\alpha_{2,6}^{11} \to x_2$$

$$\alpha_{2,7}^{12} \to x_3$$

$$\alpha_{3,4}^{10} \to x_4$$

$$\alpha_{2,5}^{10} \to x_5$$

$$\alpha_{3,8}^{14} \to x_6$$

$$\alpha_{3,9}^{15} \to x_7$$

$$\alpha_{2,10}^{15} \to x_8$$

$$\alpha_{4,6}^{13} \to x_9$$

$$\alpha_{5,6}^{14} \to x_{10}$$

$$\alpha^{12}_{3,6} \to x_{11}$$
$$\alpha^{14}_{4,7} \to x_{12}$$
$$\alpha^{14}_{2,9} \to x_{13}$$
$$\alpha^{13}_{2,8} \to x_{14}$$
$$\alpha^{11}_{3,5} \to x_{15}$$
$$\alpha^{12}_{4,5} \to x_{16}$$
$$\alpha^{13}_{3,7} \to x_{17}$$
$$\alpha^{15}_{4,8} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_4 - x_5 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_{15} - x_2 + x_5 &&= 0\\
(e_1, e_3, e_4):&\quad -x_{15} + x_4 &&= 0\\
(e_1, e_2, e_6):&\quad -x_{11} + x_2 - x_3 &&= 0\\
(e_1, e_3, e_5):&\quad -x_{11} + x_{15} - x_{16} &&= 0\\
(e_1, e_2, e_7):&\quad -x_{14} - x_{17} + x_3 &&= 0\\
(e_1, e_3, e_6):&\quad x_{11} - x_{17} - x_9 &&= 0\\
(e_1, e_4, e_5):&\quad x_{16} - x_9 &&= 0\\
(e_1, e_2, e_8):&\quad -x_{13} + x_{14} - x_6 &&= 0\\
(e_1, e_3, e_7):&\quad -x_{12} + x_{17} - x_6 &&= 0\\
(e_1, e_4, e_6):&\quad -x_{10} - x_{12} + x_9 &&= 0\\
(e_1, e_2, e_9):&\quad x_{13} - x_7 - x_8 &&= 0\\
(e_1, e_3, e_8):&\quad -x_{18} + x_6 - x_7 &&= 0\\
(e_1, e_4, e_7):&\quad -x_1 + x_{12} - x_{18} &&= 0\\
(e_1, e_5, e_6):&\quad -x_1 + x_{10} &&= 0\\
(e_2, e_3, e_4):&\quad x_{18} + x_4 x_8 - x_7 &&= 0
\end{aligned}$$

Groebner basis (18 variables, 15 linear, 1 nonlinear)

$$2x_1 - x_{16} + x_{18} = 0$$
$$4x_{16} + 2x_{17} + x_2 - 1 = 0$$
$$5x_{16} + 3x_{17} + x_3 - 1 = 0$$
$$-2x_{16} - x_{17} + x_4 = 0$$
$$2x_{16} + x_{17} + x_5 - 1 = 0$$

$$x_{16} - 2x_{17} + x_{18} + 2x_6 = 0$$
$$x_{16} - 2x_{17} + 3x_{18} + 2x_7 = 0$$
$$4x_{16} + 6x_{17} - 2x_{18} + x_8 - 1 = 0$$
$$-x_{16} + x_9 = 0$$
$$2x_{10} - x_{16} + x_{18} = 0$$
$$x_{11} - x_{16} - x_{17} = 0$$
$$2x_{12} - x_{16} - x_{18} = 0$$
$$2x_{13} + 9x_{16} + 10x_{17} - x_{18} - 2 = 0$$
$$x_{14} + 5x_{16} + 4x_{17} - 1 = 0$$
$$x_{15} - 2x_{16} - x_{17} = 0$$
$$16x_{16}^2 + 32x_{16}x_{17} - 8x_{16}x_{18} - 5x_{16} + 12x_{17}^2 - 4x_{17}x_{18} - 5x_{18} = 0$$

$\mathfrak{m}_{1A}(6, 15)$

m1A615 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_9] &= e_{15} \\
[e_3, e_8] &= -e_{15} & [e_4, e_7] &= e_{15} \\
[e_5, e_6] &= -e_{15}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(6, 15)$

m3A615 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_7] = e_{13} \\
[e_2, e_8] = 3e_{14} & [e_2, e_9] = \alpha_{2,9}^{15} e_{15} \\
[e_3, e_6] = -e_{13} & [e_3, e_7] = -2e_{14} \\
[e_3, e_8] = \alpha_{3,8}^{15} e_{15} & [e_4, e_5] = e_{13} \\
[e_4, e_6] = e_{14} & [e_4, e_7] = \alpha_{4,7}^{15} e_{15} \\
[e_5, e_6] = \alpha_{5,6}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_8): & -\alpha_{2,9}^{15} - \alpha_{3,8}^{15} + 3 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{15} - \alpha_{4,7}^{15} - 2 & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{15} - \alpha_{5,6}^{15} + 1 & = 0
\end{array}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{array}{c}
\alpha_{4,7}^{15} \to x_1 \\
\alpha_{2,9}^{15} \to x_2 \\
\alpha_{3,8}^{15} \to x_3 \\
\alpha_{5,6}^{15} \to x_4
\end{array}
$$

Jacobi Tests

$$
\begin{array}{lll}
(e_1, e_2, e_8): & -x_2 - x_3 + 3 & = 0 \\
(e_1, e_3, e_7): & -x_1 - x_3 - 2 & = 0 \\
(e_1, e_4, e_6): & -x_1 - x_4 + 1 & = 0
\end{array}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 1 = 0$$

$$x_2 + x_4 - 6 = 0$$
$$x_3 - x_4 + 3 = 0$$

$\mathfrak{m}_{5A}(6, 15)$

m5A615 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_5] &= e_{11} \\
[e_2, e_6] &= 2e_{12} & [e_2, e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_2, e_8] &= \alpha_{2,8}^{14} e_{14} & [e_2, e_9] &= \alpha_{2,9}^{15} e_{15} \\
[e_3, e_4] &= -e_{11} & [e_3, e_5] &= -e_{12} \\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_3, e_7] &= \alpha_{3,7}^{14} e_{14} \\
[e_3, e_8] &= \alpha_{3,8}^{15} e_{15} & [e_4, e_5] &= \alpha_{4,5}^{13} e_{13} \\
[e_4, e_6] &= \alpha_{4,6}^{14} e_{14} & [e_4, e_7] &= \alpha_{4,7}^{15} e_{15} \\
[e_5, e_6] &= \alpha_{5,6}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{13} - \alpha_{3,6}^{13} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{13} - \alpha_{4,5}^{13} - 1 &&= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{13} - \alpha_{2,8}^{14} - \alpha_{3,7}^{14} &&= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{13} - \alpha_{3,7}^{14} - \alpha_{4,6}^{14} &&= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{13} - \alpha_{4,6}^{14} &&= 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{14} - \alpha_{2,9}^{15} - \alpha_{3,8}^{15} &&= 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{14} - \alpha_{3,8}^{15} - \alpha_{4,7}^{15} &&= 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{14} - \alpha_{4,7}^{15} - \alpha_{5,6}^{15} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{13} \to x_1$$

$$\alpha_{2,7}^{13} \to x_2$$

$$\alpha_{4,6}^{14} \to x_3$$

$$\alpha_{5,6}^{15} \to x_4$$

$$\alpha_{4,7}^{15} \to x_5$$

$$\alpha_{3,7}^{14} \to x_6$$

$$\alpha_{3,8}^{15} \to x_7$$

$$\alpha_{2,9}^{15} \to x_8$$

$$\alpha_{3,6}^{13} \to x_9$$

$$\alpha_{2,8}^{14} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_2 - x_9 + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -x_1 - x_9 - 1 &&= 0\\
(e_1, e_2, e_7):&\quad -x_{10} + x_2 - x_6 &&= 0\\
(e_1, e_3, e_6):&\quad -x_3 - x_6 + x_9 &&= 0\\
(e_1, e_4, e_5):&\quad x_1 - x_3 &&= 0\\
(e_1, e_2, e_8):&\quad x_{10} - x_7 - x_8 &&= 0\\
(e_1, e_3, e_7):&\quad -x_5 + x_6 - x_7 &&= 0\\
(e_1, e_4, e_6):&\quad x_3 - x_4 - x_5 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$3x_1 - x_{10} + 4 = 0$$

$$-x_{10} + 3x_2 - 5 = 0$$

$$-x_{10} + 3x_3 + 4 = 0$$

$$-2x_{10} + x_4 + x_8 + 3 = 0$$

$$5x_{10} + 3x_5 - 3x_8 - 5 = 0$$

$$2x_{10} + 3x_6 - 5 = 0$$

$$-x_{10} + x_7 + x_8 = 0$$

$$x_{10} + 3x_9 - 1 = 0$$

$\mathfrak{m}_{7A}(6, 15)$

m7A615 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_3] = e_9 \\
[e_2, e_4] = e_{10} & [e_2, e_5] = \alpha_{2,5}^{11} e_{11} \\
[e_2, e_6] = \alpha_{2,6}^{12} e_{12} & [e_2, e_7] = \alpha_{2,7}^{13} e_{13} \\
[e_2, e_8] = \alpha_{2,8}^{14} e_{14} & [e_2, e_9] = \alpha_{2,9}^{15} e_{15} \\
[e_3, e_4] = \alpha_{3,4}^{11} e_{11} & [e_3, e_5] = \alpha_{3,5}^{12} e_{12} \\
[e_3, e_6] = \alpha_{3,6}^{13} e_{13} & [e_3, e_7] = \alpha_{3,7}^{14} e_{14} \\
[e_3, e_8] = \alpha_{3,8}^{15} e_{15} & [e_4, e_5] = \alpha_{4,5}^{13} e_{13} \\
[e_4, e_6] = \alpha_{4,6}^{14} e_{14} & [e_4, e_7] = \alpha_{4,7}^{15} e_{15} \\
[e_5, e_6] = \alpha_{5,6}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_4): & -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 & = 0 \\
(e_1, e_2, e_5): & \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} & = 0 \\
(e_1, e_3, e_4): & \alpha_{3,4}^{11} - \alpha_{3,5}^{12} & = 0 \\
(e_1, e_2, e_6): & \alpha_{2,6}^{12} - \alpha_{2,7}^{13} - \alpha_{3,6}^{13} & = 0 \\
(e_1, e_3, e_5): & \alpha_{3,5}^{12} - \alpha_{3,6}^{13} - \alpha_{4,5}^{13} & = 0 \\
(e_1, e_2, e_7): & \alpha_{2,7}^{13} - \alpha_{2,8}^{14} - \alpha_{3,7}^{14} & = 0 \\
(e_1, e_3, e_6): & \alpha_{3,6}^{13} - \alpha_{3,7}^{14} - \alpha_{4,6}^{14} & = 0 \\
(e_1, e_4, e_5): & \alpha_{4,5}^{13} - \alpha_{4,6}^{14} & = 0 \\
(e_1, e_2, e_8): & \alpha_{2,8}^{14} - \alpha_{2,9}^{15} - \alpha_{3,8}^{15} & = 0 \\
(e_1, e_3, e_7): & \alpha_{3,7}^{14} - \alpha_{3,8}^{15} - \alpha_{4,7}^{15} & = 0 \\
(e_1, e_4, e_6): & \alpha_{4,6}^{14} - \alpha_{4,7}^{15} - \alpha_{5,6}^{15} & = 0
\end{array}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{4,5}^{13} \to x_1$$
$$\alpha_{2,7}^{13} \to x_2$$
$$\alpha_{2,5}^{11} \to x_3$$
$$\alpha_{3,4}^{11} \to x_4$$
$$\alpha_{4,6}^{14} \to x_5$$
$$\alpha_{5,6}^{15} \to x_6$$
$$\alpha_{4,7}^{15} \to x_7$$
$$\alpha_{2,6}^{12} \to x_8$$
$$\alpha_{3,7}^{14} \to x_9$$
$$\alpha_{3,8}^{15} \to x_{10}$$
$$\alpha_{3,5}^{12} \to x_{11}$$
$$\alpha_{2,9}^{15} \to x_{12}$$
$$\alpha_{3,6}^{13} \to x_{13}$$
$$\alpha_{2,8}^{14} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4) &: \quad -x_3 - x_4 + 1 && = 0 \\
(e_1, e_2, e_5) &: \quad -x_{11} + x_3 - x_8 && = 0 \\
(e_1, e_3, e_4) &: \quad -x_{11} + x_4 && = 0 \\
(e_1, e_2, e_6) &: \quad -x_{13} - x_2 + x_8 && = 0 \\
(e_1, e_3, e_5) &: \quad -x_1 + x_{11} - x_{13} && = 0 \\
(e_1, e_2, e_7) &: \quad -x_{14} + x_2 - x_9 && = 0 \\
(e_1, e_3, e_6) &: \quad x_{13} - x_5 - x_9 && = 0 \\
(e_1, e_4, e_5) &: \quad x_1 - x_5 && = 0 \\
(e_1, e_2, e_8) &: \quad -x_{10} - x_{12} + x_{14} && = 0 \\
(e_1, e_3, e_7) &: \quad -x_{10} - x_7 + x_9 && = 0 \\
(e_1, e_4, e_6) &: \quad x_5 - x_6 - x_7 && = 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 0 nonlinear)

$$x_1 + 4x_{13} + x_{14} - 1 = 0$$

$$-5x_{13} - 2x_{14} + x_2 + 1 = 0$$
$$-3x_{13} - x_{14} + x_3 = 0$$
$$3x_{13} + x_{14} + x_4 - 1 = 0$$
$$4x_{13} + x_{14} + x_5 - 1 = 0$$
$$x_{12} + 9x_{13} + x_{14} + x_6 - 2 = 0$$
$$-x_{12} - 5x_{13} + x_7 + 1 = 0$$
$$-6x_{13} - 2x_{14} + x_8 + 1 = 0$$
$$-5x_{13} - x_{14} + x_9 + 1 = 0$$
$$x_{10} + x_{12} - x_{14} = 0$$
$$x_{11} + 3x_{13} + x_{14} - 1 = 0$$

$\mathfrak{m}_{2A}(7, 15)$

m2A715 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_7] &= e_{14} \\
[e_2, e_8] &= 3e_{15} & [e_3, e_6] &= -e_{14} \\
[e_3, e_7] &= -2e_{15} & [e_4, e_5] &= e_{14} \\
[e_4, e_6] &= e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(7, 15)$

m4A715 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_1, e_{14}] = e_{15} && [e_2, e_5] = e_{12} \\
&[e_2, e_6] = 2e_{13} && [e_2, e_7] = \alpha_{2,7}^{14} e_{14} \\
&[e_2, e_8] = \alpha_{2,8}^{15} e_{15} && [e_3, e_4] = -e_{12} \\
&[e_3, e_5] = -e_{13} && [e_3, e_6] = \alpha_{3,6}^{14} e_{14} \\
&[e_3, e_7] = \alpha_{3,7}^{15} e_{15} && [e_4, e_5] = \alpha_{4,5}^{14} e_{14} \\
&[e_4, e_6] = \alpha_{4,6}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{14} - \alpha_{3,6}^{14} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{14} - \alpha_{4,5}^{14} - 1 &&= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{14} - \alpha_{2,8}^{15} - \alpha_{3,7}^{15} &&= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{14} - \alpha_{3,7}^{15} - \alpha_{4,6}^{15} &&= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{14} - \alpha_{4,6}^{15} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,7}^{15} \to x_1$$

$$\alpha_{2,8}^{15} \to x_2$$

$$\alpha_{4,5}^{14} \to x_3$$

$$\alpha_{4,6}^{15} \to x_4$$

$$\alpha_{3,6}^{14} \to x_5$$

$$\alpha_{2,7}^{14} \to x_6$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_5 - x_6 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_3 - x_5 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad -x_1 - x_2 + x_6 &&= 0\\
(e_1, e_3, e_6): &\quad -x_1 - x_4 + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad x_3 - x_4 &&= 0
\end{aligned}
$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$x_1 + 2x_6 - 5 = 0$$
$$x_2 - 3x_6 + 5 = 0$$
$$x_3 - x_6 + 3 = 0$$
$$x_4 - x_6 + 3 = 0$$
$$x_5 + x_6 - 2 = 0$$

$\mathfrak{m}_{6A}(7, 15)$

m6A715 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{10}\\
[e_2, e_4] &= e_{11} & [e_2, e_5] &= \alpha_{2,5}^{12} e_{12}\\
[e_2, e_6] &= \alpha_{2,6}^{13} e_{13} & [e_2, e_7] &= \alpha_{2,7}^{14} e_{14}\\
[e_2, e_8] &= \alpha_{2,8}^{15} e_{15} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12}\\
[e_3, e_5] &= \alpha_{3,5}^{13} e_{13} & [e_3, e_6] &= \alpha_{3,6}^{14} e_{14}\\
[e_3, e_7] &= \alpha_{3,7}^{15} e_{15} & [e_4, e_5] &= \alpha_{4,5}^{14} e_{14}\\
[e_4, e_6] &= \alpha_{4,6}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 &&= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} &&= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{12} - \alpha_{3,5}^{13} &&= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{13} - \alpha_{2,7}^{14} - \alpha_{3,6}^{14} &&= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{13} - \alpha_{3,6}^{14} - \alpha_{4,5}^{14} &&= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{14} - \alpha_{2,8}^{15} - \alpha_{3,7}^{15} &&= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{14} - \alpha_{3,7}^{15} - \alpha_{4,6}^{15} &&= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{14} - \alpha_{4,6}^{15} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,6}^{13} &\to x_1 \\
\alpha_{3,7}^{15} &\to x_2 \\
\alpha_{2,8}^{15} &\to x_3 \\
\alpha_{2,5}^{12} &\to x_4 \\
\alpha_{4,5}^{14} &\to x_5 \\
\alpha_{4,6}^{15} &\to x_6 \\
\alpha_{3,5}^{13} &\to x_7 \\
\alpha_{3,6}^{14} &\to x_8 \\
\alpha_{3,4}^{12} &\to x_9 \\
\alpha_{2,7}^{14} &\to x_{10}
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_4 - x_9 + 1 &&= 0 \\
(e_1, e_2, e_5): &\quad -x_1 + x_4 - x_7 &&= 0 \\
(e_1, e_3, e_4): &\quad -x_7 + x_9 &&= 0 \\
(e_1, e_2, e_6): &\quad x_1 - x_{10} - x_8 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_5 + x_7 - x_8 &&= 0 \\
(e_1, e_2, e_7): &\quad x_{10} - x_2 - x_3 &&= 0 \\
(e_1, e_3, e_6): &\quad -x_2 - x_6 + x_8 &&= 0 \\
(e_1, e_4, e_5): &\quad x_5 - x_6 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + 2x_9 - 1 = 0$$
$$2x_{10} + x_2 + 5x_9 - 2 = 0$$
$$-3x_{10} + x_3 - 5x_9 + 2 = 0$$
$$x_4 + x_9 - 1 = 0$$
$$-x_{10} + x_5 - 3x_9 + 1 = 0$$
$$-x_{10} + x_6 - 3x_9 + 1 = 0$$
$$x_7 - x_9 = 0$$
$$x_{10} + x_8 + 2x_9 - 1 = 0$$

$\mathfrak{m}_{1A}(8, 15)$

m1A815 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_7] &= e_{15} \\
[e_3, e_6] &= -e_{15} & [e_4, e_5] &= e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(8, 15)$

m3A815 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_5] &= e_{13} \\
[e_2, e_6] &= 2e_{14} & [e_2, e_7] &= \alpha_{2,7}^{15} e_{15} \\
[e_3, e_4] &= -e_{13} & [e_3, e_5] &= -e_{14} \\
[e_3, e_6] &= \alpha_{3,6}^{15} e_{15} & [e_4, e_5] &= \alpha_{4,5}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{15} - \alpha_{3,6}^{15} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{15} - \alpha_{4,5}^{15} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\alpha_{2,7}^{15} \to x_1
$$
$$
\alpha_{4,5}^{15} \to x_2
$$
$$
\alpha_{3,6}^{15} \to x_3
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_3 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_2 - x_3 - 1 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$
x_1 + x_3 - 2 = 0
$$
$$
x_2 + x_3 + 1 = 0
$$

$\mathfrak{m}_{5A}(8,15)$

m5A815 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_1,e_{13}]=e_{14} \\
[e_1,e_{14}]=e_{15} & [e_2,e_3]=e_{11} \\
[e_2,e_4]=e_{12} & [e_2,e_5]=\alpha_{2,5}^{13}e_{13} \\
[e_2,e_6]=\alpha_{2,6}^{14}e_{14} & [e_2,e_7]=\alpha_{2,7}^{15}e_{15} \\
[e_3,e_4]=\alpha_{3,4}^{13}e_{13} & [e_3,e_5]=\alpha_{3,5}^{14}e_{14} \\
[e_3,e_6]=\alpha_{3,6}^{15}e_{15} & [e_4,e_5]=\alpha_{4,5}^{15}e_{15}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1,e_2,e_4): & -\alpha_{2,5}^{13}-\alpha_{3,4}^{13}+1 & =0 \\
(e_1,e_2,e_5): & \alpha_{2,5}^{13}-\alpha_{2,6}^{14}-\alpha_{3,5}^{14} & =0 \\
(e_1,e_3,e_4): & \alpha_{3,4}^{13}-\alpha_{3,5}^{14} & =0 \\
(e_1,e_2,e_6): & \alpha_{2,6}^{14}-\alpha_{2,7}^{15}-\alpha_{3,6}^{15} & =0 \\
(e_1,e_3,e_5): & \alpha_{3,5}^{14}-\alpha_{3,6}^{15}-\alpha_{4,5}^{15} & =0
\end{array}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{array}{l}
\alpha_{2,7}^{15}\to x_1 \\
\alpha_{3,5}^{14}\to x_2 \\
\alpha_{2,5}^{13}\to x_3 \\
\alpha_{3,6}^{15}\to x_4 \\
\alpha_{2,6}^{14}\to x_5 \\
\alpha_{3,4}^{13}\to x_6 \\
\alpha_{4,5}^{15}\to x_7
\end{array}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_6 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_2 + x_3 - x_5 &= 0\\
(e_1, e_3, e_4): &\quad -x_2 + x_6 &= 0\\
(e_1, e_2, e_6): &\quad -x_1 - x_4 + x_5 &= 0\\
(e_1, e_3, e_5): &\quad x_2 - x_4 - x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 + 3x_6 - x_7 - 1 = 0$$

$$x_2 - x_6 = 0$$

$$x_3 + x_6 - 1 = 0$$

$$x_4 - x_6 + x_7 = 0$$

$$x_5 + 2x_6 - 1 = 0$$

$\mathfrak{m}_{2A}(9, 15)$

m2A915 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_1, e_{14}] &= e_{15} & [e_2, e_5] &= e_{14}\\
[e_2, e_6] &= 2e_{15} & [e_3, e_4] &= -e_{14}\\
[e_3, e_5] &= -e_{15}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(9, 15)$

m4A915 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{12} \\
[e_2, e_4] &= e_{13} & [e_2, e_5] &= \alpha_{2,5}^{14} e_{14} \\
[e_2, e_6] &= \alpha_{2,6}^{15} e_{15} & [e_3, e_4] &= \alpha_{3,4}^{14} e_{14} \\
[e_3, e_5] &= \alpha_{3,5}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{14} - \alpha_{3,4}^{14} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{14} - \alpha_{2,6}^{15} - \alpha_{3,5}^{15} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{14} - \alpha_{3,5}^{15} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,6}^{15} &\to x_1 \\
\alpha_{2,5}^{14} &\to x_2 \\
\alpha_{3,5}^{15} &\to x_3 \\
\alpha_{3,4}^{14} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_4 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad -x_1 + x_2 - x_3 &= 0 \\
(e_1, e_3, e_4): &\quad -x_3 + x_4 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 + 2x_4 - 1 &= 0 \\
x_2 + x_4 - 1 &= 0 \\
x_3 - x_4 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(10, 15)$

m1A1015 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_5] &= e_{15} \\
[e_3, e_4] &= -e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(10, 15)$

m3A1015 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{13} \\
[e_2, e_4] &= e_{14} & [e_2, e_5] &= \alpha_{2,5}^{15} e_{15} \\
[e_3, e_4] &= \alpha_{3,4}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
(e_1, e_2, e_4): \quad -\alpha_{2,5}^{15} - \alpha_{3,4}^{15} + 1 = 0
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,5}^{15} &\to x_1 \\
\alpha_{3,4}^{15} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(11, 15)$

m2A1115 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{14} \\
[e_2, e_4] &= e_{15}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(12, 15)$

m1A1215 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{15}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2B}(2,6)$

m2B26 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_2,e_3]=e_5$$
$$[e_2,e_5]=e_6 \qquad [e_3,e_4]=-e_6$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_2,e_3]=e_5$$
$$[e_2,e_5]=e_6 \qquad [e_3,e_4]=\alpha_{3,4}^6 e_6$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{3,4}^6-1 \qquad =0$$

Solution 1:

$$\alpha_{3,4}^6=-1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^6\to x_1$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1-1 \qquad =0$$

Groebner basis (1 variables, 1 linear, 0 nonlinear)

$$x_1+1=0$$

Solution 1:

$$x_1=-1$$

$\mathfrak{m}_{2B}(2,8)$

m2B28 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_2,e_5]=e_7\\
&[e_2,e_7]=e_8 && [e_3,e_4]=-e_7\\
&[e_3,e_6]=\alpha_{3,6}^{8}e_8 && [e_4,e_5]=\alpha_{4,5}^{8}e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1,e_2,e_6): && -\alpha_{3,6}^{8}-1 &&=0\\
&(e_1,e_3,e_5): && -\alpha_{3,6}^{8}-\alpha_{4,5}^{8} &&=0\\
&(e_2,e_3,e_4): && \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2,8)$

m4B28 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_2,e_3]=e_5\\
&[e_2,e_4]=e_6 && [e_2,e_5]=3e_7\\
&[e_2,e_7]=e_8 && [e_3,e_4]=-2e_7\\
&[e_3,e_6]=-e_8 && [e_4,e_5]=e_8
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 & \quad & [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 & & [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 & & [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 & & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
&[e_2, e_7] = e_8 & & [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
&[e_3, e_6] = \alpha_{3,6}^{8} e_8 & & [e_4, e_5] = \alpha_{4,5}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &&= 0 \\
(e_1, e_2, e_6): &\quad -\alpha_{3,6}^{8} - 1 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{8} - \alpha_{4,5}^{8} &&= 0 \\
(e_2, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,6}^{8} + \alpha_{4,5}^{8} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,4}^{7} &= -2 \\
\alpha_{4,5}^{8} &= 1 \\
\alpha_{3,6}^{8} &= -1 \\
\alpha_{2,5}^{7} &= 3
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^{7} &\to x_1 \\
\alpha_{4,5}^{8} &\to x_2 \\
\alpha_{3,6}^{8} &\to x_3 \\
\alpha_{2,5}^{7} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_4 + 1 &&= 0 \\
(e_1, e_2, e_6): &\quad -x_3 - 1 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_2 - x_3 &&= 0 \\
(e_2, e_3, e_4): &\quad x_1 + x_2 - x_3 &&= 0
\end{aligned}
$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 2 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - 3 = 0$$

Solution 1:

$$x_1 = -2$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = 3$$

$\mathfrak{m}_{3B}(3, 8)$

m3B38 (this line included for string searching purposes)

Solution 1

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_2, e_3] &= e_6 \\ [e_2, e_4] &= e_7 & [e_2, e_7] &= e_8 \\ [e_3, e_6] &= -e_8 & [e_4, e_5] &= e_8 \end{aligned}$$

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_2, e_3] &= e_6 \\ [e_2, e_4] &= e_7 & [e_2, e_7] &= e_8 \\ [e_3, e_6] &= \alpha_{3,6}^8 e_8 & [e_4, e_5] &= \alpha_{4,5}^8 e_8 \end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned} (e_1, e_2, e_6): &\quad -\alpha_{3,6}^8 - 1 &= 0 \\ (e_1, e_3, e_5): &\quad -\alpha_{3,6}^8 - \alpha_{4,5}^8 &= 0 \end{aligned}$$

Solution 1:

$$\alpha^8_{4,5} = 1$$
$$\alpha^8_{3,6} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^8_{4,5} \to x_1$$
$$\alpha^8_{3,6} \to x_2$$

Jacobi Tests

$$\begin{aligned} (e_1, e_2, e_6): &\quad -x_2 - 1 &= 0 \\ (e_1, e_3, e_5): &\quad -x_1 - x_2 &= 0 \end{aligned}$$

Groebner basis (2 variables, 2 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -1$$

$\mathfrak{m}_{2B}(4, 8)$

m2B48 (this line included for string searching purposes)
Solution 1

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_2, e_3] &= e_7 \\ [e_2, e_7] &= e_8 & [e_3, e_6] &= -e_8 \\ [e_4, e_5] &= e_8 \end{aligned}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_7 \\
[e_2, e_7] &= e_8 & [e_3, e_6] &= \alpha_{3,6}^{8} e_8 \\
[e_4, e_5] &= \alpha_{4,5}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): \quad & -\alpha_{3,6}^{8} - 1 & = 0 \\
(e_1, e_3, e_5): \quad & -\alpha_{3,6}^{8} - \alpha_{4,5}^{8} & = 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,5}^{8} &= 1 \\
\alpha_{3,6}^{8} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{8} \to x_1$$

$$\alpha_{3,6}^{8} \to x_2$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): \quad & -x_2 - 1 & = 0 \\
(e_1, e_3, e_5): \quad & -x_1 - x_2 & = 0
\end{aligned}
$$

Groebner basis (2 variables, 2 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 + 1 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = -1$$

$\mathfrak{m}_{2B}(2,10)$

m2B210 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_2,e_7]=e_9 \\
[e_2,e_9]=e_{10} & [e_3,e_6]=-e_9 \\
[e_3,e_8]=\alpha_{3,8}^{10}e_{10} & [e_4,e_5]=e_9 \\
[e_4,e_7]=\alpha_{4,7}^{10}e_{10} & [e_5,e_6]=\alpha_{5,6}^{10}e_{10}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1,e_2,e_8): & -\alpha_{3,8}^{10}-1 & =0 \\
(e_1,e_3,e_7): & -\alpha_{3,8}^{10}-\alpha_{4,7}^{10} & =0 \\
(e_1,e_4,e_6): & -\alpha_{4,7}^{10}-\alpha_{5,6}^{10} & =0 \\
(e_2,e_3,e_6): & \text{no solutions} & \\
(e_2,e_4,e_5): & \text{no solutions} &
\end{array}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2,10)$

m4B210 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_2,e_5]=e_7 \\
[e_2,e_6]=2e_8 & [e_2,e_7]=\alpha_{2,7}^{9}e_9 \\
[e_2,e_9]=e_{10} & [e_3,e_4]=-e_7 \\
[e_3,e_5]=-e_8 & [e_3,e_6]=\alpha_{3,6}^{9}e_9 \\
[e_3,e_8]=\alpha_{3,8}^{10}e_{10} & [e_4,e_5]=\alpha_{4,5}^{9}e_9 \\
[e_4,e_7]=\alpha_{4,7}^{10}e_{10} & [e_5,e_6]=\alpha_{5,6}^{10}e_{10}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &&= 0\\
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{3,6}^{9} - 2\alpha_{3,8}^{10} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,7}^{10} &&= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{9} \to x_1$$

$$\alpha_{3,8}^{10} \to x_2$$

$$\alpha_{5,6}^{10} \to x_3$$

$$\alpha_{4,5}^{9} \to x_4$$

$$\alpha_{3,6}^{9} \to x_5$$

$$\alpha_{4,7}^{10} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_5 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_4 - x_5 - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -x_1 &&= 0\\
(e_1, e_2, e_8): &\quad -x_2 - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_6 &&= 0\\
(e_1, e_4, e_6): &\quad -x_3 - x_6 &&= 0\\
(e_2, e_3, e_6): &\quad -2x_2 + x_5 &&= 0\\
(e_2, e_4, e_5): &\quad x_4 - x_6 &&= 0
\end{aligned}$$

Groebner basis (6 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6B}(2, 10)$

m6B210 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_5 \\
[e_2, e_4] &= e_6 & [e_2, e_5] &= 0 \\
[e_2, e_6] &= -e_8 & [e_2, e_7] &= -e_9 \\
[e_2, e_9] &= e_{10} & [e_3, e_4] &= e_7 \\
[e_3, e_5] &= e_8 & [e_3, e_6] &= 0 \\
[e_3, e_8] &= -e_{10} & [e_4, e_5] &= e_9 \\
[e_4, e_7] &= e_{10} & [e_5, e_6] &= -e_{10}
\end{aligned}
$$

Solution 2

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_5 \\
[e_2, e_4] &= e_6 & [e_2, e_5] &= 2e_7 \\
[e_2, e_6] &= 3e_8 & [e_2, e_7] &= 7e_9 \\
[e_2, e_9] &= e_{10} & [e_3, e_4] &= -e_7 \\
[e_3, e_5] &= -e_8 & [e_3, e_6] &= -4e_9 \\
[e_3, e_8] &= -e_{10} & [e_4, e_5] &= 3e_9 \\
[e_4, e_7] &= e_{10} & [e_5, e_6] &= -e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
&[e_2, e_6] = \alpha_{2,6}^{8} e_8 && [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
&[e_2, e_9] = e_{10} && [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
&[e_3, e_5] = \alpha_{3,5}^{8} e_8 && [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
&[e_4, e_7] = \alpha_{4,7}^{10} e_{10} && [e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1, e_2, e_4): && -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 && = 0 \\
&(e_1, e_2, e_5): && \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} && = 0 \\
&(e_1, e_3, e_4): && \alpha_{3,4}^{7} - \alpha_{3,5}^{8} && = 0 \\
&(e_1, e_2, e_6): && \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} && = 0 \\
&(e_1, e_3, e_5): && \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} && = 0 \\
&(e_2, e_3, e_4): && \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} && = 0 \\
&(e_1, e_2, e_8): && -\alpha_{3,8}^{10} - 1 && = 0 \\
&(e_1, e_3, e_7): && -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} && = 0 \\
&(e_1, e_4, e_6): && -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} && = 0 \\
&(e_2, e_3, e_6): && -\alpha_{2,6}^{8}\alpha_{3,8}^{10} + \alpha_{3,6}^{9} - \alpha_{5,6}^{10} && = 0 \\
&(e_2, e_4, e_5): && -\alpha_{2,5}^{7}\alpha_{4,7}^{10} + \alpha_{4,5}^{9} + \alpha_{5,6}^{10} && = 0
\end{aligned}
$$

Solution 1:

$$\alpha^{9}_{2,7} = -1$$
$$\alpha^{8}_{2,6} = -1$$
$$\alpha^{7}_{2,5} = 0$$
$$\alpha^{10}_{3,8} = -1$$
$$\alpha^{10}_{5,6} = -1$$
$$\alpha^{9}_{4,5} = 1$$
$$\alpha^{9}_{3,6} = 0$$
$$\alpha^{10}_{4,7} = 1$$
$$\alpha^{7}_{3,4} = 1$$
$$\alpha^{8}_{3,5} = 1$$

Solution 2:

$$\alpha^{9}_{2,7} = 7$$
$$\alpha^{8}_{2,6} = 3$$
$$\alpha^{7}_{2,5} = 2$$
$$\alpha^{10}_{3,8} = -1$$
$$\alpha^{10}_{5,6} = -1$$
$$\alpha^{9}_{4,5} = 3$$
$$\alpha^{9}_{3,6} = -4$$
$$\alpha^{10}_{4,7} = 1$$
$$\alpha^{7}_{3,4} = -1$$
$$\alpha^{8}_{3,5} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{9}_{2,7} \to x_1$$
$$\alpha^{8}_{2,6} \to x_2$$
$$\alpha^{7}_{2,5} \to x_3$$
$$\alpha^{10}_{3,8} \to x_4$$
$$\alpha^{10}_{5,6} \to x_5$$

$$\alpha_{4,5}^{9} \to x_6$$

$$\alpha_{3,6}^{9} \to x_7$$

$$\alpha_{4,7}^{10} \to x_8$$

$$\alpha_{3,4}^{7} \to x_9$$

$$\alpha_{3,5}^{8} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_3 - x_9 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_{10} - x_2 + x_3 &&= 0\\
(e_1, e_3, e_4):&\quad -x_{10} + x_9 &&= 0\\
(e_1, e_2, e_6):&\quad -x_1 + x_2 - x_7 &&= 0\\
(e_1, e_3, e_5):&\quad x_{10} - x_6 - x_7 &&= 0\\
(e_2, e_3, e_4):&\quad x_1x_9 + x_6 - x_7 &&= 0\\
(e_1, e_2, e_8):&\quad -x_4 - 1 &&= 0\\
(e_1, e_3, e_7):&\quad -x_4 - x_8 &&= 0\\
(e_1, e_4, e_6):&\quad -x_5 - x_8 &&= 0\\
(e_2, e_3, e_6):&\quad -x_2x_4 - x_5 + x_7 &&= 0\\
(e_2, e_4, e_5):&\quad -x_3x_8 + x_5 + x_6 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 9 linear, 1 nonlinear)

$$x_1 + 4x_{10} - 3 = 0$$

$$2x_{10} + x_2 - 1 = 0$$

$$x_{10} + x_3 - 1 = 0$$

$$x_4 + 1 = 0$$

$$x_5 + 1 = 0$$

$$x_{10} + x_6 - 2 = 0$$

$$-2x_{10} + x_7 + 2 = 0$$

$$x_8 - 1 = 0$$

$$-x_{10} + x_9 = 0$$

$$x_{10}^2 - 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = -1$$
$$x_3 = 0$$
$$x_4 = -1$$
$$x_5 = -1$$
$$x_6 = 1$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = 1$$
$$x_10 = 1$$

Solution 2:

$$x_1 = 7$$
$$x_2 = 3$$
$$x_3 = 2$$
$$x_4 = -1$$
$$x_5 = -1$$
$$x_6 = 3$$
$$x_7 = -4$$
$$x_8 = 1$$
$$x_9 = -1$$
$$x_10 = -1$$

$\mathfrak{m}_{3B}(3, 10)$

m3B310 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_8 \\
&[e_2, e_6] = 2e_9 && [e_2, e_9] = e_{10} \\
&[e_3, e_4] = -e_8 && [e_3, e_5] = -e_9 \\
&[e_3, e_8] = -e_{10} && [e_4, e_7] = e_{10} \\
&[e_5, e_6] = -e_{10} &&
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_8 \\
&[e_2, e_6] = 2e_9 && [e_2, e_9] = e_{10} \\
&[e_3, e_4] = -e_8 && [e_3, e_5] = -e_9 \\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{10} e_{10} \\
&[e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &&= 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &&= 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &&= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{3,8}^{10} - 1 &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{5,6}^{10} &= -1 \\
\alpha_{4,7}^{10} &= 1 \\
\alpha_{3,8}^{10} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{5,6}^{10} &\to x_1 \\
\alpha_{4,7}^{10} &\to x_2 \\
\alpha_{3,8}^{10} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - 1 &&= 0 \\
(e_1, e_3, e_7): &\quad -x_2 - x_3 &&= 0 \\
(e_1, e_4, e_6): &\quad -x_1 - x_2 &&= 0 \\
(e_2, e_3, e_5): &\quad -x_3 - 1 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = -1$$

$\mathfrak{m}_{5B}(3, 10)$

m5B310 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_6 \\
&[e_2, e_4] = e_7 && [e_2, e_5] = \alpha_{2,5}^{8} e_8 \\
&[e_2, e_6] = \alpha_{2,6}^{9} e_9 && [e_2, e_9] = e_{10} \\
&[e_3, e_4] = \alpha_{3,4}^{8} e_8 && [e_3, e_5] = \alpha_{3,5}^{9} e_9 \\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{10} e_{10} \\
&[e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
&(e_1, e_2, e_4): && -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 && = 0 \\
&(e_1, e_2, e_5): && \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} && = 0 \\
&(e_1, e_3, e_4): && \alpha_{3,4}^{8} - \alpha_{3,5}^{9} && = 0 \\
&(e_1, e_2, e_8): && -\alpha_{3,8}^{10} - 1 && = 0 \\
&(e_1, e_3, e_7): && -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} && = 0 \\
&(e_1, e_4, e_6): && -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} && = 0 \\
&(e_2, e_3, e_5): && -\alpha_{2,5}^{8}\alpha_{3,8}^{10} + \alpha_{3,5}^{9} + \alpha_{5,6}^{10} && = 0
\end{aligned}$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha^{10}_{3,8} \to x_1$$

$$\alpha^{10}_{5,6} \to x_2$$

$$\alpha^{9}_{2,6} \to x_3$$

$$\alpha^{8}_{3,4} \to x_4$$

$$\alpha^{10}_{4,7} \to x_5$$

$$\alpha^{8}_{2,5} \to x_6$$

$$\alpha^{9}_{3,5} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_4 - x_6 + 1 &= 0\\
(e_1, e_2, e_5):&\quad -x_3 + x_6 - x_7 &= 0\\
(e_1, e_3, e_4):&\quad x_4 - x_7 &= 0\\
(e_1, e_2, e_8):&\quad -x_1 - 1 &= 0\\
(e_1, e_3, e_7):&\quad -x_1 - x_5 &= 0\\
(e_1, e_4, e_6):&\quad -x_2 - x_5 &= 0\\
(e_2, e_3, e_5):&\quad -x_1 x_6 + x_2 + x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 6 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 2x_7 - 1 = 0$$

$$x_4 - x_7 = 0$$

$$x_5 - 1 = 0$$

$$x_6 + x_7 - 1 = 0$$

$\mathfrak{m}_{2B}(4,10)$

m2B410 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_2,e_5]=e_9\\
&[e_2,e_9]=e_{10} && [e_3,e_4]=-e_9\\
&[e_3,e_8]=\alpha_{3,8}^{10}e_{10} && [e_4,e_7]=\alpha_{4,7}^{10}e_{10}\\
&[e_5,e_6]=\alpha_{5,6}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8): &\quad -\alpha_{3,8}^{10}-1 &&=0\\
(e_1,e_3,e_7): &\quad -\alpha_{3,8}^{10}-\alpha_{4,7}^{10} &&=0\\
(e_1,e_4,e_6): &\quad -\alpha_{4,7}^{10}-\alpha_{5,6}^{10} &&=0\\
(e_2,e_3,e_4): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(4,10)$

m4B410 (this line included for string searching purposes)
Solution 1

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_2,e_3]=e_7\\
&[e_2,e_4]=e_8 && [e_2,e_5]=3e_9\\
&[e_2,e_9]=e_{10} && [e_3,e_4]=-2e_9\\
&[e_3,e_8]=-e_{10} && [e_4,e_7]=e_{10}\\
&[e_5,e_6]=-e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_2, e_3] = e_7 \\
[e_2, e_4] = e_8 & [e_2, e_5] = \alpha_{2,5}^{9} e_9 \\
[e_2, e_9] = e_{10} & [e_3, e_4] = \alpha_{3,4}^{9} e_9 \\
[e_3, e_8] = \alpha_{3,8}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{10} e_{10} \\
[e_5, e_6] = \alpha_{5,6}^{10} e_{10} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_4): & -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 & = 0 \\
(e_1, e_2, e_8): & -\alpha_{3,8}^{10} - 1 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} & = 0 \\
(e_2, e_3, e_4): & \alpha_{3,4}^{9} - \alpha_{3,8}^{10} + \alpha_{4,7}^{10} & = 0
\end{array}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,8}^{10} &= -1 \\
\alpha_{5,6}^{10} &= -1 \\
\alpha_{2,5}^{9} &= 3 \\
\alpha_{4,7}^{10} &= 1 \\
\alpha_{3,4}^{9} &= -2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,8}^{10} &\to x_1 \\
\alpha_{5,6}^{10} &\to x_2 \\
\alpha_{2,5}^{9} &\to x_3 \\
\alpha_{4,7}^{10} &\to x_4 \\
\alpha_{3,4}^{9} &\to x_5
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): \quad & -x_3 - x_5 + 1 & = 0 \\
(e_1, e_2, e_8): \quad & -x_1 - 1 & = 0 \\
(e_1, e_3, e_7): \quad & -x_1 - x_4 & = 0 \\
(e_1, e_4, e_6): \quad & -x_2 - x_4 & = 0 \\
(e_2, e_3, e_4): \quad & -x_1 + x_4 + x_5 & = 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 - 3 = 0$$

$$x_4 - 1 = 0$$

$$x_5 + 2 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = -1$$

$$x_3 = 3$$

$$x_4 = 1$$

$$x_5 = -2$$

$\mathfrak{m}_{3B}(5, 10)$

m3B510 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9 & [e_2, e_9] &= e_{10} \\
[e_3, e_8] &= -e_{10} & [e_4, e_7] &= e_{10} \\
[e_5, e_6] &= -e_{10}
\end{aligned}$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_8 \\
&[e_2, e_4] = e_9 && [e_2, e_9] = e_{10} \\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{10} e_{10} \\
&[e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &&= 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &&= 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{5,6}^{10} &= -1 \\
\alpha_{4,7}^{10} &= 1 \\
\alpha_{3,8}^{10} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{5,6}^{10} &\to x_1 \\
\alpha_{4,7}^{10} &\to x_2 \\
\alpha_{3,8}^{10} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - 1 &&= 0 \\
(e_1, e_3, e_7): &\quad -x_2 - x_3 &&= 0 \\
(e_1, e_4, e_6): &\quad -x_1 - x_2 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = -1$$

$\mathfrak{m}_{2B}(6, 10)$

m2B610 (this line included for string searching purposes)

Solution 1

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_2, e_3] &= e_9 \\ [e_2, e_9] &= e_{10} & [e_3, e_8] &= -e_{10} \\ [e_4, e_7] &= e_{10} & [e_5, e_6] &= -e_{10} \end{aligned}$$

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_2, e_3] &= e_9 \\ [e_2, e_9] &= e_{10} & [e_3, e_8] &= \alpha_{3,8}^{10} e_{10} \\ [e_4, e_7] &= \alpha_{4,7}^{10} e_{10} & [e_5, e_6] &= \alpha_{5,6}^{10} e_{10} \end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned} (e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &= 0 \\ (e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &= 0 \\ (e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &= 0 \end{aligned}$$

Solution 1:

$$\alpha^{10}_{5,6} = -1$$
$$\alpha^{10}_{4,7} = 1$$
$$\alpha^{10}_{3,8} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{10}_{5,6} \to x_1$$
$$\alpha^{10}_{4,7} \to x_2$$
$$\alpha^{10}_{3,8} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_3 &&= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_2 &&= 0
\end{aligned}$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = -1$$

$\mathfrak{m}_{2B}(2, 12)$

m2B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_9] &= e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_8] &= -e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_7] &= e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_6] &= -e_{11} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0 \\
(e_2, e_3, e_8): &\quad \text{no solutions} \\
(e_2, e_4, e_7): &\quad \text{no solutions} \\
(e_2, e_5, e_6): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2, 12)$

m4B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_7] = e_9 \\
[e_2, e_8] = 3e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{11}] = e_{12} & [e_3, e_6] = -e_9 \\
[e_3, e_7] = -2e_{10} & [e_3, e_8] = \alpha_{3,8}^{11} e_{11} \\
[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} & [e_4, e_5] = e_9 \\
[e_4, e_6] = e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_9] = \alpha_{4,9}^{12} e_{12} & [e_5, e_6] = \alpha_{5,6}^{11} e_{11} \\
[e_5, e_8] = \alpha_{5,8}^{12} e_{12} & [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_8): & -\alpha_{2,9}^{11} - \alpha_{3,8}^{11} + 3 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{11} - \alpha_{4,7}^{11} - 2 & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{11} - \alpha_{5,6}^{11} + 1 & = 0 \\
(e_2, e_3, e_6): & -\alpha_{2,9}^{11} & = 0 \\
(e_2, e_4, e_5): & \alpha_{2,9}^{11} & = 0 \\
(e_1, e_2, e_{10}): & -\alpha_{3,10}^{12} - 1 & = 0 \\
(e_1, e_3, e_9): & -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} & = 0 \\
(e_1, e_4, e_8): & -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} & = 0 \\
(e_1, e_5, e_7): & -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} & = 0 \\
(e_2, e_3, e_8): & -3\alpha_{3,10}^{12} + \alpha_{3,8}^{11} & = 0 \\
(e_2, e_4, e_7): & \alpha_{4,7}^{11} - \alpha_{4,9}^{12} & = 0 \\
(e_2, e_5, e_6): & \alpha_{5,6}^{11} & = 0 \\
(e_3, e_4, e_6): & \alpha_{3,10}^{12} + \alpha_{4,9}^{12} & = 0
\end{array}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{11} \to x_1$$

$$\alpha_{5,8}^{12} \to x_2$$

$$\alpha_{4,7}^{11} \to x_3$$

$$\alpha_{5,6}^{11} \to x_4$$

$$\alpha_{3,8}^{11} \to x_5$$

$$\alpha_{4,9}^{12} \to x_6$$

$$\alpha_{3,10}^{12} \to x_7$$

$$\alpha_{6,7}^{12} \to x_8$$

Jacobi Tests

$$\begin{array}{rll}
(e_1, e_2, e_8): & -x_1 - x_5 + 3 & = 0 \\
(e_1, e_3, e_7): & -x_3 - x_5 - 2 & = 0 \\
(e_1, e_4, e_6): & -x_3 - x_4 + 1 & = 0 \\
(e_2, e_3, e_6): & -x_1 & = 0 \\
(e_2, e_4, e_5): & x_1 & = 0 \\
(e_1, e_2, e_{10}): & -x_7 - 1 & = 0 \\
(e_1, e_3, e_9): & -x_6 - x_7 & = 0 \\
(e_1, e_4, e_8): & -x_2 - x_6 & = 0 \\
(e_1, e_5, e_7): & -x_2 - x_8 & = 0 \\
(e_2, e_3, e_8): & x_5 - 3x_7 & = 0 \\
(e_2, e_4, e_7): & x_3 - x_6 & = 0 \\
(e_2, e_5, e_6): & x_4 & = 0 \\
(e_3, e_4, e_6): & x_6 + x_7 & = 0
\end{array}$$

Groebner basis (8 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6B}(2, 12)$

m6B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_5] = e_7 \\
[e_2, e_6] = 2e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{11}] = e_{12} & [e_3, e_4] = -e_7 \\
[e_3, e_5] = -e_8 & [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
[e_3, e_7] = \alpha_{3,7}^{10} e_{10} & [e_3, e_8] = \alpha_{3,8}^{11} e_{11} \\
[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_9] = \alpha_{4,9}^{12} e_{12} & [e_5, e_6] = \alpha_{5,6}^{11} e_{11} \\
[e_5, e_8] = \alpha_{5,8}^{12} e_{12} & [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0\\
(e_2, e_3, e_8): &\quad -\alpha_{2,8}^{10}\alpha_{3,10}^{12} + \alpha_{3,8}^{11} &&= 0\\
(e_2, e_4, e_7): &\quad -\alpha_{2,7}^{9}\alpha_{4,9}^{12} + \alpha_{4,7}^{11} &&= 0\\
(e_2, e_5, e_6): &\quad \alpha_{5,6}^{11} - 2\alpha_{5,8}^{12} + \alpha_{6,7}^{12} &&= 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,10}^{12}\alpha_{4,6}^{10} - \alpha_{3,6}^{9}\alpha_{4,9}^{12} - \alpha_{6,7}^{12} &&= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{11} \to x_1$$
$$\alpha_{3,7}^{10} \to x_2$$
$$\alpha_{5,8}^{12} \to x_3$$
$$\alpha_{2,7}^{9} \to x_4$$
$$\alpha_{2,8}^{10} \to x_5$$
$$\alpha_{4,5}^{9} \to x_6$$
$$\alpha_{4,7}^{11} \to x_7$$

$$\alpha^{9}_{3,6} \to x_8$$

$$\alpha^{11}_{3,8} \to x_9$$

$$\alpha^{11}_{5,6} \to x_{10}$$

$$\alpha^{12}_{4,9} \to x_{11}$$

$$\alpha^{12}_{3,10} \to x_{12}$$

$$\alpha^{10}_{4,6} \to x_{13}$$

$$\alpha^{12}_{6,7} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_4 - x_8 + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -x_6 - x_8 - 1 &&= 0\\
(e_2, e_3, e_4):&\quad -x_4 &&= 0\\
(e_1, e_2, e_7):&\quad -x_2 + x_4 - x_5 &&= 0\\
(e_1, e_3, e_6):&\quad -x_{13} - x_2 + x_8 &&= 0\\
(e_1, e_4, e_5):&\quad -x_{13} + x_6 &&= 0\\
(e_2, e_3, e_5):&\quad -x_2 - x_5 &&= 0\\
(e_1, e_2, e_8):&\quad -x_1 + x_5 - x_9 &&= 0\\
(e_1, e_3, e_7):&\quad x_2 - x_7 - x_9 &&= 0\\
(e_1, e_4, e_6):&\quad -x_{10} + x_{13} - x_7 &&= 0\\
(e_2, e_3, e_6):&\quad x_1 x_8 - 2x_9 &&= 0\\
(e_2, e_4, e_5):&\quad x_1 x_6 - x_7 &&= 0\\
(e_1, e_2, e_{10}):&\quad -x_{12} - 1 &&= 0\\
(e_1, e_3, e_9):&\quad -x_{11} - x_{12} &&= 0\\
(e_1, e_4, e_8):&\quad -x_{11} - x_3 &&= 0\\
(e_1, e_5, e_7):&\quad -x_{14} - x_3 &&= 0\\
(e_2, e_3, e_8):&\quad -x_{12} x_5 + x_9 &&= 0\\
(e_2, e_4, e_7):&\quad -x_{11} x_4 + x_7 &&= 0\\
(e_2, e_5, e_6):&\quad x_{10} + x_{14} - 2x_3 &&= 0\\
(e_3, e_4, e_6):&\quad -x_{11} x_8 + x_{12} x_{13} - x_{14} &&= 0
\end{aligned}$$

Groebner basis (14 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{8B}(2,12)$

m8B212 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_2,e_3]=e_5$$
$$[e_2,e_4]=e_6 \qquad [e_2,e_5]=e_7\left(\frac{1}{3}+\frac{2\sqrt{2}i}{3}\right)$$
$$[e_2,e_6]=e_8\left(-\frac{1}{3}+\frac{4\sqrt{2}i}{3}\right) \qquad [e_2,e_7]=e_9\left(-1+\sqrt{2}i\right)$$
$$[e_2,e_8]=e_{10}\left(-\frac{5}{3}-\frac{\sqrt{2}i}{3}\right) \qquad [e_2,e_9]=e_{11}\left(-\frac{7}{3}-\frac{2\sqrt{2}i}{3}\right)$$
$$[e_2,e_{11}]=e_{12} \qquad [e_3,e_4]=e_7\left(\frac{2}{3}-\frac{2\sqrt{2}i}{3}\right)$$
$$[e_3,e_5]=e_8\left(\frac{2}{3}-\frac{2\sqrt{2}i}{3}\right) \qquad [e_3,e_6]=e_9\left(\frac{2}{3}+\frac{\sqrt{2}i}{3}\right)$$
$$[e_3,e_7]=e_{10}\left(\frac{2}{3}+\frac{4\sqrt{2}i}{3}\right) \qquad [e_3,e_8]=e_{11}\left(\frac{2}{3}+\frac{\sqrt{2}i}{3}\right)$$
$$[e_3,e_{10}]=-e_{12} \qquad [e_4,e_5]=-\sqrt{2}ie_9$$
$$[e_4,e_6]=-\sqrt{2}ie_{10} \qquad [e_4,e_7]=\sqrt{2}ie_{11}$$
$$[e_4,e_9]=e_{12} \qquad [e_5,e_6]=-2\sqrt{2}ie_{11}$$
$$[e_5,e_8]=-e_{12} \qquad [e_6,e_7]=e_{12}$$

Solution 2

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7\left(\frac{1}{3} - \frac{2\sqrt{2}i}{3}\right)$$
$$[e_2, e_6] = e_8\left(-\frac{1}{3} - \frac{4\sqrt{2}i}{3}\right) \qquad [e_2, e_7] = e_9\left(-1 - \sqrt{2}i\right)$$
$$[e_2, e_8] = e_{10}\left(-\frac{5}{3} + \frac{\sqrt{2}i}{3}\right) \qquad [e_2, e_9] = e_{11}\left(-\frac{7}{3} + \frac{2\sqrt{2}i}{3}\right)$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = e_7\left(\frac{2}{3} + \frac{2\sqrt{2}i}{3}\right)$$
$$[e_3, e_5] = e_8\left(\frac{2}{3} + \frac{2\sqrt{2}i}{3}\right) \qquad [e_3, e_6] = e_9\left(\frac{2}{3} - \frac{\sqrt{2}i}{3}\right)$$
$$[e_3, e_7] = e_{10}\left(\frac{2}{3} - \frac{4\sqrt{2}i}{3}\right) \qquad [e_3, e_8] = e_{11}\left(\frac{2}{3} - \frac{\sqrt{2}i}{3}\right)$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = \sqrt{2}ie_9$$
$$[e_4, e_6] = \sqrt{2}ie_{10} \qquad [e_4, e_7] = -\sqrt{2}ie_{11}$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_6] = 2\sqrt{2}ie_{11}$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Solution 3

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7 \left(1 - \frac{\sqrt{10}}{5}\right)$$
$$[e_2, e_6] = e_8 \left(1 - \frac{2\sqrt{10}}{5}\right) \qquad [e_2, e_7] = e_9 \left(\frac{5}{3} - \frac{2\sqrt{10}}{3}\right)$$
$$[e_2, e_8] = e_{10} \left(3 - \sqrt{10}\right) \qquad [e_2, e_9] = e_{11} \left(7 - 2\sqrt{10}\right)$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = \frac{\sqrt{10}e_7}{5}$$
$$[e_3, e_5] = \frac{\sqrt{10}e_8}{5} \qquad [e_3, e_6] = e_9 \left(-\frac{2}{3} + \frac{4\sqrt{10}}{15}\right)$$
$$[e_3, e_7] = e_{10} \left(-\frac{4}{3} + \frac{\sqrt{10}}{3}\right) \qquad [e_3, e_8] = e_{11} \left(-4 + \sqrt{10}\right)$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = e_9 \left(\frac{2}{3} - \frac{\sqrt{10}}{15}\right)$$
$$[e_4, e_6] = e_{10} \left(\frac{2}{3} - \frac{\sqrt{10}}{15}\right) \qquad [e_4, e_7] = e_{11} \left(\frac{8}{3} - \frac{2\sqrt{10}}{3}\right)$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_6] = e_{11} \left(-2 + \frac{3\sqrt{10}}{5}\right)$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Solution 4

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7 \left(\frac{\sqrt{10}}{5} + 1\right)$$
$$[e_2, e_6] = e_8 \left(1 + \frac{2\sqrt{10}}{5}\right) \qquad [e_2, e_7] = e_9 \left(\frac{5}{3} + \frac{2\sqrt{10}}{3}\right)$$
$$[e_2, e_8] = e_{10} \left(3 + \sqrt{10}\right) \qquad [e_2, e_9] = e_{11} \left(2\sqrt{10} + 7\right)$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = -\frac{\sqrt{10}e_7}{5}$$
$$[e_3, e_5] = -\frac{\sqrt{10}e_8}{5} \qquad [e_3, e_6] = e_9 \left(-\frac{4\sqrt{10}}{15} - \frac{2}{3}\right)$$
$$[e_3, e_7] = e_{10} \left(-\frac{4}{3} - \frac{\sqrt{10}}{3}\right) \qquad [e_3, e_8] = e_{11} \left(-4 - \sqrt{10}\right)$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = e_9 \left(\frac{\sqrt{10}}{15} + \frac{2}{3}\right)$$
$$[e_4, e_6] = e_{10} \left(\frac{\sqrt{10}}{15} + \frac{2}{3}\right) \qquad [e_4, e_7] = e_{11} \left(\frac{2\sqrt{10}}{3} + \frac{8}{3}\right)$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_6] = e_{11} \left(-2 - \frac{3\sqrt{10}}{5}\right)$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{11}] = e_{12} & [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
[e_3, e_5] = \alpha_{3,5}^{8} e_8 & [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
[e_3, e_7] = \alpha_{3,7}^{10} e_{10} & [e_3, e_8] = \alpha_{3,8}^{11} e_{11} \\
[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_9] = \alpha_{4,9}^{12} e_{12} & [e_5, e_6] = \alpha_{5,6}^{11} e_{11} \\
[e_5, e_8] = \alpha_{5,8}^{12} e_{12} & [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha_{2,5}^{7}-\alpha_{3,4}^{7}+1 &&=0\\
(e_1,e_2,e_5): &\quad \alpha_{2,5}^{7}-\alpha_{2,6}^{8}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_3,e_4): &\quad \alpha_{3,4}^{7}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_2,e_6): &\quad \alpha_{2,6}^{8}-\alpha_{2,7}^{9}-\alpha_{3,6}^{9} &&=0\\
(e_1,e_3,e_5): &\quad \alpha_{3,5}^{8}-\alpha_{3,6}^{9}-\alpha_{4,5}^{9} &&=0\\
(e_2,e_3,e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7}-\alpha_{3,6}^{9}+\alpha_{4,5}^{9} &&=0\\
(e_1,e_2,e_7): &\quad \alpha_{2,7}^{9}-\alpha_{2,8}^{10}-\alpha_{3,7}^{10} &&=0\\
(e_1,e_3,e_6): &\quad \alpha_{3,6}^{9}-\alpha_{3,7}^{10}-\alpha_{4,6}^{10} &&=0\\
(e_1,e_4,e_5): &\quad \alpha_{4,5}^{9}-\alpha_{4,6}^{10} &&=0\\
(e_2,e_3,e_5): &\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10}+\alpha_{2,8}^{10}\alpha_{3,5}^{8} &&=0\\
(e_1,e_2,e_8): &\quad \alpha_{2,8}^{10}-\alpha_{2,9}^{11}-\alpha_{3,8}^{11} &&=0\\
(e_1,e_3,e_7): &\quad \alpha_{3,7}^{10}-\alpha_{3,8}^{11}-\alpha_{4,7}^{11} &&=0\\
(e_1,e_4,e_6): &\quad \alpha_{4,6}^{10}-\alpha_{4,7}^{11}-\alpha_{5,6}^{11} &&=0\\
(e_2,e_3,e_6): &\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11}+\alpha_{2,9}^{11}\alpha_{3,6}^{9}-\alpha_{5,6}^{11} &&=0\\
(e_2,e_4,e_5): &\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11}+\alpha_{2,9}^{11}\alpha_{4,5}^{9}+\alpha_{5,6}^{11} &&=0\\
(e_1,e_2,e_{10}): &\quad -\alpha_{3,10}^{12}-1 &&=0\\
(e_1,e_3,e_9): &\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &&=0\\
(e_1,e_4,e_8): &\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &&=0\\
(e_1,e_5,e_7): &\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &&=0\\
(e_2,e_3,e_8): &\quad -\alpha_{2,8}^{10}\alpha_{3,10}^{12}+\alpha_{3,8}^{11}-\alpha_{5,8}^{12} &&=0\\
(e_2,e_4,e_7): &\quad -\alpha_{2,7}^{9}\alpha_{4,9}^{12}+\alpha_{4,7}^{11}-\alpha_{6,7}^{12} &&=0\\
(e_2,e_5,e_6): &\quad \alpha_{2,5}^{7}\alpha_{6,7}^{12}-\alpha_{2,6}^{8}\alpha_{5,8}^{12}+\alpha_{5,6}^{11} &&=0\\
(e_3,e_4,e_6): &\quad \alpha_{3,10}^{12}\alpha_{4,6}^{10}+\alpha_{3,4}^{7}\alpha_{6,7}^{12}-\alpha_{3,6}^{9}\alpha_{4,9}^{12} &&=0
\end{aligned}$$

Solution 1:

$$\alpha_{2,9}^{11} = -7/3 - 2 * sqrt(2) * I/3$$
$$\alpha_{3,7}^{10} = 2/3 + 4 * sqrt(2) * I/3$$
$$\alpha_{5,8}^{12} = -1$$
$$\alpha_{2,7}^{9} = -1 + sqrt(2) * I$$
$$\alpha_{2,6}^{8} = -1/3 + 4 * sqrt(2) * I/3$$
$$\alpha_{2,5}^{7} = 1/3 + 2 * sqrt(2) * I/3$$
$$\alpha_{2,8}^{10} = -5/3 - sqrt(2) * I/3$$
$$\alpha_{4,5}^{9} = -sqrt(2) * I$$
$$\alpha_{4,7}^{11} = sqrt(2) * I$$
$$\alpha_{3,6}^{9} = 2/3 + sqrt(2) * I/3$$
$$\alpha_{3,8}^{11} = 2/3 + sqrt(2) * I/3$$
$$\alpha_{5,6}^{11} = -2 * sqrt(2) * I$$
$$\alpha_{3,10}^{12} = -1$$
$$\alpha_{3,4}^{7} = 2/3 - 2 * sqrt(2) * I/3$$
$$\alpha_{4,9}^{12} = 1$$
$$\alpha_{3,5}^{8} = 2/3 - 2 * sqrt(2) * I/3$$
$$\alpha_{4,6}^{10} = -sqrt(2) * I$$
$$\alpha_{6,7}^{12} = 1$$

Solution 2:

$$\alpha^{11}_{2,9} = -7/3 + 2 * sqrt(2) * I/3$$
$$\alpha^{10}_{3,7} = 2/3 - 4 * sqrt(2) * I/3$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{9}_{2,7} = -1 - sqrt(2) * I$$
$$\alpha^{8}_{2,6} = -1/3 - 4 * sqrt(2) * I/3$$
$$\alpha^{7}_{2,5} = 1/3 - 2 * sqrt(2) * I/3$$
$$\alpha^{10}_{2,8} = -5/3 + sqrt(2) * I/3$$
$$\alpha^{9}_{4,5} = sqrt(2) * I$$
$$\alpha^{11}_{4,7} = -sqrt(2) * I$$
$$\alpha^{9}_{3,6} = 2/3 - sqrt(2) * I/3$$
$$\alpha^{11}_{3,8} = 2/3 - sqrt(2) * I/3$$
$$\alpha^{11}_{5,6} = 2 * sqrt(2) * I$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{7}_{3,4} = 2/3 + 2 * sqrt(2) * I/3$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{8}_{3,5} = 2/3 + 2 * sqrt(2) * I/3$$
$$\alpha^{10}_{4,6} = sqrt(2) * I$$
$$\alpha^{12}_{6,7} = 1$$

Solution 3:

$$\alpha_{2,9}^{11} = 7 - 2 * sqrt(10)$$
$$\alpha_{3,7}^{10} = -4/3 + sqrt(10)/3$$
$$\alpha_{5,8}^{12} = -1$$
$$\alpha_{2,7}^{9} = 5/3 - 2 * sqrt(10)/3$$
$$\alpha_{2,6}^{8} = 1 - 2 * sqrt(10)/5$$
$$\alpha_{2,5}^{7} = 1 - sqrt(10)/5$$
$$\alpha_{2,8}^{10} = 3 - sqrt(10)$$
$$\alpha_{4,5}^{9} = 2/3 - sqrt(10)/15$$
$$\alpha_{4,7}^{11} = 8/3 - 2 * sqrt(10)/3$$
$$\alpha_{3,6}^{9} = -2/3 + 4 * sqrt(10)/15$$
$$\alpha_{3,8}^{11} = -4 + sqrt(10)$$
$$\alpha_{5,6}^{11} = -2 + 3 * sqrt(10)/5$$
$$\alpha_{3,10}^{12} = -1$$
$$\alpha_{3,4}^{7} = sqrt(10)/5$$
$$\alpha_{4,9}^{12} = 1$$
$$\alpha_{3,5}^{8} = sqrt(10)/5$$
$$\alpha_{4,6}^{10} = 2/3 - sqrt(10)/15$$
$$\alpha_{6,7}^{12} = 1$$

Solution 4:

$$\alpha_{2,9}^{11} = 2 * sqrt(10) + 7$$
$$\alpha_{3,7}^{10} = -4/3 - sqrt(10)/3$$
$$\alpha_{5,8}^{12} = -1$$
$$\alpha_{2,7}^{9} = 5/3 + 2 * sqrt(10)/3$$
$$\alpha_{2,6}^{8} = 1 + 2 * sqrt(10)/5$$
$$\alpha_{2,5}^{7} = sqrt(10)/5 + 1$$
$$\alpha_{2,8}^{10} = 3 + sqrt(10)$$
$$\alpha_{4,5}^{9} = sqrt(10)/15 + 2/3$$
$$\alpha_{4,7}^{11} = 2 * sqrt(10)/3 + 8/3$$
$$\alpha_{3,6}^{9} = -4 * sqrt(10)/15 - 2/3$$
$$\alpha_{3,8}^{11} = -4 - sqrt(10)$$
$$\alpha_{5,6}^{11} = -2 - 3 * sqrt(10)/5$$
$$\alpha_{3,10}^{12} = -1$$
$$\alpha_{3,4}^{7} = -sqrt(10)/5$$
$$\alpha_{4,9}^{12} = 1$$
$$\alpha_{3,5}^{8} = -sqrt(10)/5$$
$$\alpha_{4,6}^{10} = sqrt(10)/15 + 2/3$$
$$\alpha_{6,7}^{12} = 1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{11} \to x_1$$
$$\alpha_{3,7}^{10} \to x_2$$
$$\alpha_{5,8}^{12} \to x_3$$
$$\alpha_{2,7}^{9} \to x_4$$
$$\alpha_{2,6}^{8} \to x_5$$
$$\alpha_{2,5}^{7} \to x_6$$
$$\alpha_{2,8}^{10} \to x_7$$
$$\alpha_{4,5}^{9} \to x_8$$
$$\alpha_{4,7}^{11} \to x_9$$

$$\alpha^{9}_{3,6} \to x_{10}$$
$$\alpha^{11}_{3,8} \to x_{11}$$
$$\alpha^{11}_{5,6} \to x_{12}$$
$$\alpha^{12}_{3,10} \to x_{13}$$
$$\alpha^{7}_{3,4} \to x_{14}$$
$$\alpha^{12}_{4,9} \to x_{15}$$
$$\alpha^{8}_{3,5} \to x_{16}$$
$$\alpha^{10}_{4,6} \to x_{17}$$
$$\alpha^{12}_{6,7} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{14} - x_6 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{16} - x_5 + x_6 &&= 0\\
(e_1, e_3, e_4): &\quad x_{14} - x_{16} &&= 0\\
(e_1, e_2, e_6): &\quad -x_{10} - x_4 + x_5 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{10} + x_{16} - x_8 &&= 0\\
(e_2, e_3, e_4): &\quad -x_{10} + x_{14}x_4 + x_8 &&= 0\\
(e_1, e_2, e_7): &\quad -x_2 + x_4 - x_7 &&= 0\\
(e_1, e_3, e_6): &\quad x_{10} - x_{17} - x_2 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{17} + x_8 &&= 0\\
(e_2, e_3, e_5): &\quad x_{16}x_7 - x_2x_6 &&= 0\\
(e_1, e_2, e_8): &\quad -x_1 - x_{11} + x_7 &&= 0\\
(e_1, e_3, e_7): &\quad -x_{11} + x_2 - x_9 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{12} + x_{17} - x_9 &&= 0\\
(e_2, e_3, e_6): &\quad x_1x_{10} - x_{11}x_5 - x_{12} &&= 0\\
(e_2, e_4, e_5): &\quad x_1x_8 + x_{12} - x_6x_9 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_{13} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_{13} - x_{15} &&= 0\\
(e_1, e_4, e_8): &\quad -x_{15} - x_3 &&= 0\\
(e_1, e_5, e_7): &\quad -x_{18} - x_3 &&= 0\\
(e_2, e_3, e_8): &\quad x_{11} - x_{13}x_7 - x_3 &&= 0\\
(e_2, e_4, e_7): &\quad -x_{15}x_4 - x_{18} + x_9 &&= 0\\
(e_2, e_5, e_6): &\quad x_{12} + x_{18}x_6 - x_3x_5 &&= 0\\
(e_3, e_4, e_6): &\quad -x_{10}x_{15} + x_{13}x_{17} + x_{14}x_{18} &&= 0
\end{aligned}$$

Groebner basis (18 variables, 5 linear, 13 nonlinear)

$$3x_1 - 30x_{17}^3 + 10x_{17}^2 - 62x_{17} + 27 = 0$$
$$15x_{17}^3 - 5x_{17}^2 + 46x_{17} + 12x_2 - 18 = 0$$
$$x_3 + 1 = 0$$
$$-15x_{17}^3 + 5x_{17}^2 - 26x_{17} + 4x_4 + 14 = 0$$
$$-15x_{17}^3 + 5x_{17}^2 - 22x_{17} + 6x_5 + 12 = 0$$
$$-15x_{17}^3 + 5x_{17}^2 - 22x_{17} + 12x_6 + 6 = 0$$
$$-15x_{17}^3 + 5x_{17}^2 - 31x_{17} + 3x_7 + 15 = 0$$
$$-x_{17} + x_8 = 0$$
$$-15x_{17}^3 + 5x_{17}^2 - 26x_{17} + 4x_9 + 10 = 0$$
$$12x_{10} + 15x_{17}^3 - 5x_{17}^2 + 34x_{17} - 18 = 0$$
$$3x_{11} + 15x_{17}^3 - 5x_{17}^2 + 31x_{17} - 12 = 0$$
$$4x_{12} + 15x_{17}^3 - 5x_{17}^2 + 22x_{17} - 10 = 0$$
$$x_{13} + 1 = 0$$
$$12x_{14} + 15x_{17}^3 - 5x_{17}^2 + 22x_{17} - 18 = 0$$
$$x_{15} - 1 = 0$$
$$12x_{16} + 15x_{17}^3 - 5x_{17}^2 + 22x_{17} - 18 = 0$$
$$15x_{17}^4 - 20x_{17}^3 + 36x_{17}^2 - 40x_{17} + 12 = 0$$
$$x_{18} - 1 = 0$$

Solution 1:

$$x_1 = -7/3 - 2 * sqrt(2) * I/3$$
$$x_2 = 2/3 + 4 * sqrt(2) * I/3$$
$$x_3 = -1$$
$$x_4 = -1 + sqrt(2) * I$$
$$x_5 = -1/3 + 4 * sqrt(2) * I/3$$
$$x_6 = 1/3 + 2 * sqrt(2) * I/3$$
$$x_7 = -5/3 - sqrt(2) * I/3$$
$$x_8 = -sqrt(2) * I$$
$$x_9 = sqrt(2) * I$$
$$x_10 = 2/3 + sqrt(2) * I/3$$
$$x_11 = 2/3 + sqrt(2) * I/3$$

$$x_12 = -2 * sqrt(2) * I$$

$$x_13 = -1$$

$$x_14 = 2/3 - 2 * sqrt(2) * I/3$$

$$x_15 = 1$$

$$x_16 = 2/3 - 2 * sqrt(2) * I/3$$

$$x_17 = -sqrt(2) * I$$

$$x_18 = 1$$

Solution 2:

$$x_1 = -7/3 + 2 * sqrt(2) * I/3$$

$$x_2 = 2/3 - 4 * sqrt(2) * I/3$$

$$x_3 = -1$$

$$x_4 = -1 - sqrt(2) * I$$

$$x_5 = -1/3 - 4 * sqrt(2) * I/3$$

$$x_6 = 1/3 - 2 * sqrt(2) * I/3$$

$$x_7 = -5/3 + sqrt(2) * I/3$$

$$x_8 = sqrt(2) * I$$

$$x_9 = -sqrt(2) * I$$

$$x_10 = 2/3 - sqrt(2) * I/3$$

$$x_11 = 2/3 - sqrt(2) * I/3$$

$$x_12 = 2 * sqrt(2) * I$$

$$x_13 = -1$$

$$x_14 = 2/3 + 2 * sqrt(2) * I/3$$

$$x_15 = 1$$

$$x_16 = 2/3 + 2 * sqrt(2) * I/3$$

$$x_17 = sqrt(2) * I$$

$$x_18 = 1$$

Solution 3:

$$x_1 = 7 - 2 * sqrt(10)$$

$$x_2 = -4/3 + sqrt(10)/3$$

$$x_3 = -1$$

$$x_4 = 5/3 - 2 * sqrt(10)/3$$

$$x_5 = 1 - 2 * sqrt(10)/5$$

$$x_6 = 1 - sqrt(10)/5$$
$$x_7 = 3 - sqrt(10)$$
$$x_8 = 2/3 - sqrt(10)/15$$
$$x_9 = 8/3 - 2 * sqrt(10)/3$$
$$x_10 = -2/3 + 4 * sqrt(10)/15$$
$$x_11 = -4 + sqrt(10)$$
$$x_12 = -2 + 3 * sqrt(10)/5$$
$$x_13 = -1$$
$$x_14 = sqrt(10)/5$$
$$x_15 = 1$$
$$x_16 = sqrt(10)/5$$
$$x_17 = 2/3 - sqrt(10)/15$$
$$x_18 = 1$$

Solution 4:

$$x_1 = 2 * sqrt(10) + 7$$
$$x_2 = -4/3 - sqrt(10)/3$$
$$x_3 = -1$$
$$x_4 = 5/3 + 2 * sqrt(10)/3$$
$$x_5 = 1 + 2 * sqrt(10)/5$$
$$x_6 = sqrt(10)/5 + 1$$
$$x_7 = 3 + sqrt(10)$$
$$x_8 = sqrt(10)/15 + 2/3$$
$$x_9 = 2 * sqrt(10)/3 + 8/3$$
$$x_10 = -4 * sqrt(10)/15 - 2/3$$
$$x_11 = -4 - sqrt(10)$$
$$x_12 = -2 - 3 * sqrt(10)/5$$
$$x_13 = -1$$
$$x_14 = -sqrt(10)/5$$
$$x_15 = 1$$
$$x_16 = -sqrt(10)/5$$
$$x_17 = sqrt(10)/15 + 2/3$$
$$x_18 = 1$$

$\mathfrak{m}_{3B}(3,12)$

m3B312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_7] &= e_{10}\\
[e_2,e_8] &= 3e_{11} & [e_2,e_{11}] &= e_{12}\\
[e_3,e_6] &= -e_{10} & [e_3,e_7] &= -2e_{11}\\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_5] &= e_{10}\\
[e_4,e_6] &= e_{11} & [e_4,e_9] &= \alpha_{4,9}^{12}e_{12}\\
[e_5,e_8] &= \alpha_{5,8}^{12}e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}):&\quad -\alpha_{3,10}^{12}-1 &&= 0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &&= 0\\
(e_1,e_4,e_8):&\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &&= 0\\
(e_1,e_5,e_7):&\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &&= 0\\
(e_2,e_3,e_7):&\quad -\alpha_{3,10}^{12}-2 &&= 0\\
(e_2,e_4,e_6):&\quad \text{no solutions} &&\\
(e_3,e_4,e_5):&\quad \alpha_{3,10}^{12} &&= 0
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{5B}(3,12)$

m5B312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= 2e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= -e_8 & [e_3, e_5] &= -e_9 \\
[e_3, e_6] &= \alpha_{3,6}^{10} e_{10} & [e_3, e_7] &= \alpha_{3,7}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &= 0 \\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_7): &\quad -\alpha_{2,7}^{10}\alpha_{3,10}^{12} + \alpha_{3,7}^{11} &= 0 \\
(e_2, e_4, e_6): &\quad \alpha_{4,6}^{11} - 2\alpha_{4,9}^{12} &= 0 \\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{12}\alpha_{4,5}^{10} + \alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{5,8}^{12} \to x_1$$

$$\alpha^{11}_{2,8} \to x_2$$

$$\alpha^{10}_{3,6} \to x_3$$

$$\alpha^{11}_{4,6} \to x_4$$

$$\alpha^{11}_{3,7} \to x_5$$

$$\alpha^{10}_{4,5} \to x_6$$

$$\alpha^{12}_{4,9} \to x_7$$

$$\alpha^{12}_{3,10} \to x_8$$

$$\alpha^{10}_{2,7} \to x_9$$

$$\alpha^{12}_{6,7} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_3 - x_9 + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -x_3 - x_6 - 1 &&= 0\\
(e_1, e_2, e_7):&\quad -x_2 - x_5 + x_9 &&= 0\\
(e_1, e_3, e_6):&\quad x_3 - x_4 - x_5 &&= 0\\
(e_1, e_4, e_5):&\quad -x_4 + x_6 &&= 0\\
(e_2, e_3, e_4):&\quad -x_2 &&= 0\\
(e_1, e_2, e_{10}):&\quad -x_8 - 1 &&= 0\\
(e_1, e_3, e_9):&\quad -x_7 - x_8 &&= 0\\
(e_1, e_4, e_8):&\quad -x_1 - x_7 &&= 0\\
(e_1, e_5, e_7):&\quad -x_1 - x_{10} &&= 0\\
(e_2, e_3, e_7):&\quad x_5 - x_8 x_9 &&= 0\\
(e_2, e_4, e_6):&\quad x_4 - 2x_7 &&= 0\\
(e_3, e_4, e_5):&\quad -x_1 + x_6 x_8 + x_7 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{7B}(3, 12)$

m7B312 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = e_8$$
$$[e_2, e_6] = e_9 \qquad [e_2, e_7] = e_{10}$$
$$[e_2, e_8] = e_{11} \qquad [e_2, e_{11}] = e_{12}$$
$$[e_3, e_4] = 0 \qquad [e_3, e_5] = 0$$
$$[e_3, e_6] = 0 \qquad [e_3, e_7] = 0$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = 0$$
$$[e_4, e_6] = 0 \qquad [e_4, e_9] = e_{12}$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Solution 2

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{8e_8}{5}$$
$$[e_2, e_6] = \frac{11e_9}{5} \qquad [e_2, e_7] = 4e_{10}$$
$$[e_2, e_8] = 7e_{11} \qquad [e_2, e_{11}] = e_{12}$$
$$[e_3, e_4] = -\frac{3e_8}{5} \qquad [e_3, e_5] = -\frac{3e_9}{5}$$
$$[e_3, e_6] = -\frac{9e_{10}}{5} \qquad [e_3, e_7] = -3e_{11}$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = \frac{6e_{10}}{5}$$
$$[e_4, e_6] = \frac{6e_{11}}{5} \qquad [e_4, e_9] = e_{12}$$
$$[e_5, e_8] = -e_{12} \qquad [e_6, e_7] = e_{12}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] &= \alpha_{2,6}^{9} e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= \alpha_{3,4}^{8} e_8 & [e_3, e_5] &= \alpha_{3,5}^{9} e_9 \\
[e_3, e_6] &= \alpha_{3,6}^{10} e_{10} & [e_3, e_7] &= \alpha_{3,7}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):& \quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0 \\
(e_1, e_2, e_5):& \quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_3, e_4):& \quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_2, e_6):& \quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &= 0 \\
(e_1, e_3, e_5):& \quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &= 0 \\
(e_1, e_2, e_7):& \quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0 \\
(e_1, e_3, e_6):& \quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0 \\
(e_1, e_4, e_5):& \quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0 \\
(e_2, e_3, e_4):& \quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &= 0 \\
(e_1, e_2, e_{10}):& \quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9):& \quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8):& \quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7):& \quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_7):& \quad -\alpha_{2,7}^{10}\alpha_{3,10}^{12} + \alpha_{3,7}^{11} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_4, e_6):& \quad -\alpha_{2,6}^{9}\alpha_{4,9}^{12} + \alpha_{4,6}^{11} + \alpha_{6,7}^{12} &= 0 \\
(e_3, e_4, e_5):& \quad \alpha_{3,10}^{12}\alpha_{4,5}^{10} + \alpha_{3,4}^{8}\alpha_{5,8}^{12} - \alpha_{3,5}^{9}\alpha_{4,9}^{12} &= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{11}_{2,8} = 1$$
$$\alpha^{10}_{3,6} = 0$$
$$\alpha^{11}_{4,6} = 0$$
$$\alpha^{11}_{3,7} = 0$$
$$\alpha^{9}_{2,6} = 1$$
$$\alpha^{10}_{4,5} = 0$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{8}_{3,4} = 0$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{10}_{2,7} = 1$$
$$\alpha^{8}_{2,5} = 1$$
$$\alpha^{9}_{3,5} = 0$$

Solution 2:

$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{11}_{2,8} = 7$$
$$\alpha^{10}_{3,6} = -9/5$$
$$\alpha^{11}_{4,6} = 6/5$$
$$\alpha^{11}_{3,7} = -3$$
$$\alpha^{9}_{2,6} = 11/5$$
$$\alpha^{10}_{4,5} = 6/5$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{8}_{3,4} = -3/5$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{10}_{2,7} = 4$$
$$\alpha^{8}_{2,5} = 8/5$$
$$\alpha^{9}_{3,5} = -3/5$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{5,8}^{12} \to x_1$$
$$\alpha_{2,8}^{11} \to x_2$$
$$\alpha_{3,6}^{10} \to x_3$$
$$\alpha_{4,6}^{11} \to x_4$$
$$\alpha_{3,7}^{11} \to x_5$$
$$\alpha_{2,6}^{9} \to x_6$$
$$\alpha_{4,5}^{10} \to x_7$$
$$\alpha_{6,7}^{12} \to x_8$$
$$\alpha_{3,4}^{8} \to x_9$$
$$\alpha_{4,9}^{12} \to x_{10}$$
$$\alpha_{3,10}^{12} \to x_{11}$$
$$\alpha_{2,7}^{10} \to x_{12}$$
$$\alpha_{2,5}^{8} \to x_{13}$$
$$\alpha_{3,5}^{9} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{13}-x_9+1 &&=0\\
(e_1,e_2,e_5):&\quad x_{13}-x_{14}-x_6 &&=0\\
(e_1,e_3,e_4):&\quad -x_{14}+x_9 &&=0\\
(e_1,e_2,e_6):&\quad -x_{12}-x_3+x_6 &&=0\\
(e_1,e_3,e_5):&\quad x_{14}-x_3-x_7 &&=0\\
(e_1,e_2,e_7):&\quad x_{12}-x_2-x_5 &&=0\\
(e_1,e_3,e_6):&\quad x_3-x_4-x_5 &&=0\\
(e_1,e_4,e_5):&\quad -x_4+x_7 &&=0\\
(e_2,e_3,e_4):&\quad x_2x_9+x_4-x_5 &&=0\\
(e_1,e_2,e_{10}):&\quad -x_{11}-1 &&=0\\
(e_1,e_3,e_9):&\quad -x_{10}-x_{11} &&=0\\
(e_1,e_4,e_8):&\quad -x_1-x_{10} &&=0\\
(e_1,e_5,e_7):&\quad -x_1-x_8 &&=0\\
(e_2,e_3,e_7):&\quad -x_{11}x_{12}+x_5-x_8 &&=0\\
(e_2,e_4,e_6):&\quad -x_{10}x_6+x_4+x_8 &&=0\\
(e_3,e_4,e_5):&\quad x_1x_9-x_{10}x_{14}+x_{11}x_7 &&=0
\end{aligned}$$

Groebner basis (14 variables, 13 linear, 1 nonlinear)

$$x_1 + 1 = 0$$
$$10x_{14} + x_2 - 1 = 0$$
$$-3x_{14} + x_3 = 0$$
$$2x_{14} + x_4 = 0$$
$$-5x_{14} + x_5 = 0$$
$$2x_{14} + x_6 - 1 = 0$$
$$2x_{14} + x_7 = 0$$
$$x_8 - 1 = 0$$
$$-x_{14} + x_9 = 0$$
$$x_{10} - 1 = 0$$
$$x_{11} + 1 = 0$$
$$x_{12} + 5x_{14} - 1 = 0$$
$$x_{13} + x_{14} - 1 = 0$$
$$5x_{14}^2 + 3x_{14} = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 0$$
$$x_6 = 1$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = 0$$
$$x_10 = 1$$
$$x_11 = -1$$
$$x_12 = 1$$
$$x_13 = 1$$
$$x_14 = 0$$

Solution 2:

$$x_1 = -1$$

$$x_2 = 7$$
$$x_3 = -9/5$$
$$x_4 = 6/5$$
$$x_5 = -3$$
$$x_6 = 11/5$$
$$x_7 = 6/5$$
$$x_8 = 1$$
$$x_9 = -3/5$$
$$x_10 = 1$$
$$x_11 = -1$$
$$x_12 = 4$$
$$x_13 = 8/5$$
$$x_14 = -3/5$$

$\mathfrak{m}_{2B}(4, 12)$

m2B412 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_7] = e_{11}$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_6] = -e_{11}$$
$$[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} \qquad [e_4, e_5] = e_{11}$$
$$[e_4, e_9] = \alpha_{4,9}^{12} e_{12} \qquad [e_5, e_8] = \alpha_{5,8}^{12} e_{12}$$
$$[e_6, e_7] = \alpha_{6,7}^{12} e_{12}$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_{10}) : \quad -\alpha_{3,10}^{12} - 1 = 0$$
$$(e_1, e_3, e_9) : \quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} = 0$$
$$(e_1, e_4, e_8) : \quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} = 0$$
$$(e_1, e_5, e_7) : \quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} = 0$$
$$(e_2, e_3, e_6) : \quad \text{no solutions}$$
$$(e_2, e_4, e_5) : \quad \text{no solutions}$$

There are no solutions.

$\mathfrak{m}_{4B}(4, 12)$

m4B412 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_5] = e_9$$
$$[e_2, e_6] = 2e_{10} \qquad [e_2, e_7] = 4e_{11}$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = -e_9$$
$$[e_3, e_5] = -e_{10} \qquad [e_3, e_6] = -2e_{11}$$
$$[e_3, e_{10}] = -e_{12} \qquad [e_4, e_5] = e_{11}$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_8] = -e_{12}$$
$$[e_6, e_7] = e_{12}$$

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_5] = e_9$$
$$[e_2, e_6] = 2e_{10} \qquad [e_2, e_7] = \alpha_{2,7}^{11} e_{11}$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_4] = -e_9$$
$$[e_3, e_5] = -e_{10} \qquad [e_3, e_6] = \alpha_{3,6}^{11} e_{11}$$
$$[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} \qquad [e_4, e_5] = \alpha_{4,5}^{11} e_{11}$$
$$[e_4, e_9] = \alpha_{4,9}^{12} e_{12} \qquad [e_5, e_8] = \alpha_{5,8}^{12} e_{12}$$
$$[e_6, e_7] = \alpha_{6,7}^{12} e_{12}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &= 0\\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0\\
(e_2, e_3, e_6): &\quad -2\alpha_{3,10}^{12} + \alpha_{3,6}^{11} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,9}^{12} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{5,8}^{12} &= -1\\
\alpha_{3,6}^{11} &= -2\\
\alpha_{4,9}^{12} &= 1\\
\alpha_{3,10}^{12} &= -1\\
\alpha_{4,5}^{11} &= 1\\
\alpha_{2,7}^{11} &= 4\\
\alpha_{6,7}^{12} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{5,8}^{12} &\to x_1\\
\alpha_{3,6}^{11} &\to x_2\\
\alpha_{4,9}^{12} &\to x_3\\
\alpha_{3,10}^{12} &\to x_4\\
\alpha_{4,5}^{11} &\to x_5\\
\alpha_{2,7}^{11} &\to x_6\\
\alpha_{6,7}^{12} &\to x_7
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6):& \quad -x_2 - x_6 + 2 && = 0 \\
(e_1, e_3, e_5):& \quad -x_2 - x_5 - 1 && = 0 \\
(e_1, e_2, e_{10}):& \quad -x_4 - 1 && = 0 \\
(e_1, e_3, e_9):& \quad -x_3 - x_4 && = 0 \\
(e_1, e_4, e_8):& \quad -x_1 - x_3 && = 0 \\
(e_1, e_5, e_7):& \quad -x_1 - x_7 && = 0 \\
(e_2, e_3, e_6):& \quad x_2 - 2x_4 && = 0 \\
(e_2, e_4, e_5):& \quad -x_3 + x_5 && = 0
\end{aligned}
$$

Groebner basis (7 variables, 7 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 2 = 0$$

$$x_3 - 1 = 0$$

$$x_4 + 1 = 0$$

$$x_5 - 1 = 0$$

$$x_6 - 4 = 0$$

$$x_7 - 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = -2$$

$$x_3 = 1$$

$$x_4 = -1$$

$$x_5 = 1$$

$$x_6 = 4$$

$$x_7 = 1$$

$\mathfrak{m}_{6B}(4,12)$

m6B412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_7 \\
[e_2,e_4] &= e_8 & [e_2,e_5] &= \alpha_{2,5}^{9} e_9 \\
[e_2,e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2,e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_2,e_{11}] &= e_{12} & [e_3,e_4] &= \alpha_{3,4}^{9} e_9 \\
[e_3,e_5] &= \alpha_{3,5}^{10} e_{10} & [e_3,e_6] &= \alpha_{3,6}^{11} e_{11} \\
[e_3,e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4,e_5] &= \alpha_{4,5}^{11} e_{11} \\
[e_4,e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5,e_8] &= \alpha_{5,8}^{12} e_{12} \\
[e_6,e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &&= 0 \\
(e_1,e_2,e_5): &\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &&= 0 \\
(e_1,e_3,e_4): &\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &&= 0 \\
(e_1,e_2,e_6): &\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &&= 0 \\
(e_1,e_3,e_5): &\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &&= 0 \\
(e_1,e_2,e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0 \\
(e_1,e_3,e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0 \\
(e_1,e_4,e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0 \\
(e_1,e_5,e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0 \\
(e_2,e_3,e_6): &\quad -\alpha_{2,6}^{10}\alpha_{3,10}^{12} + \alpha_{3,6}^{11} + \alpha_{6,7}^{12} &&= 0 \\
(e_2,e_4,e_5): &\quad -\alpha_{2,5}^{9}\alpha_{4,9}^{12} + \alpha_{4,5}^{11} + \alpha_{5,8}^{12} &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{5,8}^{12} \to x_1$$

$$\alpha_{2,6}^{10} \to x_2$$

$$\alpha_{3,5}^{10} \to x_3$$
$$\alpha_{3,6}^{11} \to x_4$$
$$\alpha_{4,9}^{12} \to x_5$$
$$\alpha_{2,5}^{9} \to x_6$$
$$\alpha_{3,10}^{12} \to x_7$$
$$\alpha_{3,4}^{9} \to x_8$$
$$\alpha_{4,5}^{11} \to x_9$$
$$\alpha_{2,7}^{11} \to x_{10}$$
$$\alpha_{6,7}^{12} \to x_{11}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_6 - x_8 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_2 - x_3 + x_6 &&= 0\\
(e_1, e_3, e_4):&\quad -x_3 + x_8 &&= 0\\
(e_1, e_2, e_6):&\quad -x_{10} + x_2 - x_4 &&= 0\\
(e_1, e_3, e_5):&\quad x_3 - x_4 - x_9 &&= 0\\
(e_1, e_2, e_{10}):&\quad -x_7 - 1 &&= 0\\
(e_1, e_3, e_9):&\quad -x_5 - x_7 &&= 0\\
(e_1, e_4, e_8):&\quad -x_1 - x_5 &&= 0\\
(e_1, e_5, e_7):&\quad -x_1 - x_{11} &&= 0\\
(e_2, e_3, e_6):&\quad x_{11} - x_2 x_7 + x_4 &&= 0\\
(e_2, e_4, e_5):&\quad x_1 - x_5 x_6 + x_9 &&= 0
\end{aligned}$$

Groebner basis (11 variables, 10 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$-x_{10} + 2x_2 + 1 = 0$$
$$x_{10} + 4x_3 - 3 = 0$$
$$x_{10} + 2x_4 + 1 = 0$$
$$x_5 - 1 = 0$$
$$-x_{10} + 4x_6 - 1 = 0$$
$$x_7 + 1 = 0$$
$$x_{10} + 4x_8 - 3 = 0$$
$$-x_{10} + 4x_9 - 5 = 0$$
$$x_{11} - 1 = 0$$

$\mathfrak{m}_{3B}(5,12)$

m3B512 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_{10}\\
[e_2,e_6] &= 2e_{11} & [e_2,e_{11}] &= e_{12}\\
[e_3,e_4] &= -e_{10} & [e_3,e_5] &= -e_{11}\\
[e_3,e_{10}] &= -e_{12} & [e_4,e_9] &= e_{12}\\
[e_5,e_8] &= -e_{12} & [e_6,e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_{10}\\
[e_2,e_6] &= 2e_{11} & [e_2,e_{11}] &= e_{12}\\
[e_3,e_4] &= -e_{10} & [e_3,e_5] &= -e_{11}\\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_9] &= \alpha_{4,9}^{12}e_{12}\\
[e_5,e_8] &= \alpha_{5,8}^{12}e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}): &\quad -\alpha_{3,10}^{12}-1 &&= 0\\
(e_1,e_3,e_9): &\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &&= 0\\
(e_1,e_4,e_8): &\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &&= 0\\
(e_1,e_5,e_7): &\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &&= 0\\
(e_2,e_3,e_5): &\quad -\alpha_{3,10}^{12}-1 &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{6,7} = 1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{3,10} \to x_1$$
$$\alpha^{12}_{5,8} \to x_2$$
$$\alpha^{12}_{4,9} \to x_3$$
$$\alpha^{12}_{6,7} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}):\quad & -x_1 - 1 && = 0\\
(e_1, e_3, e_9):\quad & -x_1 - x_3 && = 0\\
(e_1, e_4, e_8):\quad & -x_2 - x_3 && = 0\\
(e_1, e_5, e_7):\quad & -x_2 - x_4 && = 0\\
(e_2, e_3, e_5):\quad & -x_1 - 1 && = 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 + 1 = 0$$
$$x_3 - 1 = 0$$
$$x_4 - 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = -1$$
$$x_3 = 1$$
$$x_4 = 1$$

$\mathfrak{m}_{5B}(5, 12)$

m5B512 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9 & [e_2, e_5] &= \alpha_{2,5}^{10} e_{10} \\
[e_2, e_6] &= \alpha_{2,6}^{11} e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= \alpha_{3,4}^{10} e_{10} & [e_3, e_5] &= \alpha_{3,5}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &= 0 \\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{10}\alpha_{3,10}^{12} + \alpha_{3,5}^{11} + \alpha_{5,8}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{5,8}^{12} &\to x_1 \\
\alpha_{2,6}^{11} &\to x_2 \\
\alpha_{3,4}^{10} &\to x_3 \\
\alpha_{2,5}^{10} &\to x_4 \\
\alpha_{4,9}^{12} &\to x_5
\end{aligned}
$$

$$\alpha_{3,5}^{11} \to x_6$$
$$\alpha_{3,10}^{12} \to x_7$$
$$\alpha_{6,7}^{12} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_4 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_2 + x_4 - x_6 &= 0\\
(e_1, e_3, e_4): &\quad x_3 - x_6 &= 0\\
(e_1, e_2, e_{10}): &\quad -x_7 - 1 &= 0\\
(e_1, e_3, e_9): &\quad -x_5 - x_7 &= 0\\
(e_1, e_4, e_8): &\quad -x_1 - x_5 &= 0\\
(e_1, e_5, e_7): &\quad -x_1 - x_8 &= 0\\
(e_2, e_3, e_5): &\quad x_1 - x_4 x_7 + x_6 &= 0
\end{aligned}$$

Groebner basis (8 variables, 7 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 + 2x_6 - 1 = 0$$
$$x_3 - x_6 = 0$$
$$x_4 + x_6 - 1 = 0$$
$$x_5 - 1 = 0$$
$$x_7 + 1 = 0$$
$$x_8 - 1 = 0$$

$\mathfrak{m}_{2B}(6, 12)$

m2B612 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= -e_{11}\\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12}\\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0\\
(e_2, e_3, e_4): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(6, 12)$

m4B612 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_9\\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= 3e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= -2e_{11}\\
[e_3, e_{10}] &= -e_{12} & [e_4, e_9] &= e_{12}\\
[e_5, e_8] &= -e_{12} & [e_6, e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_9\\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= \alpha_{2,5}^{11} e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= \alpha_{3,4}^{11} e_{11}\\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12}\\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha^{11}_{2,5} - \alpha^{11}_{3,4} + 1 && = 0\\
(e_1, e_2, e_{10}): &\quad -\alpha^{12}_{3,10} - 1 && = 0\\
(e_1, e_3, e_9): &\quad -\alpha^{12}_{3,10} - \alpha^{12}_{4,9} && = 0\\
(e_1, e_4, e_8): &\quad -\alpha^{12}_{4,9} - \alpha^{12}_{5,8} && = 0\\
(e_1, e_5, e_7): &\quad -\alpha^{12}_{5,8} - \alpha^{12}_{6,7} && = 0\\
(e_2, e_3, e_4): &\quad -\alpha^{12}_{3,10} + \alpha^{11}_{3,4} + \alpha^{12}_{4,9} && = 0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha^{12}_{5,8} &= -1\\
\alpha^{11}_{2,5} &= 3\\
\alpha^{11}_{3,4} &= -2\\
\alpha^{12}_{4,9} &= 1\\
\alpha^{12}_{3,10} &= -1\\
\alpha^{12}_{6,7} &= 1
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha^{12}_{5,8} &\to x_1\\
\alpha^{11}_{2,5} &\to x_2\\
\alpha^{11}_{3,4} &\to x_3\\
\alpha^{12}_{4,9} &\to x_4\\
\alpha^{12}_{3,10} &\to x_5\\
\alpha^{12}_{6,7} &\to x_6
\end{aligned}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_3 + 1 && = 0\\
(e_1, e_2, e_{10}): &\quad -x_5 - 1 && = 0\\
(e_1, e_3, e_9): &\quad -x_4 - x_5 && = 0\\
(e_1, e_4, e_8): &\quad -x_1 - x_4 && = 0\\
(e_1, e_5, e_7): &\quad -x_1 - x_6 && = 0\\
(e_2, e_3, e_4): &\quad x_3 + x_4 - x_5 && = 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 3 = 0$$
$$x_3 + 2 = 0$$
$$x_4 - 1 = 0$$
$$x_5 + 1 = 0$$
$$x_6 - 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 3$$
$$x_3 = -2$$
$$x_4 = 1$$
$$x_5 = -1$$
$$x_6 = 1$$

$\mathfrak{m}_{3B}(7, 12)$

m3B712 (this line included for string searching purposes)

Solution 1

$$\begin{array}{cc}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_3] = e_{10} \\
[e_2, e_4] = e_{11} & [e_2, e_{11}] = e_{12} \\
[e_3, e_{10}] = -e_{12} & [e_4, e_9] = e_{12} \\
[e_5, e_8] = -e_{12} & [e_6, e_7] = e_{12}
\end{array}$$

Original brackets:

$$
\begin{array}{rlrl}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rll}
(e_1, e_2, e_{10}): & -\alpha_{3,10}^{12} - 1 & = 0 \\
(e_1, e_3, e_9): & -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} & = 0 \\
(e_1, e_4, e_8): & -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} & = 0 \\
(e_1, e_5, e_7): & -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} & = 0
\end{array}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,10}^{12} &= -1 \\
\alpha_{5,8}^{12} &= -1 \\
\alpha_{4,9}^{12} &= 1 \\
\alpha_{6,7}^{12} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,10}^{12} &\to x_1 \\
\alpha_{5,8}^{12} &\to x_2 \\
\alpha_{4,9}^{12} &\to x_3 \\
\alpha_{6,7}^{12} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}) : &\quad -x_1 - 1 &= 0\\
(e_1, e_3, e_9) : &\quad -x_1 - x_3 &= 0\\
(e_1, e_4, e_8) : &\quad -x_2 - x_3 &= 0\\
(e_1, e_5, e_7) : &\quad -x_2 - x_4 &= 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 - 1 = 0$$

$$x_4 - 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = -1$$

$$x_3 = 1$$

$$x_4 = 1$$

$\mathfrak{m}_{2B}(8, 12)$

m2B812 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_{10}] &= -e_{12}\\
[e_4, e_9] &= e_{12} & [e_5, e_8] &= -e_{12}\\
[e_6, e_7] &= e_{12}
\end{aligned}$$

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_8] &= \alpha_{5,8}^{12} e_{12} \\
[e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha_{3,10}^{12} &= -1 \\
\alpha_{5,8}^{12} &= -1 \\
\alpha_{4,9}^{12} &= 1 \\
\alpha_{6,7}^{12} &= 1
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha_{3,10}^{12} &\to x_1 \\
\alpha_{5,8}^{12} &\to x_2 \\
\alpha_{4,9}^{12} &\to x_3 \\
\alpha_{6,7}^{12} &\to x_4
\end{aligned}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_1 - x_3 &&= 0\\
(e_1, e_4, e_8): &\quad -x_2 - x_3 &&= 0\\
(e_1, e_5, e_7): &\quad -x_2 - x_4 &&= 0
\end{aligned}
$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 - 1 = 0$$

$$x_4 - 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = -1$$

$$x_3 = 1$$

$$x_4 = 1$$

$\mathfrak{m}_{2B}(2, 14)$

m2B214 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_{11}] &= e_{13}\\
[e_2, e_{13}] &= e_{14} & [e_3, e_{10}] &= -e_{13}\\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_9] &= e_{13}\\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_8] &= -e_{13}\\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_7] &= e_{13}\\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{12}):& \quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}):& \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}):& \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9):& \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8):& \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_{10}):& \quad \text{no solutions}\\
(e_2, e_4, e_9):& \quad \text{no solutions}\\
(e_2, e_5, e_8):& \quad \text{no solutions}\\
(e_2, e_6, e_7):& \quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(2, 14)$

m4B214 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_9] &= e_{11}\\
[e_2, e_{10}] &= 4e_{12} & [e_2, e_{11}] &= \alpha_{2,11}^{13} e_{13}\\
[e_2, e_{13}] &= e_{14} & [e_3, e_8] &= -e_{11}\\
[e_3, e_9] &= -3e_{12} & [e_3, e_{10}] &= \alpha_{3,10}^{13} e_{13}\\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_7] &= e_{11}\\
[e_4, e_8] &= 2e_{12} & [e_4, e_9] &= \alpha_{4,9}^{13} e_{13}\\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_6] &= -e_{11}\\
[e_5, e_7] &= -e_{12} & [e_5, e_8] &= \alpha_{5,8}^{13} e_{13}\\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_7] &= \alpha_{6,7}^{13} e_{13}\\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}):&\quad -\alpha_{2,11}^{13} - \alpha_{3,10}^{13} + 4 &&= 0\\
(e_1, e_3, e_9):&\quad -\alpha_{3,10}^{13} - \alpha_{4,9}^{13} - 3 &&= 0\\
(e_1, e_4, e_8):&\quad -\alpha_{4,9}^{13} - \alpha_{5,8}^{13} + 2 &&= 0\\
(e_1, e_5, e_7):&\quad -\alpha_{5,8}^{13} - \alpha_{6,7}^{13} - 1 &&= 0\\
(e_2, e_3, e_8):&\quad -\alpha_{2,11}^{13} &&= 0\\
(e_2, e_4, e_7):&\quad \alpha_{2,11}^{13} &&= 0\\
(e_2, e_5, e_6):&\quad -\alpha_{2,11}^{13} &&= 0\\
(e_1, e_2, e_{12}):&\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}):&\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}):&\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9):&\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8):&\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_{10}):&\quad \alpha_{3,10}^{13} - 4\alpha_{3,12}^{14} &&= 0\\
(e_2, e_4, e_9):&\quad -\alpha_{4,11}^{14} + \alpha_{4,9}^{13} &&= 0\\
(e_2, e_5, e_8):&\quad \alpha_{5,8}^{13} &&= 0\\
(e_2, e_6, e_7):&\quad \alpha_{6,7}^{13} &&= 0\\
(e_3, e_4, e_8):&\quad 2\alpha_{3,12}^{14} + \alpha_{4,11}^{14} &&= 0\\
(e_3, e_5, e_7):&\quad -\alpha_{3,12}^{14} &&= 0\\
(e_4, e_5, e_6):&\quad -\alpha_{4,11}^{14} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,12}^{14} \to x_1$$

$$\alpha_{5,10}^{14} \to x_2$$

$$\alpha_{7,8}^{14} \to x_3$$

$$\alpha_{5,8}^{13} \to x_4$$

$$\alpha_{3,10}^{13} \to x_5$$

$$\alpha_{4,9}^{13} \to x_6$$

$$\alpha_{6,9}^{14} \to x_7$$

$$\alpha_{4,11}^{14} \to x_8$$

$$\alpha_{2,11}^{13} \to x_9$$

$$\alpha_{6,7}^{13} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_5 - x_9 + 4 &= 0\\
(e_1, e_3, e_9): &\quad -x_5 - x_6 - 3 &= 0\\
(e_1, e_4, e_8): &\quad -x_4 - x_6 + 2 &= 0\\
(e_1, e_5, e_7): &\quad -x_{10} - x_4 - 1 &= 0\\
(e_2, e_3, e_8): &\quad -x_9 &= 0\\
(e_2, e_4, e_7): &\quad x_9 &= 0\\
(e_2, e_5, e_6): &\quad -x_9 &= 0\\
(e_1, e_2, e_{12}): &\quad -x_1 - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_8 &= 0\\
(e_1, e_4, e_{10}): &\quad -x_2 - x_8 &= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_7 &= 0\\
(e_1, e_6, e_8): &\quad -x_3 - x_7 &= 0\\
(e_2, e_3, e_{10}): &\quad -4x_1 + x_5 &= 0\\
(e_2, e_4, e_9): &\quad x_6 - x_8 &= 0\\
(e_2, e_5, e_8): &\quad x_4 &= 0\\
(e_2, e_6, e_7): &\quad x_{10} &= 0\\
(e_3, e_4, e_8): &\quad 2x_1 + x_8 &= 0\\
(e_3, e_5, e_7): &\quad -x_1 &= 0\\
(e_4, e_5, e_6): &\quad -x_8 &= 0
\end{aligned}$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{10B}(2, 14)$

m10B214 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_2, e_{13}] = e_{14} & [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
[e_3, e_5] = \alpha_{3,5}^{8} e_8 & [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
[e_3, e_7] = \alpha_{3,7}^{10} e_{10} & [e_3, e_8] = \alpha_{3,8}^{11} e_{11} \\
[e_3, e_9] = \alpha_{3,9}^{12} e_{12} & [e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_8] = \alpha_{4,8}^{12} e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{11} e_{11} \\
[e_5, e_7] = \alpha_{5,7}^{12} e_{12} & [e_5, e_8] = \alpha_{5,8}^{13} e_{13} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13} \\
[e_6, e_9] = \alpha_{6,9}^{14} e_{14} & [e_7, e_8] = \alpha_{7,8}^{14} e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{7}_{2,5}-\alpha^{7}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{7}_{2,5}-\alpha^{8}_{2,6}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{7}_{3,4}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{8}_{2,6}-\alpha^{9}_{2,7}-\alpha^{9}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{8}_{3,5}-\alpha^{9}_{3,6}-\alpha^{9}_{4,5} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{9}_{2,7}\alpha^{7}_{3,4}-\alpha^{9}_{3,6}+\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{9}_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{9}_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{9}_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha^{8}_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^{9}_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^{9}_{4,5}+\alpha^{11}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{12}_{2,10}+\alpha^{11}_{2,9}-\alpha^{12}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{11}_{3,8}-\alpha^{12}_{3,9}-\alpha^{12}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{11}_{4,7}-\alpha^{12}_{4,8}-\alpha^{12}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{11}_{5,6}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_3,e_7):&\quad \alpha^{12}_{2,10}\alpha^{10}_{3,7}-\alpha^{9}_{2,7}\alpha^{12}_{3,9}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_4,e_6):&\quad \alpha^{12}_{2,10}\alpha^{10}_{4,6}-\alpha^{8}_{2,6}\alpha^{12}_{4,8} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^{7}_{3,4}\alpha^{12}_{5,7}-\alpha^{8}_{3,5}\alpha^{12}_{4,8}+\alpha^{12}_{3,9}\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha^{12}_{2,10}-\alpha^{13}_{2,11}-\alpha^{13}_{3,10} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{13}_{3,10}+\alpha^{12}_{3,9}-\alpha^{13}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad \alpha^{12}_{4,8}-\alpha^{13}_{4,9}-\alpha^{13}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad \alpha^{12}_{5,7}-\alpha^{13}_{5,8}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_3,e_8):&\quad \alpha^{13}_{2,11}\alpha^{11}_{3,8}-\alpha^{10}_{2,8}\alpha^{13}_{3,10}-\alpha^{13}_{5,8} &&=0\\
(e_2,e_4,e_7):&\quad \alpha^{13}_{2,11}\alpha^{11}_{4,7}-\alpha^{9}_{2,7}\alpha^{13}_{4,9}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_5,e_6):&\quad \alpha^{13}_{2,11}\alpha^{11}_{5,6}+\alpha^{7}_{2,5}\alpha^{13}_{6,7}-\alpha^{8}_{2,6}\alpha^{13}_{5,8} &&=0\\
(e_3,e_4,e_6):&\quad \alpha^{13}_{3,10}\alpha^{10}_{4,6}+\alpha^{7}_{3,4}\alpha^{13}_{6,7}-\alpha^{9}_{3,6}\alpha^{13}_{4,9} &&=0\\
(e_1,e_2,e_{12}):&\quad -\alpha^{14}_{3,12}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -\alpha^{14}_{3,12}-\alpha^{14}_{4,11} &&=0\\
(e_1,e_4,e_{10}):&\quad -\alpha^{14}_{4,11}-\alpha^{14}_{5,10} &&=0\\
(e_1,e_5,e_9):&\quad -\alpha^{14}_{5,10}-\alpha^{14}_{6,9} &&=0\\
(e_1,e_6,e_8):&\quad -\alpha^{14}_{6,9}-\alpha^{14}_{7,8} &&=0\\
(e_2,e_3,e_{10}):&\quad -\alpha^{12}_{2,10}\alpha^{14}_{3,12}+\alpha^{13}_{3,10}-\alpha^{14}_{5,10} &&=0\\
(e_2,e_4,e_9):&\quad -\alpha^{11}_{2,9}\alpha^{14}_{4,11}+\alpha^{13}_{4,9}-\alpha^{14}_{6,9} &&=0\\
(e_2,e_5,e_8):&\quad -\alpha^{7}_{2,5}\alpha^{14}_{7,8}-\alpha^{10}_{2,8}\alpha^{14}_{5,10}+\alpha^{13}_{5,8} &&=0\\
(e_2,e_6,e_7):&\quad \alpha^{8}_{2,6}\alpha^{14}_{7,8}-\alpha^{9}_{2,7}\alpha^{14}_{6,9}+\alpha^{13}_{6,7} &&=0\\
(e_3,e_4,e_8):&\quad \alpha^{14}_{3,12}\alpha^{12}_{4,8}-\alpha^{7}_{3,4}\alpha^{14}_{7,8}-\alpha^{11}_{3,8}\alpha^{14}_{4,11} &&=0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{3,10} \to x_1$$

$$\alpha^{13}_{4,9} \to x_2$$

$$\alpha^{9}_{3,6} \to x_3$$

$$\alpha^{13}_{2,11} \to x_4$$

$$\alpha^{7}_{3,4} \to x_5$$

$$\alpha^{11}_{2,9} \to x_6$$

$$\alpha^{14}_{3,12} \to x_7$$

$$\alpha^{14}_{7,8} \to x_8$$

$$\alpha^{9}_{2,7} \to x_9$$

$$\alpha^{8}_{2,6} \to x_{10}$$

$$\alpha^{7}_{2,5} \to x_{11}$$

$$\alpha^{14}_{4,11} \to x_{12}$$

$$\alpha^{12}_{5,7} \to x_{13}$$

$$\alpha^{13}_{6,7} \to x_{14}$$

$$\alpha^{8}_{3,5} \to x_{15}$$

$$\alpha^{10}_{4,6} \to x_{16}$$

$$\alpha^{12}_{3,9} \to x_{17}$$

$$\alpha^{10}_{3,7} \to x_{18}$$

$$\alpha^{14}_{5,10} \to x_{19}$$

$$\alpha^{12}_{2,10} \to x_{20}$$

$$\alpha^{10}_{2,8} \to x_{21}$$

$$\alpha^{14}_{6,9} \to x_{22}$$

$$\alpha^{11}_{4,7} \to x_{23}$$

$$\alpha^{11}_{5,6} \to x_{24}$$

$$\alpha^{13}_{5,8} \to x_{25}$$

$$\alpha^{9}_{4,5} \to x_{26}$$

$$\alpha_{4,8}^{12} \to x_{27}$$

$$\alpha_{3,8}^{11} \to x_{28}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{11}-x_5+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{10}+x_{11}-x_{15} &&=0\\
(e_1,e_3,e_4):&\quad -x_{15}+x_5 &&=0\\
(e_1,e_2,e_6):&\quad x_{10}-x_3-x_9 &&=0\\
(e_1,e_3,e_5):&\quad x_{15}-x_{26}-x_3 &&=0\\
(e_2,e_3,e_4):&\quad x_{26}-x_3+x_5x_9 &&=0\\
(e_1,e_2,e_7):&\quad -x_{18}-x_{21}+x_9 &&=0\\
(e_1,e_3,e_6):&\quad -x_{16}-x_{18}+x_3 &&=0\\
(e_1,e_4,e_5):&\quad -x_{16}+x_{26} &&=0\\
(e_2,e_3,e_5):&\quad -x_{11}x_{18}+x_{15}x_{21} &&=0\\
(e_1,e_2,e_8):&\quad x_{21}-x_{28}-x_6 &&=0\\
(e_1,e_3,e_7):&\quad x_{18}-x_{23}-x_{28} &&=0\\
(e_1,e_4,e_6):&\quad x_{16}-x_{23}-x_{24} &&=0\\
(e_2,e_3,e_6):&\quad -x_{10}x_{28}-x_{24}+x_3x_6 &&=0\\
(e_2,e_4,e_5):&\quad -x_{11}x_{23}+x_{24}+x_{26}x_6 &&=0\\
(e_1,e_2,e_9):&\quad -x_{17}-x_{20}+x_6 &&=0\\
(e_1,e_3,e_8):&\quad -x_{17}-x_{27}+x_{28} &&=0\\
(e_1,e_4,e_7):&\quad -x_{13}+x_{23}-x_{27} &&=0\\
(e_1,e_5,e_6):&\quad -x_{13}+x_{24} &&=0\\
(e_2,e_3,e_7):&\quad -x_{13}-x_{17}x_9+x_{18}x_{20} &&=0\\
(e_2,e_4,e_6):&\quad -x_{10}x_{27}+x_{16}x_{20} &&=0\\
(e_3,e_4,e_5):&\quad x_{13}x_5-x_{15}x_{27}+x_{17}x_{26} &&=0\\
(e_1,e_2,e_{10}):&\quad -x_1+x_{20}-x_4 &&=0\\
(e_1,e_3,e_9):&\quad -x_1+x_{17}-x_2 &&=0\\
(e_1,e_4,e_8):&\quad -x_2-x_{25}+x_{27} &&=0\\
(e_1,e_5,e_7):&\quad x_{13}-x_{14}-x_{25} &&=0\\
(e_2,e_3,e_8):&\quad -x_1x_{21}-x_{25}+x_{28}x_4 &&=0\\
(e_2,e_4,e_7):&\quad -x_{14}-x_2x_9+x_{23}x_4 &&=0\\
(e_2,e_5,e_6):&\quad -x_{10}x_{25}+x_{11}x_{14}+x_{24}x_4 &&=0\\
(e_3,e_4,e_6):&\quad x_1x_{16}+x_{14}x_5-x_2x_3 &&=0\\
(e_1,e_2,e_{12}):&\quad -x_7-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -x_{12}-x_7 &&=0\\
(e_1,e_4,e_{10}):&\quad -x_{12}-x_{19} &&=0\\
(e_1,e_5,e_9):&\quad -x_{19}-x_{22} &&=0\\
(e_1,e_6,e_8):&\quad -x_{22}-x_8 &&=0\\
(e_2,e_3,e_{10}):&\quad x_1-x_{19}-x_{20}x_7 &&=0\\
(e_2,e_4,e_9):&\quad -x_{12}x_6+x_2-x_{22} &&=0\\
(e_2,e_5,e_8):&\quad -x_{11}x_8-x_{19}x_{21}+x_{25} &&=0\\
(e_2,e_6,e_7):&\quad x_{10}x_8+x_{14}-x_{22}x_9 &&=0\\
(e_3,e_4,e_8):&\quad -x_{12}x_{28}+x_{27}x_7-x_5x_8 &&=0\\
(e_3,e_5,e_7):&\quad x_{13}x_7+x_{15}x_8-x_{18}x_{19} &&=0\\
(e_4,e_5,e_6):&\quad x_{12}x_{24}-x_{16}x_{19}+x_{22}x_{26} &&=0
\end{aligned}
$$

Groebner basis (28 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{3B}(3, 14)$

m3B314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_9] &= e_{12} \\
[e_2, e_{10}] &= 4e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_8] &= -e_{12} & [e_3, e_9] &= -3e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_7] &= e_{12} \\
[e_4, e_8] &= 2e_{13} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_6] &= -e_{12} & [e_5, e_7] &= -e_{13} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}) &: \quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}) &: \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}) &: \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9) &: \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8) &: \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_9) &: \quad -\alpha_{3,12}^{14} - 3 &&= 0 \\
(e_2, e_4, e_8) &: \quad \text{no solutions} \\
(e_2, e_5, e_7) &: \quad \text{no solutions} \\
(e_3, e_4, e_7) &: \quad \alpha_{3,12}^{14} &&= 0 \\
(e_3, e_5, e_6) &: \quad -\alpha_{3,12}^{14} &&= 0
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{5B}(3,14)$

m5B314 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_7]=e_{10}$$
$$[e_2,e_8]=3e_{11} \qquad [e_2,e_9]=\alpha_{2,9}^{12}e_{12}$$
$$[e_2,e_{10}]=\alpha_{2,10}^{13}e_{13} \qquad [e_2,e_{13}]=e_{14}$$
$$[e_3,e_6]=-e_{10} \qquad [e_3,e_7]=-2e_{11}$$
$$[e_3,e_8]=\alpha_{3,8}^{12}e_{12} \qquad [e_3,e_9]=\alpha_{3,9}^{13}e_{13}$$
$$[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} \qquad [e_4,e_5]=e_{10}$$
$$[e_4,e_6]=e_{11} \qquad [e_4,e_7]=\alpha_{4,7}^{12}e_{12}$$
$$[e_4,e_8]=\alpha_{4,8}^{13}e_{13} \qquad [e_4,e_{11}]=\alpha_{4,11}^{14}e_{14}$$
$$[e_5,e_6]=\alpha_{5,6}^{12}e_{12} \qquad [e_5,e_7]=\alpha_{5,7}^{13}e_{13}$$
$$[e_5,e_{10}]=\alpha_{5,10}^{14}e_{14} \qquad [e_6,e_9]=\alpha_{6,9}^{14}e_{14}$$
$$[e_7,e_8]=\alpha_{7,8}^{14}e_{14}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,10}^{13} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13} &&= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_9): &\quad -\alpha_{2,9}^{12}\alpha_{3,12}^{14} + \alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_8): &\quad -3\alpha_{4,11}^{14} + \alpha_{4,8}^{13} &&= 0\\
(e_2, e_5, e_7): &\quad -\alpha_{5,10}^{14} + \alpha_{5,7}^{13} &&= 0\\
(e_3, e_4, e_7): &\quad \alpha_{3,12}^{14}\alpha_{4,7}^{12} + 2\alpha_{4,11}^{14} &&= 0\\
(e_3, e_5, e_6): &\quad \alpha_{3,12}^{14}\alpha_{5,6}^{12} + \alpha_{5,10}^{14} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{4,8}^{13} \to x_1\\
\alpha_{3,12}^{14} \to x_2\\
\alpha_{5,10}^{14} \to x_3\\
\alpha_{7,8}^{14} \to x_4\\
\alpha_{3,9}^{13} \to x_5\\
\alpha_{3,8}^{12} \to x_6\\
\alpha_{5,7}^{13} \to x_7\\
\alpha_{6,9}^{14} \to x_8
\end{gathered}
$$

$$\alpha_{4,11}^{14} \to x_9$$

$$\alpha_{2,10}^{13} \to x_{10}$$

$$\alpha_{2,9}^{12} \to x_{11}$$

$$\alpha_{4,7}^{12} \to x_{12}$$

$$\alpha_{5,6}^{12} \to x_{13}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_{11} - x_6 + 3 && = 0 \\
(e_1, e_3, e_7): &\quad -x_{12} - x_6 - 2 && = 0 \\
(e_1, e_4, e_6): &\quad -x_{12} - x_{13} + 1 && = 0 \\
(e_1, e_2, e_9): &\quad -x_{10} + x_{11} - x_5 && = 0 \\
(e_1, e_3, e_8): &\quad -x_1 - x_5 + x_6 && = 0 \\
(e_1, e_4, e_7): &\quad -x_1 + x_{12} - x_7 && = 0 \\
(e_1, e_5, e_6): &\quad x_{13} - x_7 && = 0 \\
(e_2, e_3, e_6): &\quad -x_{10} && = 0 \\
(e_2, e_4, e_5): &\quad x_{10} && = 0 \\
(e_1, e_2, e_{12}): &\quad -x_2 - 1 && = 0 \\
(e_1, e_3, e_{11}): &\quad -x_2 - x_9 && = 0 \\
(e_1, e_4, e_{10}): &\quad -x_3 - x_9 && = 0 \\
(e_1, e_5, e_9): &\quad -x_3 - x_8 && = 0 \\
(e_1, e_6, e_8): &\quad -x_4 - x_8 && = 0 \\
(e_2, e_3, e_9): &\quad -x_{11}x_2 + x_5 && = 0 \\
(e_2, e_4, e_8): &\quad x_1 - 3x_9 && = 0 \\
(e_2, e_5, e_7): &\quad -x_3 + x_7 && = 0 \\
(e_3, e_4, e_7): &\quad x_{12}x_2 + 2x_9 && = 0 \\
(e_3, e_5, e_6): &\quad x_{13}x_2 + x_3 && = 0
\end{aligned}$$

Groebner basis (13 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{7B}(3, 14)$

m7B314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_2, e_5] = e_8 \\
&[e_2, e_6] = 2e_9 && [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
&[e_2, e_8] = \alpha_{2,8}^{11} e_{11} && [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
&[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} && [e_2, e_{13}] = e_{14} \\
&[e_3, e_4] = -e_8 && [e_3, e_5] = -e_9 \\
&[e_3, e_6] = \alpha_{3,6}^{10} e_{10} && [e_3, e_7] = \alpha_{3,7}^{11} e_{11} \\
&[e_3, e_8] = \alpha_{3,8}^{12} e_{12} && [e_3, e_9] = \alpha_{3,9}^{13} e_{13} \\
&[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} && [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
&[e_4, e_6] = \alpha_{4,6}^{11} e_{11} && [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
&[e_4, e_8] = \alpha_{4,8}^{13} e_{13} && [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
&[e_5, e_6] = \alpha_{5,6}^{12} e_{12} && [e_5, e_7] = \alpha_{5,7}^{13} e_{13} \\
&[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} && [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
&[e_7, e_8] = \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - 2\alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_9): &\quad -\alpha_{2,9}^{12}\alpha_{3,12}^{14} + \alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_8): &\quad -\alpha_{2,8}^{11}\alpha_{4,11}^{14} + \alpha_{4,8}^{13} &&= 0\\
(e_2, e_5, e_7): &\quad -\alpha_{2,7}^{10}\alpha_{5,10}^{14} + \alpha_{5,7}^{13} + \alpha_{7,8}^{14} &&= 0\\
(e_3, e_4, e_7): &\quad \alpha_{3,12}^{14}\alpha_{4,7}^{12} - \alpha_{3,7}^{11}\alpha_{4,11}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_3, e_5, e_6): &\quad \alpha_{3,12}^{14}\alpha_{5,6}^{12} - \alpha_{3,6}^{10}\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,8}^{13} \to x_1$$

$$\alpha_{3,12}^{14} \to x_2$$

$$\alpha_{5,10}^{14} \to x_3$$

$$\alpha_{7,8}^{14} \to x_4$$

$$\alpha_{2,8}^{11} \to x_5$$

$$\alpha_{3,9}^{13} \to x_6$$

$$\alpha_{3,6}^{10} \to x_7$$

$$\alpha_{4,6}^{11} \to x_8$$

$$\alpha_{3,8}^{12} \to x_9$$

$$\alpha_{5,7}^{13} \to x_{10}$$

$$\alpha_{3,7}^{11} \to x_{11}$$

$$\alpha_{2,9}^{12} \to x_{12}$$

$$\alpha_{6,9}^{14} \to x_{13}$$

$$\alpha_{4,5}^{10} \to x_{14}$$

$$\alpha_{4,11}^{14} \to x_{15}$$

$$\alpha_{2,10}^{13} \to x_{16}$$

$$\alpha_{2,7}^{10} \to x_{17}$$

$$\alpha_{4,7}^{12} \to x_{18}$$

$$\alpha_{5,6}^{12} \to x_{19}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{17} - x_7 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_{14} - x_7 - 1 &= 0\\
(e_1, e_2, e_7): &\quad -x_{11} + x_{17} - x_5 &= 0\\
(e_1, e_3, e_6): &\quad -x_{11} + x_7 - x_8 &= 0\\
(e_1, e_4, e_5): &\quad x_{14} - x_8 &= 0\\
(e_2, e_3, e_4): &\quad -x_5 &= 0\\
(e_1, e_2, e_8): &\quad -x_{12} + x_5 - x_9 &= 0\\
(e_1, e_3, e_7): &\quad x_{11} - x_{18} - x_9 &= 0\\
(e_1, e_4, e_6): &\quad -x_{18} - x_{19} + x_8 &= 0\\
(e_2, e_3, e_5): &\quad -x_{12} - x_9 &= 0\\
(e_1, e_2, e_9): &\quad x_{12} - x_{16} - x_6 &= 0\\
(e_1, e_3, e_8): &\quad -x_1 - x_6 + x_9 &= 0\\
(e_1, e_4, e_7): &\quad -x_1 - x_{10} + x_{18} &= 0\\
(e_1, e_5, e_6): &\quad -x_{10} + x_{19} &= 0\\
(e_2, e_3, e_6): &\quad x_{16}x_7 - 2x_6 &= 0\\
(e_2, e_4, e_5): &\quad -x_1 + x_{14}x_{16} &= 0\\
(e_1, e_2, e_{12}): &\quad -x_2 - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -x_{15} - x_2 &= 0\\
(e_1, e_4, e_{10}): &\quad -x_{15} - x_3 &= 0\\
(e_1, e_5, e_9): &\quad -x_{13} - x_3 &= 0\\
(e_1, e_6, e_8): &\quad -x_{13} - x_4 &= 0\\
(e_2, e_3, e_9): &\quad -x_{12}x_2 + x_6 &= 0\\
(e_2, e_4, e_8): &\quad x_1 - x_{15}x_5 &= 0\\
(e_2, e_5, e_7): &\quad x_{10} - x_{17}x_3 + x_4 &= 0\\
(e_3, e_4, e_7): &\quad -x_{11}x_{15} + x_{18}x_2 - x_4 &= 0\\
(e_3, e_5, e_6): &\quad -x_{13} + x_{19}x_2 - x_3x_7 &= 0
\end{aligned}
$$

Groebner basis (19 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{9B}(3, 14)$

m9B314 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= -2e_8 \\
[e_2, e_6] &= -5e_9 & [e_2, e_7] &= -5e_{10} \\
[e_2, e_8] &= -2e_{11} & [e_2, e_9] &= e_{12} \\
[e_2, e_{10}] &= e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_4] &= 3e_8 & [e_3, e_5] &= 3e_9 \\
[e_3, e_6] &= 0 & [e_3, e_7] &= -3e_{11} \\
[e_3, e_8] &= -3e_{12} & [e_3, e_9] &= 0 \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_5] &= 3e_{10} \\
[e_4, e_6] &= 3e_{11} & [e_4, e_7] &= 0 \\
[e_4, e_8] &= -3e_{13} & [e_4, e_{11}] &= e_{14} \\
[e_5, e_6] &= 3e_{12} & [e_5, e_7] &= 3e_{13} \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}
$$

Solution 2

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{e_8}{4}$$
$$[e_2, e_6] = -\frac{e_9}{2} \qquad [e_2, e_7] = -\frac{23e_{10}}{28}$$
$$[e_2, e_8] = -\frac{5e_{11}}{7} \qquad [e_2, e_9] = -\frac{5e_{12}}{4}$$
$$[e_2, e_{10}] = -\frac{7e_{13}}{2} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_4] = \frac{3e_8}{4} \qquad [e_3, e_5] = \frac{3e_9}{4}$$
$$[e_3, e_6] = \frac{9e_{10}}{28} \qquad [e_3, e_7] = -\frac{3e_{11}}{28}$$
$$[e_3, e_8] = \frac{15e_{12}}{28} \qquad [e_3, e_9] = \frac{9e_{13}}{4}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_5] = \frac{3e_{10}}{7}$$
$$[e_4, e_6] = \frac{3e_{11}}{7} \qquad [e_4, e_7] = -\frac{9e_{12}}{14}$$
$$[e_4, e_8] = -\frac{12e_{13}}{7} \qquad [e_4, e_{11}] = e_{14}$$
$$[e_5, e_6] = \frac{15e_{12}}{14} \qquad [e_5, e_7] = \frac{15e_{13}}{14}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Solution 3

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_6 \\
[e_2, e_4] = e_7 & [e_2, e_5] = e_8 \\
[e_2, e_6] = e_9 & [e_2, e_7] = e_{10} \\
[e_2, e_8] = e_{11} & [e_2, e_9] = e_{12} \\
[e_2, e_{10}] = e_{13} & [e_2, e_{13}] = e_{14} \\
[e_3, e_4] = 0 & [e_3, e_5] = 0 \\
[e_3, e_6] = 0 & [e_3, e_7] = 0 \\
[e_3, e_8] = 0 & [e_3, e_9] = 0 \\
[e_3, e_{12}] = -e_{14} & [e_4, e_5] = 0 \\
[e_4, e_6] = 0 & [e_4, e_7] = 0 \\
[e_4, e_8] = 0 & [e_4, e_{11}] = e_{14} \\
[e_5, e_6] = 0 & [e_5, e_7] = 0 \\
[e_5, e_{10}] = -e_{14} & [e_6, e_9] = e_{14} \\
[e_7, e_8] = -e_{14} &
\end{array}
$$

Solution 4

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{10e_8}{7}$$
$$[e_2, e_6] = \frac{13e_9}{7} \qquad [e_2, e_7] = \frac{19e_{10}}{7}$$
$$[e_2, e_8] = 4e_{11} \qquad [e_2, e_9] = 7e_{12}$$
$$[e_2, e_{10}] = 13e_{13} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_4] = -\frac{3e_8}{7} \qquad [e_3, e_5] = -\frac{3e_9}{7}$$
$$[e_3, e_6] = -\frac{6e_{10}}{7} \qquad [e_3, e_7] = -\frac{9e_{11}}{7}$$
$$[e_3, e_8] = -3e_{12} \qquad [e_3, e_9] = -6e_{13}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_5] = \frac{3e_{10}}{7}$$
$$[e_4, e_6] = \frac{3e_{11}}{7} \qquad [e_4, e_7] = \frac{12e_{12}}{7}$$
$$[e_4, e_8] = 3e_{13} \qquad [e_4, e_{11}] = e_{14}$$
$$[e_5, e_6] = -\frac{9e_{12}}{7} \qquad [e_5, e_7] = -\frac{9e_{13}}{7}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] &= \alpha_{2,6}^{9} e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_2, e_9] &= \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] &= \alpha_{2,10}^{13} e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_4] &= \alpha_{3,4}^{8} e_8 & [e_3, e_5] &= \alpha_{3,5}^{9} e_9 \\
[e_3, e_6] &= \alpha_{3,6}^{10} e_{10} & [e_3, e_7] &= \alpha_{3,7}^{11} e_{11} \\
[e_3, e_8] &= \alpha_{3,8}^{12} e_{12} & [e_3, e_9] &= \alpha_{3,9}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11} & [e_4, e_7] &= \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] &= \alpha_{4,8}^{13} e_{13} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_6] &= \alpha_{5,6}^{12} e_{12} & [e_5, e_7] &= \alpha_{5,7}^{13} e_{13} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{8}_{2,5}-\alpha^{8}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{8}_{2,5}-\alpha^{9}_{2,6}-\alpha^{9}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{8}_{3,4}-\alpha^{9}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{9}_{2,6}-\alpha^{10}_{2,7}-\alpha^{10}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{9}_{3,5}-\alpha^{10}_{3,6}-\alpha^{10}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{10}_{2,7}-\alpha^{11}_{2,8}-\alpha^{11}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{10}_{3,6}-\alpha^{11}_{3,7}-\alpha^{11}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{10}_{4,5}-\alpha^{11}_{4,6} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{11}_{2,8}\alpha^{8}_{3,4}-\alpha^{11}_{3,7}+\alpha^{11}_{4,6} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{11}_{2,8}-\alpha^{12}_{2,9}-\alpha^{12}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{11}_{3,7}-\alpha^{12}_{3,8}-\alpha^{12}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{11}_{4,6}-\alpha^{12}_{4,7}-\alpha^{12}_{5,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{8}_{2,5}\alpha^{12}_{3,8}+\alpha^{12}_{2,9}\alpha^{9}_{3,5}+\alpha^{12}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{13}_{2,10}+\alpha^{12}_{2,9}-\alpha^{13}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{12}_{3,8}-\alpha^{13}_{3,9}-\alpha^{13}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{12}_{4,7}-\alpha^{13}_{4,8}-\alpha^{13}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{12}_{5,6}-\alpha^{13}_{5,7} &&=0\\
(e_2,e_3,e_6):&\quad \alpha^{13}_{2,10}\alpha^{10}_{3,6}-\alpha^{9}_{2,6}\alpha^{13}_{3,9} &&=0\\
(e_2,e_4,e_5):&\quad \alpha^{13}_{2,10}\alpha^{10}_{4,5}-\alpha^{8}_{2,5}\alpha^{13}_{4,8}+\alpha^{13}_{5,7} &&=0\\
(e_1,e_2,e_{12}):&\quad -\alpha^{14}_{3,12}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -\alpha^{14}_{3,12}-\alpha^{14}_{4,11} &&=0\\
(e_1,e_4,e_{10}):&\quad -\alpha^{14}_{4,11}-\alpha^{14}_{5,10} &&=0\\
(e_1,e_5,e_9):&\quad -\alpha^{14}_{5,10}-\alpha^{14}_{6,9} &&=0\\
(e_1,e_6,e_8):&\quad -\alpha^{14}_{6,9}-\alpha^{14}_{7,8} &&=0\\
(e_2,e_3,e_9):&\quad -\alpha^{12}_{2,9}\alpha^{14}_{3,12}+\alpha^{13}_{3,9}-\alpha^{14}_{6,9} &&=0\\
(e_2,e_4,e_8):&\quad -\alpha^{11}_{2,8}\alpha^{14}_{4,11}+\alpha^{13}_{4,8}-\alpha^{14}_{7,8} &&=0\\
(e_2,e_5,e_7):&\quad \alpha^{8}_{2,5}\alpha^{14}_{7,8}-\alpha^{10}_{2,7}\alpha^{14}_{5,10}+\alpha^{13}_{5,7} &&=0\\
(e_3,e_4,e_7):&\quad \alpha^{14}_{3,12}\alpha^{12}_{4,7}+\alpha^{8}_{3,4}\alpha^{14}_{7,8}-\alpha^{11}_{3,7}\alpha^{14}_{4,11} &&=0\\
(e_3,e_5,e_6):&\quad \alpha^{14}_{3,12}\alpha^{12}_{5,6}+\alpha^{9}_{3,5}\alpha^{14}_{6,9}-\alpha^{10}_{3,6}\alpha^{14}_{5,10} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha^{11}_{2,8} = -2$$
$$\alpha^{10}_{3,6} = 0$$
$$\alpha^{12}_{2,9} = 1$$
$$\alpha^{9}_{3,5} = 3$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{10}_{4,5} = 3$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{8}_{3,4} = 3$$
$$\alpha^{12}_{5,6} = 3$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{13}_{3,9} = 0$$
$$\alpha^{13}_{5,7} = 3$$
$$\alpha^{9}_{2,6} = -5$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{10}_{2,7} = -5$$
$$\alpha^{8}_{2,5} = -2$$
$$\alpha^{11}_{4,6} = 3$$
$$\alpha^{11}_{3,7} = -3$$
$$\alpha^{13}_{2,10} = 1$$
$$\alpha^{12}_{3,8} = -3$$
$$\alpha^{12}_{4,7} = 0$$
$$\alpha^{13}_{4,8} = -3$$

Solution 2:

$$\alpha_{2,8}^{11} = -5/7$$
$$\alpha_{3,6}^{10} = 9/28$$
$$\alpha_{2,9}^{12} = -5/4$$
$$\alpha_{3,5}^{9} = 3/4$$
$$\alpha_{3,12}^{14} = -1$$
$$\alpha_{7,8}^{14} = -1$$
$$\alpha_{4,5}^{10} = 3/7$$
$$\alpha_{4,11}^{14} = 1$$
$$\alpha_{3,4}^{8} = 3/4$$
$$\alpha_{5,6}^{12} = 15/14$$
$$\alpha_{5,10}^{14} = -1$$
$$\alpha_{3,9}^{13} = 9/4$$
$$\alpha_{5,7}^{13} = 15/14$$
$$\alpha_{2,6}^{9} = -1/2$$
$$\alpha_{6,9}^{14} = 1$$
$$\alpha_{2,7}^{10} = -23/28$$
$$\alpha_{2,5}^{8} = 1/4$$
$$\alpha_{4,6}^{11} = 3/7$$
$$\alpha_{3,7}^{11} = -3/28$$
$$\alpha_{2,10}^{13} = -7/2$$
$$\alpha_{3,8}^{12} = 15/28$$
$$\alpha_{4,7}^{12} = -9/14$$
$$\alpha_{4,8}^{13} = -12/7$$

Solution 3:

$$\alpha_{2,8}^{11} = 1$$
$$\alpha_{3,6}^{10} = 0$$
$$\alpha_{2,9}^{12} = 1$$
$$\alpha_{3,5}^{9} = 0$$
$$\alpha_{3,12}^{14} = -1$$
$$\alpha_{7,8}^{14} = -1$$
$$\alpha_{4,5}^{10} = 0$$
$$\alpha_{4,11}^{14} = 1$$
$$\alpha_{3,4}^{8} = 0$$
$$\alpha_{5,6}^{12} = 0$$
$$\alpha_{5,10}^{14} = -1$$
$$\alpha_{3,9}^{13} = 0$$
$$\alpha_{5,7}^{13} = 0$$
$$\alpha_{2,6}^{9} = 1$$
$$\alpha_{6,9}^{14} = 1$$
$$\alpha_{2,7}^{10} = 1$$
$$\alpha_{2,5}^{8} = 1$$
$$\alpha_{4,6}^{11} = 0$$
$$\alpha_{3,7}^{11} = 0$$
$$\alpha_{2,10}^{13} = 1$$
$$\alpha_{3,8}^{12} = 0$$
$$\alpha_{4,7}^{12} = 0$$
$$\alpha_{4,8}^{13} = 0$$

Solution 4:

$$\alpha_{2,8}^{11} = 4$$
$$\alpha_{3,6}^{10} = -6/7$$
$$\alpha_{2,9}^{12} = 7$$
$$\alpha_{3,5}^{9} = -3/7$$
$$\alpha_{3,12}^{14} = -1$$
$$\alpha_{7,8}^{14} = -1$$
$$\alpha_{4,5}^{10} = 3/7$$
$$\alpha_{4,11}^{14} = 1$$
$$\alpha_{3,4}^{8} = -3/7$$
$$\alpha_{5,6}^{12} = -9/7$$
$$\alpha_{5,10}^{14} = -1$$
$$\alpha_{3,9}^{13} = -6$$
$$\alpha_{5,7}^{13} = -9/7$$
$$\alpha_{2,6}^{9} = 13/7$$
$$\alpha_{6,9}^{14} = 1$$
$$\alpha_{2,7}^{10} = 19/7$$
$$\alpha_{2,5}^{8} = 10/7$$
$$\alpha_{4,6}^{11} = 3/7$$
$$\alpha_{3,7}^{11} = -9/7$$
$$\alpha_{2,10}^{13} = 13$$
$$\alpha_{3,8}^{12} = -3$$
$$\alpha_{4,7}^{12} = 12/7$$
$$\alpha_{4,8}^{13} = 3$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,8}^{11} \to x_1$$

$$\alpha_{3,6}^{10} \to x_2$$

$$\alpha_{2,9}^{12} \to x_3$$

$$\alpha_{3,5}^{9} \to x_4$$

$$\alpha_{3,12}^{14} \to x_5$$

$$\alpha_{7,8}^{14} \to x_6$$

$$\alpha_{4,5}^{10} \to x_7$$

$$\alpha_{4,11}^{14} \to x_8$$

$$\alpha_{3,4}^{8} \to x_9$$

$$\alpha_{5,6}^{12} \to x_{10}$$

$$\alpha_{5,10}^{14} \to x_{11}$$

$$\alpha_{3,9}^{13} \to x_{12}$$

$$\alpha_{5,7}^{13} \to x_{13}$$

$$\alpha_{2,6}^{9} \to x_{14}$$

$$\alpha_{6,9}^{14} \to x_{15}$$

$$\alpha_{2,7}^{10} \to x_{16}$$

$$\alpha_{2,5}^{8} \to x_{17}$$

$$\alpha_{4,6}^{11} \to x_{18}$$

$$\alpha_{3,7}^{11} \to x_{19}$$

$$\alpha_{2,10}^{13} \to x_{20}$$

$$\alpha_{3,8}^{12} \to x_{21}$$

$$\alpha_{4,7}^{12} \to x_{22}$$

$$\alpha_{4,8}^{13} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{17}-x_9+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{14}+x_{17}-x_4 &&=0\\
(e_1,e_3,e_4):&\quad -x_4+x_9 &&=0\\
(e_1,e_2,e_6):&\quad x_{14}-x_{16}-x_2 &&=0\\
(e_1,e_3,e_5):&\quad -x_2+x_4-x_7 &&=0\\
(e_1,e_2,e_7):&\quad -x_1+x_{16}-x_{19} &&=0\\
(e_1,e_3,e_6):&\quad -x_{18}-x_{19}+x_2 &&=0\\
(e_1,e_4,e_5):&\quad -x_{18}+x_7 &&=0\\
(e_2,e_3,e_4):&\quad x_1x_9+x_{18}-x_{19} &&=0\\
(e_1,e_2,e_8):&\quad x_1-x_{21}-x_3 &&=0\\
(e_1,e_3,e_7):&\quad x_{19}-x_{21}-x_{22} &&=0\\
(e_1,e_4,e_6):&\quad -x_{10}+x_{18}-x_{22} &&=0\\
(e_2,e_3,e_5):&\quad x_{10}-x_{17}x_{21}+x_3x_4 &&=0\\
(e_1,e_2,e_9):&\quad -x_{12}-x_{20}+x_3 &&=0\\
(e_1,e_3,e_8):&\quad -x_{12}+x_{21}-x_{23} &&=0\\
(e_1,e_4,e_7):&\quad -x_{13}+x_{22}-x_{23} &&=0\\
(e_1,e_5,e_6):&\quad x_{10}-x_{13} &&=0\\
(e_2,e_3,e_6):&\quad -x_{12}x_{14}+x_2x_{20} &&=0\\
(e_2,e_4,e_5):&\quad x_{13}-x_{17}x_{23}+x_{20}x_7 &&=0\\
(e_1,e_2,e_{12}):&\quad -x_5-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -x_5-x_8 &&=0\\
(e_1,e_4,e_{10}):&\quad -x_{11}-x_8 &&=0\\
(e_1,e_5,e_9):&\quad -x_{11}-x_{15} &&=0\\
(e_1,e_6,e_8):&\quad -x_{15}-x_6 &&=0\\
(e_2,e_3,e_9):&\quad x_{12}-x_{15}-x_3x_5 &&=0\\
(e_2,e_4,e_8):&\quad -x_1x_8+x_{23}-x_6 &&=0\\
(e_2,e_5,e_7):&\quad -x_{11}x_{16}+x_{13}+x_{17}x_6 &&=0\\
(e_3,e_4,e_7):&\quad -x_{19}x_8+x_{22}x_5+x_6x_9 &&=0\\
(e_3,e_5,e_6):&\quad x_{10}x_5-x_{11}x_2+x_{15}x_4 &&=0
\end{aligned}
$$

Groebner basis (23 variables, 6 linear, 17 nonlinear)

$$x_1-x_{23}-1=0$$

$$336x_2-7x_{23}^3+16x_{23}^2+111x_{23}=0$$

$$7x_{23}^3-16x_{23}^2-111x_{23}+48x_3-48=0$$

$$7x_{23}^3 - 16x_{23}^2 + x_{23} + 112x_4 = 0$$
$$x_5 + 1 = 0$$
$$x_6 + 1 = 0$$
$$7x_{23}^3 - 16x_{23}^2 - 27x_{23} + 84x_7 = 0$$
$$x_8 - 1 = 0$$
$$7x_{23}^3 - 16x_{23}^2 + x_{23} + 112x_9 = 0$$
$$168x_{10} + 7x_{23}^3 - 16x_{23}^2 + 57x_{23} = 0$$
$$x_{11} + 1 = 0$$
$$48x_{12} - 7x_{23}^3 + 16x_{23}^2 + 111x_{23} = 0$$
$$168x_{13} + 7x_{23}^3 - 16x_{23}^2 + 57x_{23} = 0$$
$$56x_{14} - 7x_{23}^3 + 16x_{23}^2 - x_{23} - 56 = 0$$
$$x_{15} - 1 = 0$$
$$336x_{16} - 35x_{23}^3 + 80x_{23}^2 - 117x_{23} - 336 = 0$$
$$112x_{17} - 7x_{23}^3 + 16x_{23}^2 - x_{23} - 112 = 0$$
$$84x_{18} + 7x_{23}^3 - 16x_{23}^2 - 27x_{23} = 0$$
$$336x_{19} - 35x_{23}^3 + 80x_{23}^2 + 219x_{23} = 0$$
$$24x_{20} + 7x_{23}^3 - 16x_{23}^2 - 111x_{23} - 24 = 0$$
$$48x_{21} - 7x_{23}^3 + 16x_{23}^2 + 63x_{23} = 0$$
$$168x_{22} + 7x_{23}^3 - 16x_{23}^2 - 111x_{23} = 0$$
$$7x_{23}^4 + 12x_{23}^3 - 63x_{23}^2 - 108x_{23} = 0$$

Solution 1:

$$x_1 = -2$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_4 = 3$$
$$x_5 = -1$$
$$x_6 = -1$$
$$x_7 = 3$$
$$x_8 = 1$$
$$x_9 = 3$$
$$x_10 = 3$$
$$x_11 = -1$$

$$x_12 = 0$$
$$x_13 = 3$$
$$x_14 = -5$$
$$x_15 = 1$$
$$x_16 = -5$$
$$x_17 = -2$$
$$x_18 = 3$$
$$x_19 = -3$$
$$x_20 = 1$$
$$x_21 = -3$$
$$x_22 = 0$$
$$x_23 = -3$$

Solution 2:

$$x_1 = -5/7$$
$$x_2 = 9/28$$
$$x_3 = -5/4$$
$$x_4 = 3/4$$
$$x_5 = -1$$
$$x_6 = -1$$
$$x_7 = 3/7$$
$$x_8 = 1$$
$$x_9 = 3/4$$
$$x_10 = 15/14$$
$$x_11 = -1$$
$$x_12 = 9/4$$
$$x_13 = 15/14$$
$$x_14 = -1/2$$
$$x_15 = 1$$
$$x_16 = -23/28$$
$$x_17 = 1/4$$
$$x_18 = 3/7$$
$$x_19 = -3/28$$

$$x_20 = -7/2$$
$$x_21 = 15/28$$
$$x_22 = -9/14$$
$$x_23 = -12/7$$

Solution 3:

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_4 = 0$$
$$x_5 = -1$$
$$x_6 = -1$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = 0$$
$$x_10 = 0$$
$$x_11 = -1$$
$$x_12 = 0$$
$$x_13 = 0$$
$$x_14 = 1$$
$$x_15 = 1$$
$$x_16 = 1$$
$$x_17 = 1$$
$$x_18 = 0$$
$$x_19 = 0$$
$$x_20 = 1$$
$$x_21 = 0$$
$$x_22 = 0$$
$$x_23 = 0$$

Solution 4:

$$x_1 = 4$$
$$x_2 = -6/7$$
$$x_3 = 7$$

$$x_4 = -3/7$$
$$x_5 = -1$$
$$x_6 = -1$$
$$x_7 = 3/7$$
$$x_8 = 1$$
$$x_9 = -3/7$$
$$x_10 = -9/7$$
$$x_11 = -1$$
$$x_12 = -6$$
$$x_13 = -9/7$$
$$x_14 = 13/7$$
$$x_15 = 1$$
$$x_16 = 19/7$$
$$x_17 = 10/7$$
$$x_18 = 3/7$$
$$x_19 = -9/7$$
$$x_20 = 13$$
$$x_21 = -3$$
$$x_22 = 12/7$$
$$x_23 = 3$$

$\mathfrak{m}_{2B}(4, 14)$

m2B414 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_9] = e_{13}$$
$$[e_2, e_{13}] = e_{14} \qquad [e_3, e_8] = -e_{13}$$
$$[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} \qquad [e_4, e_7] = e_{13}$$
$$[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \qquad [e_5, e_6] = -e_{13}$$
$$[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} \qquad [e_6, e_9] = \alpha_{6,9}^{14} e_{14}$$
$$[e_7, e_8] = \alpha_{7,8}^{14} e_{14}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha^{14}_{3,12} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha^{14}_{3,12} - \alpha^{14}_{4,11} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha^{14}_{4,11} - \alpha^{14}_{5,10} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha^{14}_{5,10} - \alpha^{14}_{6,9} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha^{14}_{6,9} - \alpha^{14}_{7,8} &&= 0\\
(e_2, e_3, e_8): &\quad \text{no solutions}\\
(e_2, e_4, e_7): &\quad \text{no solutions}\\
(e_2, e_5, e_6): &\quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(4, 14)$

m4B414 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{11}\\
[e_2, e_8] &= 3e_{12} & [e_2, e_9] &= 6e_{13}\\
[e_2, e_{13}] &= e_{14} & [e_3, e_6] &= -e_{11}\\
[e_3, e_7] &= -2e_{12} & [e_3, e_8] &= -3e_{13}\\
[e_3, e_{12}] &= -e_{14} & [e_4, e_5] &= e_{11}\\
[e_4, e_6] &= e_{12} & [e_4, e_7] &= e_{13}\\
[e_4, e_{11}] &= e_{14} & [e_5, e_6] &= 0\\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14}\\
[e_7, e_8] &= -e_{14}
\end{aligned}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_7] = e_{11} \\
[e_2, e_8] = 3e_{12} & [e_2, e_9] = \alpha_{2,9}^{13} e_{13} \\
[e_2, e_{13}] = e_{14} & [e_3, e_6] = -e_{11} \\
[e_3, e_7] = -2e_{12} & [e_3, e_8] = \alpha_{3,8}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_5] = e_{11} \\
[e_4, e_6] = e_{12} & [e_4, e_7] = \alpha_{4,7}^{13} e_{13} \\
[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{13} e_{13} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] = \alpha_{7,8}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rll}
(e_1, e_2, e_8): & -\alpha_{2,9}^{13} - \alpha_{3,8}^{13} + 3 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{13} - \alpha_{4,7}^{13} - 2 & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{13} - \alpha_{5,6}^{13} + 1 & = 0 \\
(e_1, e_2, e_{12}): & -\alpha_{3,12}^{14} - 1 & = 0 \\
(e_1, e_3, e_{11}): & -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} & = 0 \\
(e_1, e_4, e_{10}): & -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} & = 0 \\
(e_1, e_5, e_9): & -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} & = 0 \\
(e_1, e_6, e_8): & -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} & = 0 \\
(e_2, e_3, e_8): & -3\alpha_{3,12}^{14} + \alpha_{3,8}^{13} & = 0 \\
(e_2, e_4, e_7): & -\alpha_{4,11}^{14} + \alpha_{4,7}^{13} & = 0 \\
(e_2, e_5, e_6): & \alpha_{5,6}^{13} & = 0 \\
(e_3, e_4, e_6): & \alpha_{3,12}^{14} + \alpha_{4,11}^{14} & = 0
\end{array}
$$

Solution 1:

$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{13}_{3,8} = -3$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{13}_{4,7} = 1$$
$$\alpha^{13}_{2,9} = 6$$
$$\alpha^{13}_{5,6} = 0$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{14}_{3,12} \to x_1$$
$$\alpha^{14}_{5,10} \to x_2$$
$$\alpha^{14}_{7,8} \to x_3$$
$$\alpha^{13}_{3,8} \to x_4$$
$$\alpha^{14}_{6,9} \to x_5$$
$$\alpha^{14}_{4,11} \to x_6$$
$$\alpha^{13}_{4,7} \to x_7$$
$$\alpha^{13}_{2,9} \to x_8$$
$$\alpha^{13}_{5,6} \to x_9$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_4 - x_8 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_4 - x_7 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_7 - x_9 + 1 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_6 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_2 - x_6 &&= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_5 &&= 0\\
(e_1, e_6, e_8): &\quad -x_3 - x_5 &&= 0\\
(e_2, e_3, e_8): &\quad -3x_1 + x_4 &&= 0\\
(e_2, e_4, e_7): &\quad -x_6 + x_7 &&= 0\\
(e_2, e_5, e_6): &\quad x_9 &&= 0\\
(e_3, e_4, e_6): &\quad x_1 + x_6 &&= 0
\end{aligned}$$

Groebner basis (9 variables, 9 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 + 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 + 3 = 0$$
$$x_5 - 1 = 0$$
$$x_6 - 1 = 0$$
$$x_7 - 1 = 0$$
$$x_8 - 6 = 0$$
$$x_9 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = -1$$
$$x_3 = -1$$
$$x_4 = -3$$
$$x_5 = 1$$
$$x_6 = 1$$
$$x_7 = 1$$
$$x_8 = 6$$
$$x_9 = 0$$

$\mathfrak{m}_{6B}(4,14)$

m6B414 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_9$$
$$[e_2,e_6]=2e_{10} \qquad [e_2,e_7]=\frac{5e_{11}}{3}$$
$$[e_2,e_8]=0 \qquad [e_2,e_9]=0$$
$$[e_2,e_{13}]=e_{14} \qquad [e_3,e_4]=-e_9$$
$$[e_3,e_5]=-e_{10} \qquad [e_3,e_6]=\frac{e_{11}}{3}$$
$$[e_3,e_7]=\frac{5e_{12}}{3} \qquad [e_3,e_8]=0$$
$$[e_3,e_{12}]=-e_{14} \qquad [e_4,e_5]=-\frac{4e_{11}}{3}$$
$$[e_4,e_6]=-\frac{4e_{12}}{3} \qquad [e_4,e_7]=\frac{5e_{13}}{3}$$
$$[e_4,e_{11}]=e_{14} \qquad [e_5,e_6]=-3e_{13}$$
$$[e_5,e_{10}]=-e_{14} \qquad [e_6,e_9]=e_{14}$$
$$[e_7,e_8]=-e_{14}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_5] = e_9 \\
[e_2, e_6] = 2e_{10} & [e_2, e_7] = \alpha_{2,7}^{11} e_{11} \\
[e_2, e_8] = \alpha_{2,8}^{12} e_{12} & [e_2, e_9] = \alpha_{2,9}^{13} e_{13} \\
[e_2, e_{13}] = e_{14} & [e_3, e_4] = -e_9 \\
[e_3, e_5] = -e_{10} & [e_3, e_6] = \alpha_{3,6}^{11} e_{11} \\
[e_3, e_7] = \alpha_{3,7}^{12} e_{12} & [e_3, e_8] = \alpha_{3,8}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_5] = \alpha_{4,5}^{11} e_{11} \\
[e_4, e_6] = \alpha_{4,6}^{12} e_{12} & [e_4, e_7] = \alpha_{4,7}^{13} e_{13} \\
[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{13} e_{13} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] = \alpha_{7,8}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,9}^{13} &= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_8): &\quad -\alpha_{2,8}^{12}\alpha_{3,12}^{14} + \alpha_{3,8}^{13} &= 0\\
(e_2, e_4, e_7): &\quad -\alpha_{2,7}^{11}\alpha_{4,11}^{14} + \alpha_{4,7}^{13} &= 0\\
(e_2, e_5, e_6): &\quad -2\alpha_{5,10}^{14} + \alpha_{5,6}^{13} + \alpha_{6,9}^{14} &= 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,12}^{14}\alpha_{4,6}^{12} - \alpha_{3,6}^{11}\alpha_{4,11}^{14} - \alpha_{6,9}^{14} &= 0
\end{aligned}
$$

Solution 1:

$$\alpha_{3,7}^{12} = 5/3$$
$$\alpha_{3,12}^{14} = -1$$
$$\alpha_{5,10}^{14} = -1$$
$$\alpha_{7,8}^{14} = -1$$
$$\alpha_{3,8}^{13} = 0$$
$$\alpha_{2,8}^{12} = 0$$
$$\alpha_{4,6}^{12} = -4/3$$
$$\alpha_{6,9}^{14} = 1$$
$$\alpha_{3,6}^{11} = 1/3$$
$$\alpha_{4,11}^{14} = 1$$
$$\alpha_{4,7}^{13} = 5/3$$
$$\alpha_{2,9}^{13} = 0$$
$$\alpha_{5,6}^{13} = -3$$
$$\alpha_{4,5}^{11} = -4/3$$
$$\alpha_{2,7}^{11} = 5/3$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,7}^{12} \to x_1$$
$$\alpha_{3,12}^{14} \to x_2$$
$$\alpha_{5,10}^{14} \to x_3$$
$$\alpha_{7,8}^{14} \to x_4$$
$$\alpha_{3,8}^{13} \to x_5$$
$$\alpha_{2,8}^{12} \to x_6$$
$$\alpha_{4,6}^{12} \to x_7$$
$$\alpha_{6,9}^{14} \to x_8$$
$$\alpha_{3,6}^{11} \to x_9$$
$$\alpha_{4,11}^{14} \to x_{10}$$
$$\alpha_{4,7}^{13} \to x_{11}$$
$$\alpha_{2,9}^{13} \to x_{12}$$

$$\alpha^{13}_{5,6} \to x_{13}$$

$$\alpha^{11}_{4,5} \to x_{14}$$

$$\alpha^{11}_{2,7} \to x_{15}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{15} - x_9 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_{14} - x_9 - 1 &= 0\\
(e_1, e_2, e_7): &\quad -x_1 + x_{15} - x_6 &= 0\\
(e_1, e_3, e_6): &\quad -x_1 - x_7 + x_9 &= 0\\
(e_1, e_4, e_5): &\quad x_{14} - x_7 &= 0\\
(e_1, e_2, e_8): &\quad -x_{12} - x_5 + x_6 &= 0\\
(e_1, e_3, e_7): &\quad x_1 - x_{11} - x_5 &= 0\\
(e_1, e_4, e_6): &\quad -x_{11} - x_{13} + x_7 &= 0\\
(e_2, e_3, e_4): &\quad -x_{12} &= 0\\
(e_1, e_2, e_{12}): &\quad -x_2 - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -x_{10} - x_2 &= 0\\
(e_1, e_4, e_{10}): &\quad -x_{10} - x_3 &= 0\\
(e_1, e_5, e_9): &\quad -x_3 - x_8 &= 0\\
(e_1, e_6, e_8): &\quad -x_4 - x_8 &= 0\\
(e_2, e_3, e_8): &\quad -x_2x_6 + x_5 &= 0\\
(e_2, e_4, e_7): &\quad -x_{10}x_{15} + x_{11} &= 0\\
(e_2, e_5, e_6): &\quad x_{13} - 2x_3 + x_8 &= 0\\
(e_3, e_4, e_6): &\quad -x_{10}x_9 + x_2x_7 - x_8 &= 0
\end{aligned}$$

Groebner basis (15 variables, 15 linear, 0 nonlinear)

$$3x_1 - 5 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 + 1 = 0$$

$$x_5 = 0$$

$$x_6 = 0$$

$$3x_7 + 4 = 0$$

$$x_8 - 1 = 0$$

$$3x_9 - 1 = 0$$
$$x_{10} - 1 = 0$$
$$3x_{11} - 5 = 0$$
$$x_{12} = 0$$
$$x_{13} + 3 = 0$$
$$3x_{14} + 4 = 0$$
$$3x_{15} - 5 = 0$$

Solution 1:

$$x_1 = 5/3$$
$$x_2 = -1$$
$$x_3 = -1$$
$$x_4 = -1$$
$$x_5 = 0$$
$$x_6 = 0$$
$$x_7 = -4/3$$
$$x_8 = 1$$
$$x_9 = 1/3$$
$$x_10 = 1$$
$$x_11 = 5/3$$
$$x_12 = 0$$
$$x_13 = -3$$
$$x_14 = -4/3$$
$$x_15 = 5/3$$

$\mathfrak{m}_{8B}(4, 14)$

m8B414 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_7$$
$$[e_2, e_4] = e_8 \qquad [e_2, e_5] = \alpha_{2,5}^{9} e_9$$
$$[e_2, e_6] = \alpha_{2,6}^{10} e_{10} \qquad [e_2, e_7] = \alpha_{2,7}^{11} e_{11}$$
$$[e_2, e_8] = \alpha_{2,8}^{12} e_{12} \qquad [e_2, e_9] = \alpha_{2,9}^{13} e_{13}$$
$$[e_2, e_{13}] = e_{14} \qquad [e_3, e_4] = \alpha_{3,4}^{9} e_9$$
$$[e_3, e_5] = \alpha_{3,5}^{10} e_{10} \qquad [e_3, e_6] = \alpha_{3,6}^{11} e_{11}$$
$$[e_3, e_7] = \alpha_{3,7}^{12} e_{12} \qquad [e_3, e_8] = \alpha_{3,8}^{13} e_{13}$$
$$[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} \qquad [e_4, e_5] = \alpha_{4,5}^{11} e_{11}$$
$$[e_4, e_6] = \alpha_{4,6}^{12} e_{12} \qquad [e_4, e_7] = \alpha_{4,7}^{13} e_{13}$$
$$[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \qquad [e_5, e_6] = \alpha_{5,6}^{13} e_{13}$$
$$[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} \qquad [e_6, e_9] = \alpha_{6,9}^{14} e_{14}$$
$$[e_7, e_8] = \alpha_{7,8}^{14} e_{14}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,9}^{13}\alpha_{3,4}^{9} - \alpha_{3,8}^{13} + \alpha_{4,7}^{13} &= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_8): &\quad -\alpha_{2,8}^{12}\alpha_{3,12}^{14} + \alpha_{3,8}^{13} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_4, e_7): &\quad -\alpha_{2,7}^{11}\alpha_{4,11}^{14} + \alpha_{4,7}^{13} + \alpha_{7,8}^{14} &= 0\\
(e_2, e_5, e_6): &\quad \alpha_{2,5}^{9}\alpha_{6,9}^{14} - \alpha_{2,6}^{10}\alpha_{5,10}^{14} + \alpha_{5,6}^{13} &= 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,12}^{14}\alpha_{4,6}^{12} + \alpha_{3,4}^{9}\alpha_{6,9}^{14} - \alpha_{3,6}^{11}\alpha_{4,11}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,7}^{12} \to x_1$$
$$\alpha_{3,12}^{14} \to x_2$$
$$\alpha_{5,10}^{14} \to x_3$$
$$\alpha_{7,8}^{14} \to x_4$$
$$\alpha_{3,8}^{13} \to x_5$$
$$\alpha_{2,6}^{10} \to x_6$$

$$\alpha^{12}_{2,8} \to x_7$$

$$\alpha^{12}_{4,6} \to x_8$$

$$\alpha^{10}_{3,5} \to x_9$$

$$\alpha^{11}_{3,6} \to x_{10}$$

$$\alpha^{14}_{4,11} \to x_{11}$$

$$\alpha^{14}_{6,9} \to x_{12}$$

$$\alpha^{9}_{2,5} \to x_{13}$$

$$\alpha^{13}_{4,7} \to x_{14}$$

$$\alpha^{13}_{2,9} \to x_{15}$$

$$\alpha^{9}_{3,4} \to x_{16}$$

$$\alpha^{13}_{5,6} \to x_{17}$$

$$\alpha^{11}_{4,5} \to x_{18}$$

$$\alpha^{11}_{2,7} \to x_{19}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -x_{13}-x_{16}+1 &&=0\\
(e_1,e_2,e_5): &\quad x_{13}-x_6-x_9 &&=0\\
(e_1,e_3,e_4): &\quad x_{16}-x_9 &&=0\\
(e_1,e_2,e_6): &\quad -x_{10}-x_{19}+x_6 &&=0\\
(e_1,e_3,e_5): &\quad -x_{10}-x_{18}+x_9 &&=0\\
(e_1,e_2,e_7): &\quad -x_1+x_{19}-x_7 &&=0\\
(e_1,e_3,e_6): &\quad -x_1+x_{10}-x_8 &&=0\\
(e_1,e_4,e_5): &\quad x_{18}-x_8 &&=0\\
(e_1,e_2,e_8): &\quad -x_{15}-x_5+x_7 &&=0\\
(e_1,e_3,e_7): &\quad x_1-x_{14}-x_5 &&=0\\
(e_1,e_4,e_6): &\quad -x_{14}-x_{17}+x_8 &&=0\\
(e_2,e_3,e_4): &\quad x_{14}+x_{15}x_{16}-x_5 &&=0\\
(e_1,e_2,e_{12}): &\quad -x_2-1 &&=0\\
(e_1,e_3,e_{11}): &\quad -x_{11}-x_2 &&=0\\
(e_1,e_4,e_{10}): &\quad -x_{11}-x_3 &&=0\\
(e_1,e_5,e_9): &\quad -x_{12}-x_3 &&=0\\
(e_1,e_6,e_8): &\quad -x_{12}-x_4 &&=0\\
(e_2,e_3,e_8): &\quad -x_2x_7-x_4+x_5 &&=0\\
(e_2,e_4,e_7): &\quad -x_{11}x_{19}+x_{14}+x_4 &&=0\\
(e_2,e_5,e_6): &\quad x_{12}x_{13}+x_{17}-x_3x_6 &&=0\\
(e_3,e_4,e_6): &\quad -x_{10}x_{11}+x_{12}x_{16}+x_2x_8 &&=0
\end{aligned}
$$

Groebner basis (19 variables, 17 linear, 1 nonlinear)

$$3x_1+5x_{18}+x_{19}-1=0$$

$$x_2+1=0$$

$$x_3+1=0$$

$$x_4+1=0$$

$$5x_{18}+4x_{19}+3x_5+2=0$$

$$2x_{18}-2x_{19}+3x_6-1=0$$

$$-5x_{18}-4x_{19}+3x_7+1=0$$

$$-x_{18}+x_8=0$$

$$-x_{18}+x_{19}+3x_9-1=0$$

$$3x_{10}+2x_{18}+x_{19}-1=0$$

$$x_{11} - 1 = 0$$
$$x_{12} - 1 = 0$$
$$3x_{13} + x_{18} - x_{19} - 2 = 0$$
$$x_{14} - x_{19} - 1 = 0$$
$$3x_{15} - 10x_{18} - 8x_{19} - 1 = 0$$
$$3x_{16} - x_{18} + x_{19} - 1 = 0$$
$$x_{17} - x_{18} + x_{19} + 1 = 0$$
$$5x_{18}^2 - x_{18}x_{19} + 13x_{18} - 4x_{19}^2 + 14x_{19} + 8 = 0$$

$\mathfrak{m}_{3B}(5, 14)$

m3B514 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{12} \\
[e_2, e_8] &= 3e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_6] &= -e_{12} & [e_3, e_7] &= -2e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= e_{12} \\
[e_4, e_6] &= e_{13} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_7): &\quad -\alpha_{3,12}^{14} - 2 &&= 0 \\
(e_2, e_4, e_6): &\quad \text{no solutions} && \\
(e_3, e_4, e_5): &\quad \alpha_{3,12}^{14} &&= 0
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{5B}(5,14)$

m5B514 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_{10}$$
$$[e_2,e_6]=2e_{11} \qquad [e_2,e_7]=5e_{12}$$
$$[e_2,e_8]=10e_{13} \qquad [e_2,e_{13}]=e_{14}$$
$$[e_3,e_4]=-e_{10} \qquad [e_3,e_5]=-e_{11}$$
$$[e_3,e_6]=-3e_{12} \qquad [e_3,e_7]=-5e_{13}$$
$$[e_3,e_{12}]=-e_{14} \qquad [e_4,e_5]=2e_{12}$$
$$[e_4,e_6]=2e_{13} \qquad [e_4,e_{11}]=e_{14}$$
$$[e_5,e_{10}]=-e_{14} \qquad [e_6,e_9]=e_{14}$$
$$[e_7,e_8]=-e_{14}$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_{10}$$
$$[e_2,e_6]=2e_{11} \qquad [e_2,e_7]=\alpha_{2,7}^{12}e_{12}$$
$$[e_2,e_8]=\alpha_{2,8}^{13}e_{13} \qquad [e_2,e_{13}]=e_{14}$$
$$[e_3,e_4]=-e_{10} \qquad [e_3,e_5]=-e_{11}$$
$$[e_3,e_6]=\alpha_{3,6}^{12}e_{12} \qquad [e_3,e_7]=\alpha_{3,7}^{13}e_{13}$$
$$[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} \qquad [e_4,e_5]=\alpha_{4,5}^{12}e_{12}$$
$$[e_4,e_6]=\alpha_{4,6}^{13}e_{13} \qquad [e_4,e_{11}]=\alpha_{4,11}^{14}e_{14}$$
$$[e_5,e_{10}]=\alpha_{5,10}^{14}e_{14} \qquad [e_6,e_9]=\alpha_{6,9}^{14}e_{14}$$
$$[e_7,e_8]=\alpha_{7,8}^{14}e_{14}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &&= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_7): &\quad -\alpha_{2,7}^{12}\alpha_{3,12}^{14} + \alpha_{3,7}^{13} &&= 0\\
(e_2, e_4, e_6): &\quad -2\alpha_{4,11}^{14} + \alpha_{4,6}^{13} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,12}^{14}\alpha_{4,5}^{12} + \alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha_{3,12}^{14} &= -1\\
\alpha_{5,10}^{14} &= -1\\
\alpha_{7,8}^{14} &= -1\\
\alpha_{2,7}^{12} &= 5\\
\alpha_{6,9}^{14} &= 1\\
\alpha_{4,6}^{13} &= 2\\
\alpha_{4,11}^{14} &= 1\\
\alpha_{3,6}^{12} &= -3\\
\alpha_{2,8}^{13} &= 10\\
\alpha_{4,5}^{12} &= 2\\
\alpha_{3,7}^{13} &= -5
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha_{3,12}^{14} &\to x_1\\
\alpha_{5,10}^{14} &\to x_2
\end{aligned}$$

$$\alpha_{7,8}^{14} \to x_3$$

$$\alpha_{2,7}^{12} \to x_4$$

$$\alpha_{6,9}^{14} \to x_5$$

$$\alpha_{4,6}^{13} \to x_6$$

$$\alpha_{4,11}^{14} \to x_7$$

$$\alpha_{3,6}^{12} \to x_8$$

$$\alpha_{2,8}^{13} \to x_9$$

$$\alpha_{4,5}^{12} \to x_{10}$$

$$\alpha_{3,7}^{13} \to x_{11}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_4 - x_8 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{10} - x_8 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{11} + x_4 - x_9 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{11} - x_6 + x_8 &&= 0\\
(e_1, e_4, e_5): &\quad x_{10} - x_6 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_7 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_2 - x_7 &&= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_5 &&= 0\\
(e_1, e_6, e_8): &\quad -x_3 - x_5 &&= 0\\
(e_2, e_3, e_7): &\quad -x_1x_4 + x_{11} &&= 0\\
(e_2, e_4, e_6): &\quad x_6 - 2x_7 &&= 0\\
(e_3, e_4, e_5): &\quad x_1x_{10} - x_2 + x_7 &&= 0
\end{aligned}$$

Groebner basis (11 variables, 11 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 - 5 = 0$$

$$x_5 - 1 = 0$$

$$x_6 - 2 = 0$$

$$x_7 - 1 = 0$$
$$x_8 + 3 = 0$$
$$x_9 - 10 = 0$$
$$x_{10} - 2 = 0$$
$$x_{11} + 5 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = -1$$
$$x_3 = -1$$
$$x_4 = 5$$
$$x_5 = 1$$
$$x_6 = 2$$
$$x_7 = 1$$
$$x_8 = -3$$
$$x_9 = 10$$
$$x_10 = 2$$
$$x_11 = -5$$

$\mathfrak{m}_{7B}(5, 14)$

m7B514 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9 & [e_2, e_5] &= \alpha_{2,5}^{10} e_{10} \\
[e_2, e_6] &= \alpha_{2,6}^{11} e_{11} & [e_2, e_7] &= \alpha_{2,7}^{12} e_{12} \\
[e_2, e_8] &= \alpha_{2,8}^{13} e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_4] &= \alpha_{3,4}^{10} e_{10} & [e_3, e_5] &= \alpha_{3,5}^{11} e_{11} \\
[e_3, e_6] &= \alpha_{3,6}^{12} e_{12} & [e_3, e_7] &= \alpha_{3,7}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= \alpha_{4,5}^{12} e_{12} \\
[e_4, e_6] &= \alpha_{4,6}^{13} e_{13} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_7): &\quad -\alpha_{2,7}^{12}\alpha_{3,12}^{14} + \alpha_{3,7}^{13} + \alpha_{7,8}^{14} &= 0\\
(e_2, e_4, e_6): &\quad -\alpha_{2,6}^{11}\alpha_{4,11}^{14} + \alpha_{4,6}^{13} + \alpha_{6,9}^{14} &= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,12}^{14}\alpha_{4,5}^{12} + \alpha_{3,4}^{10}\alpha_{5,10}^{14} - \alpha_{3,5}^{11}\alpha_{4,11}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,12}^{14} \to x_1$$

$$\alpha_{5,10}^{14} \to x_2$$

$$\alpha_{7,8}^{14} \to x_3$$

$$\alpha_{2,6}^{11} \to x_4$$

$$\alpha_{2,7}^{12} \to x_5$$

$$\alpha_{3,4}^{10} \to x_6$$

$$\alpha_{2,5}^{10} \to x_7$$

$$\alpha_{6,9}^{14} \to x_8$$

$$\alpha_{4,6}^{13} \to x_9$$

$$\alpha_{4,11}^{14} \to x_{10}$$

$$\alpha_{3,6}^{12} \to x_{11}$$

$$\alpha_{2,8}^{13} \to x_{12}$$

$$\alpha_{3,5}^{11} \to x_{13}$$

$$\alpha_{4,5}^{12} \to x_{14}$$

$$\alpha_{3,7}^{13} \to x_{15}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_6 - x_7 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_{13} - x_4 + x_7 &= 0\\
(e_1, e_3, e_4): &\quad -x_{13} + x_6 &= 0\\
(e_1, e_2, e_6): &\quad -x_{11} + x_4 - x_5 &= 0\\
(e_1, e_3, e_5): &\quad -x_{11} + x_{13} - x_{14} &= 0\\
(e_1, e_2, e_7): &\quad -x_{12} - x_{15} + x_5 &= 0\\
(e_1, e_3, e_6): &\quad x_{11} - x_{15} - x_9 &= 0\\
(e_1, e_4, e_5): &\quad x_{14} - x_9 &= 0\\
(e_1, e_2, e_{12}): &\quad -x_1 - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_{10} &= 0\\
(e_1, e_4, e_{10}): &\quad -x_{10} - x_2 &= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_8 &= 0\\
(e_1, e_6, e_8): &\quad -x_3 - x_8 &= 0\\
(e_2, e_3, e_7): &\quad -x_1 x_5 + x_{15} + x_3 &= 0\\
(e_2, e_4, e_6): &\quad -x_{10} x_4 + x_8 + x_9 &= 0\\
(e_3, e_4, e_5): &\quad x_1 x_{14} - x_{10} x_{13} + x_2 x_6 &= 0
\end{aligned}$$

Groebner basis (15 variables, 14 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$

$$2x_{15} + 5x_4 - 5 = 0$$

$$x_{15} + x_5 - 1 = 0$$

$$-x_{15} + 5x_6 = 0$$

$$x_{15} + 5x_7 - 5 = 0$$

$$x_8 - 1 = 0$$

$$2x_{15} + 5x_9 = 0$$
$$x_{10} - 1 = 0$$
$$5x_{11} - 3x_{15} = 0$$
$$x_{12} + 2x_{15} - 1 = 0$$
$$5x_{13} - x_{15} = 0$$
$$5x_{14} + 2x_{15} = 0$$

$\mathfrak{m}_{2B}(6, 14)$

m2B614 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_6] &= -e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= e_{13} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} \\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{12}) &: \quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}) &: \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}) &: \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9) &: \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8) &: \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_6) &: \quad \text{no solutions} \\
(e_2, e_4, e_5) &: \quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(6,14)$

m4B614 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_{11}$$
$$[e_2,e_6]=2e_{12} \qquad [e_2,e_7]=4e_{13}$$
$$[e_2,e_{13}]=e_{14} \qquad [e_3,e_4]=-e_{11}$$
$$[e_3,e_5]=-e_{12} \qquad [e_3,e_6]=-2e_{13}$$
$$[e_3,e_{12}]=-e_{14} \qquad [e_4,e_5]=e_{13}$$
$$[e_4,e_{11}]=e_{14} \qquad [e_5,e_{10}]=-e_{14}$$
$$[e_6,e_9]=e_{14} \qquad [e_7,e_8]=-e_{14}$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_{11}$$
$$[e_2,e_6]=2e_{12} \qquad [e_2,e_7]=\alpha_{2,7}^{13}e_{13}$$
$$[e_2,e_{13}]=e_{14} \qquad [e_3,e_4]=-e_{11}$$
$$[e_3,e_5]=-e_{12} \qquad [e_3,e_6]=\alpha_{3,6}^{13}e_{13}$$
$$[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} \qquad [e_4,e_5]=\alpha_{4,5}^{13}e_{13}$$
$$[e_4,e_{11}]=\alpha_{4,11}^{14}e_{14} \qquad [e_5,e_{10}]=\alpha_{5,10}^{14}e_{14}$$
$$[e_6,e_9]=\alpha_{6,9}^{14}e_{14} \qquad [e_7,e_8]=\alpha_{7,8}^{14}e_{14}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha^{13}_{2,7} - \alpha^{13}_{3,6} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha^{13}_{3,6} - \alpha^{13}_{4,5} - 1 &&= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha^{14}_{3,12} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha^{14}_{3,12} - \alpha^{14}_{4,11} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha^{14}_{4,11} - \alpha^{14}_{5,10} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha^{14}_{5,10} - \alpha^{14}_{6,9} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha^{14}_{6,9} - \alpha^{14}_{7,8} &&= 0\\
(e_2, e_3, e_6): &\quad -2\alpha^{14}_{3,12} + \alpha^{13}_{3,6} &&= 0\\
(e_2, e_4, e_5): &\quad -\alpha^{14}_{4,11} + \alpha^{13}_{4,5} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha^{14}_{3,12} &= -1\\
\alpha^{14}_{5,10} &= -1\\
\alpha^{14}_{7,8} &= -1\\
\alpha^{13}_{4,5} &= 1\\
\alpha^{13}_{2,7} &= 4\\
\alpha^{14}_{6,9} &= 1\\
\alpha^{14}_{4,11} &= 1\\
\alpha^{13}_{3,6} &= -2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha^{14}_{3,12} &\to x_1\\
\alpha^{14}_{5,10} &\to x_2\\
\alpha^{14}_{7,8} &\to x_3\\
\alpha^{13}_{4,5} &\to x_4\\
\alpha^{13}_{2,7} &\to x_5\\
\alpha^{14}_{6,9} &\to x_6\\
\alpha^{14}_{4,11} &\to x_7
\end{aligned}
$$

$$\alpha^{13}_{3,6} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_5 - x_8 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_4 - x_8 - 1 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_7 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_2 - x_7 &&= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_6 &&= 0\\
(e_1, e_6, e_8): &\quad -x_3 - x_6 &&= 0\\
(e_2, e_3, e_6): &\quad -2x_1 + x_8 &&= 0\\
(e_2, e_4, e_5): &\quad x_4 - x_7 &&= 0
\end{aligned}$$

Groebner basis (8 variables, 8 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 + 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - 1 = 0$$
$$x_5 - 4 = 0$$
$$x_6 - 1 = 0$$
$$x_7 - 1 = 0$$
$$x_8 + 2 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = -1$$
$$x_3 = -1$$
$$x_4 = 1$$
$$x_5 = 4$$
$$x_6 = 1$$
$$x_7 = 1$$
$$x_8 = -2$$

$\mathfrak{m}_{6B}(6, 14)$

m6B614 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_9 \\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= \alpha_{2,5}^{11} e_{11} \\
[e_2, e_6] &= \alpha_{2,6}^{12} e_{12} & [e_2, e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_4] &= \alpha_{3,4}^{11} e_{11} \\
[e_3, e_5] &= \alpha_{3,5}^{12} e_{12} & [e_3, e_6] &= \alpha_{3,6}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= \alpha_{4,5}^{13} e_{13} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} \\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{12} - \alpha_{2,7}^{13} - \alpha_{3,6}^{13} &= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{12} - \alpha_{3,6}^{13} - \alpha_{4,5}^{13} &= 0 \\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0 \\
(e_2, e_3, e_6): &\quad -\alpha_{2,6}^{12}\alpha_{3,12}^{14} + \alpha_{3,6}^{13} + \alpha_{6,9}^{14} &= 0 \\
(e_2, e_4, e_5): &\quad -\alpha_{2,5}^{11}\alpha_{4,11}^{14} + \alpha_{4,5}^{13} + \alpha_{5,10}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha^{14}_{3,12} \to x_1$$

$$\alpha^{14}_{5,10} \to x_2$$

$$\alpha^{14}_{7,8} \to x_3$$

$$\alpha^{13}_{4,5} \to x_4$$

$$\alpha^{13}_{2,7} \to x_5$$

$$\alpha^{11}_{2,5} \to x_6$$

$$\alpha^{11}_{3,4} \to x_7$$

$$\alpha^{12}_{2,6} \to x_8$$

$$\alpha^{14}_{6,9} \to x_9$$

$$\alpha^{14}_{4,11} \to x_{10}$$

$$\alpha^{12}_{3,5} \to x_{11}$$

$$\alpha^{13}_{3,6} \to x_{12}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_6 - x_7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{11} + x_6 - x_8 &&= 0\\
(e_1, e_3, e_4): &\quad -x_{11} + x_7 &&= 0\\
(e_1, e_2, e_6): &\quad -x_{12} - x_5 + x_8 &&= 0\\
(e_1, e_3, e_5): &\quad x_{11} - x_{12} - x_4 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_{10} &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_{10} - x_2 &&= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_9 &&= 0\\
(e_1, e_6, e_8): &\quad -x_3 - x_9 &&= 0\\
(e_2, e_3, e_6): &\quad -x_1x_8 + x_{12} + x_9 &&= 0\\
(e_2, e_4, e_5): &\quad -x_{10}x_6 + x_2 + x_4 &&= 0
\end{aligned}$$

Groebner basis (12 variables, 11 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$
$$x_{12} + 2x_4 - 2 = 0$$
$$2x_{12} + x_5 + 1 = 0$$
$$x_{12} + 2x_6 = 0$$
$$-x_{12} + 2x_7 - 2 = 0$$
$$x_{12} + x_8 + 1 = 0$$
$$x_9 - 1 = 0$$
$$x_{10} - 1 = 0$$
$$2x_{11} - x_{12} - 2 = 0$$

$\mathfrak{m}_{3B}(7, 14)$

m3B714 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_5] = e_{12}$$
$$[e_2, e_6] = 2e_{13} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_4] = -e_{12} \qquad [e_3, e_5] = -e_{13}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_{11}] = e_{14}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_5] = e_{12} \\
[e_2, e_6] = 2e_{13} & [e_2, e_{13}] = e_{14} \\
[e_3, e_4] = -e_{12} & [e_3, e_5] = -e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] = \alpha_{7,8}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rll}
(e_1, e_2, e_{12}): & -\alpha_{3,12}^{14} - 1 & = 0 \\
(e_1, e_3, e_{11}): & -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} & = 0 \\
(e_1, e_4, e_{10}): & -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} & = 0 \\
(e_1, e_5, e_9): & -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} & = 0 \\
(e_1, e_6, e_8): & -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} & = 0 \\
(e_2, e_3, e_5): & -\alpha_{3,12}^{14} - 1 & = 0
\end{array}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,12}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1 \\
\alpha_{7,8}^{14} &= -1 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{4,11}^{14} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\alpha_{3,12}^{14} \to x_1
$$

$$
\alpha_{5,10}^{14} \to x_2
$$

$$\alpha_{7,8}^{14} \to x_3$$

$$\alpha_{6,9}^{14} \to x_4$$

$$\alpha_{4,11}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}) : &\quad -x_1 - 1 &= 0\\
(e_1, e_3, e_{11}) : &\quad -x_1 - x_5 &= 0\\
(e_1, e_4, e_{10}) : &\quad -x_2 - x_5 &= 0\\
(e_1, e_5, e_9) : &\quad -x_2 - x_4 &= 0\\
(e_1, e_6, e_8) : &\quad -x_3 - x_4 &= 0\\
(e_2, e_3, e_5) : &\quad -x_1 - 1 &= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 - 1 = 0$$

$$x_5 - 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = -1$$

$$x_3 = -1$$

$$x_4 = 1$$

$$x_5 = 1$$

$\mathfrak{m}_{5B}(7, 14)$

m5B714 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_{10} \\
[e_2, e_4] = e_{11} & [e_2, e_5] = \alpha_{2,5}^{12} e_{12} \\
[e_2, e_6] = \alpha_{2,6}^{13} e_{13} & [e_2, e_{13}] = e_{14} \\
[e_3, e_4] = \alpha_{3,4}^{12} e_{12} & [e_3, e_5] = \alpha_{3,5}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] = \alpha_{7,8}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rlr}
(e_1, e_2, e_4): & -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 & = 0 \\
(e_1, e_2, e_5): & \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} & = 0 \\
(e_1, e_3, e_4): & \alpha_{3,4}^{12} - \alpha_{3,5}^{13} & = 0 \\
(e_1, e_2, e_{12}): & -\alpha_{3,12}^{14} - 1 & = 0 \\
(e_1, e_3, e_{11}): & -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} & = 0 \\
(e_1, e_4, e_{10}): & -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} & = 0 \\
(e_1, e_5, e_9): & -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} & = 0 \\
(e_1, e_6, e_8): & -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} & = 0 \\
(e_2, e_3, e_5): & -\alpha_{2,5}^{12}\alpha_{3,12}^{14} + \alpha_{3,5}^{13} + \alpha_{5,10}^{14} & = 0
\end{array}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,12}^{14} \to x_1$$

$$\alpha_{2,6}^{13} \to x_2$$

$$\alpha_{5,10}^{14} \to x_3$$

$$\alpha_{7,8}^{14} \to x_4$$

$$\alpha_{2,5}^{12} \to x_5$$

$$\alpha_{6,9}^{14} \to x_6$$

$$\alpha_{3,5}^{13} \to x_7$$

$$\alpha_{4,11}^{14} \to x_8$$

$$\alpha_{3,4}^{12} \to x_9$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_5 - x_9 + 1 && = 0\\
(e_1, e_2, e_5): &\quad -x_2 + x_5 - x_7 && = 0\\
(e_1, e_3, e_4): &\quad -x_7 + x_9 && = 0\\
(e_1, e_2, e_{12}): &\quad -x_1 - 1 && = 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_8 && = 0\\
(e_1, e_4, e_{10}): &\quad -x_3 - x_8 && = 0\\
(e_1, e_5, e_9): &\quad -x_3 - x_6 && = 0\\
(e_1, e_6, e_8): &\quad -x_4 - x_6 && = 0\\
(e_2, e_3, e_5): &\quad -x_1x_5 + x_3 + x_7 && = 0
\end{aligned}$$

Groebner basis (9 variables, 8 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 2x_9 - 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 + 1 = 0$$

$$x_5 + x_9 - 1 = 0$$

$$x_6 - 1 = 0$$

$$x_7 - x_9 = 0$$

$$x_8 - 1 = 0$$

$\mathfrak{m}_{2B}(8,14)$

m2B814 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_2,e_5] &= e_{13} \\
[e_2,e_{13}] &= e_{14} & [e_3,e_4] &= -e_{13} \\
[e_3,e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4,e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5,e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6,e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7,e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1,e_3,e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1,e_4,e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1,e_5,e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1,e_6,e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2,e_3,e_4): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(8,14)$

m4B814 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{11} \\
[e_2, e_4] &= e_{12} & [e_2, e_5] &= 3e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_4] &= -2e_{13} \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_{11}] &= e_{14} \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{11} \\
[e_2, e_4] &= e_{12} & [e_2, e_5] &= \alpha_{2,5}^{13} e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_4] &= \alpha_{3,4}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 &&= 0 \\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{3,12}^{14} + \alpha_{3,4}^{13} + \alpha_{4,11}^{14} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{13}_{2,5} = 3$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{13}_{3,4} = -2$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{14}_{3,12} \to x_1$$
$$\alpha^{14}_{5,10} \to x_2$$
$$\alpha^{14}_{7,8} \to x_3$$
$$\alpha^{13}_{2,5} \to x_4$$
$$\alpha^{14}_{6,9} \to x_5$$
$$\alpha^{14}_{4,11} \to x_6$$
$$\alpha^{13}_{3,4} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_4 - x_7 + 1 &= 0\\
(e_1, e_2, e_{12}): &\quad -x_1 - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_6 &= 0\\
(e_1, e_4, e_{10}): &\quad -x_2 - x_6 &= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_5 &= 0\\
(e_1, e_6, e_8): &\quad -x_3 - x_5 &= 0\\
(e_2, e_3, e_4): &\quad -x_1 + x_6 + x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 7 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$
$$x_4 - 3 = 0$$
$$x_5 - 1 = 0$$
$$x_6 - 1 = 0$$
$$x_7 + 2 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = -1$$
$$x_3 = -1$$
$$x_4 = 3$$
$$x_5 = 1$$
$$x_6 = 1$$
$$x_7 = -2$$

$\mathfrak{m}_{3B}(9, 14)$

m3B914 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{12} \\
[e_2, e_4] &= e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_{11}] &= e_{14} \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{12} \\
[e_2, e_4] &= e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}) : &\quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}) : &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}) : &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9) : &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8) : &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,12}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1 \\
\alpha_{7,8}^{14} &= -1 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{4,11}^{14} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,12}^{14} &\to x_1 \\
\alpha_{5,10}^{14} &\to x_2 \\
\alpha_{7,8}^{14} &\to x_3 \\
\alpha_{6,9}^{14} &\to x_4
\end{aligned}
$$

$$\alpha_{4,11}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}): &\quad -x_1 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_5 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_2 - x_5 &&= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_4 &&= 0\\
(e_1, e_6, e_8): &\quad -x_3 - x_4 &&= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 + 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - 1 = 0$$
$$x_5 - 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = -1$$
$$x_3 = -1$$
$$x_4 = 1$$
$$x_5 = 1$$

$\mathfrak{m}_{2B}(10, 14)$

m2B1014 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{13}\\
[e_2, e_{13}] &= e_{14} & [e_3, e_{12}] &= -e_{14}\\
[e_4, e_{11}] &= e_{14} & [e_5, e_{10}] &= -e_{14}\\
[e_6, e_9] &= e_{14} & [e_7, e_8] &= -e_{14}
\end{aligned}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_{13} \\
[e_2, e_{13}] = e_{14} & [e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} \\
[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} & [e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} \\
[e_6, e_9] = \alpha_{6,9}^{14} e_{14} & [e_7, e_8] = \alpha_{7,8}^{14} e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rlr}
(e_1, e_2, e_{12}): & -\alpha_{3,12}^{14} - 1 & = 0 \\
(e_1, e_3, e_{11}): & -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} & = 0 \\
(e_1, e_4, e_{10}): & -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} & = 0 \\
(e_1, e_5, e_9): & -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} & = 0 \\
(e_1, e_6, e_8): & -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} & = 0
\end{array}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,12}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1 \\
\alpha_{7,8}^{14} &= -1 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{4,11}^{14} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,12}^{14} &\to x_1 \\
\alpha_{5,10}^{14} &\to x_2 \\
\alpha_{7,8}^{14} &\to x_3 \\
\alpha_{6,9}^{14} &\to x_4
\end{aligned}
$$

$$\alpha_{4,11}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}) : &\quad -x_1 - 1 && = 0 \\
(e_1, e_3, e_{11}) : &\quad -x_1 - x_5 && = 0 \\
(e_1, e_4, e_{10}) : &\quad -x_2 - x_5 && = 0 \\
(e_1, e_5, e_9) : &\quad -x_2 - x_4 && = 0 \\
(e_1, e_6, e_8) : &\quad -x_3 - x_4 && = 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 - 1 = 0$$

$$x_5 - 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = -1$$

$$x_3 = -1$$

$$x_4 = 1$$

$$x_5 = 1$$