Computation of positively graded filiform Lie algebras over Q

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April 8, 2021

Summary table (starting on the next page)

Explanation of table

- Column 1 (search) - A character string for text searching purposes
- Column 2 (algebra) - The subclass of positively graded filiform Lie algebra(s)
- Column 3 (Jac) - A check indicates that the class is nonempty
- Column 4 (sol) - Number of Lie algebras in the class

search	algebra	Jac	sol
m1A25	$\mathfrak{m}_{1A}(2,5)$	$\surd$	1
m2A26	$\mathfrak{m}_{2A}(2,6)$	$\surd$	1
m1A36	$\mathfrak{m}_{1A}(3,6)$	$\surd$	1
m1A27	$\mathfrak{m}_{1A}(2,7)$	$\surd$	1
m3A27	$\mathfrak{m}_{3A}(2,7)$	$\surd$	∞
m2A37	$\mathfrak{m}_{2A}(3,7)$	$\surd$	1
m1A47	$\mathfrak{m}_{1A}(4,7)$	$\surd$	1
m2A28	$\mathfrak{m}_{2A}(2,8)$	$\surd$	1
m4A28	$\mathfrak{m}_{4A}(2,8)$	$\surd$	∞
m1A38	$\mathfrak{m}_{1A}(3,8)$	$\surd$	1
m3A38	$\mathfrak{m}_{3A}(3,8)$	$\surd$	∞
m2A48	$\mathfrak{m}_{2A}(4,8)$	$\surd$	1
m1A58	$\mathfrak{m}_{1A}(5,8)$	$\surd$	1
m1A29	$\mathfrak{m}_{1A}(2,9)$	$\surd$	1
m3A29	$\mathfrak{m}_{3A}(2,9)$	$\surd$	1
m5A29	$\mathfrak{m}_{5A}(2,9)$	$\surd$	∞
m2A39	$\mathfrak{m}_{2A}(3,9)$	$\surd$	1
m4A39	$\mathfrak{m}_{4A}(3,9)$	$\surd$	∞
m1A49	$\mathfrak{m}_{1A}(4,9)$	$\surd$	1
m3A49	$\mathfrak{m}_{3A}(4,9)$	$\surd$	∞
m2A59	$\mathfrak{m}_{2A}(5,9)$	$\surd$	1
m1A69	$\mathfrak{m}_{1A}(6,9)$	$\surd$	1
m2A210	$\mathfrak{m}_{2A}(2,10)$	$\surd$	1
m4A210	$\mathfrak{m}_{4A}(2,10)$	$\surd$	1
m6A210	$\mathfrak{m}_{6A}(2,10)$	$\surd$	∞
m1A310	$\mathfrak{m}_{1A}(3,10)$	$\surd$	1
m3A310	$\mathfrak{m}_{3A}(3,10)$	$\surd$	∞
m5A310	$\mathfrak{m}_{5A}(3,10)$	$\surd$	∞
m2A410	$\mathfrak{m}_{2A}(4,10)$	$\surd$	1
m4A410	$\mathfrak{m}_{4A}(4,10)$	$\surd$	∞
m1A510	$\mathfrak{m}_{1A}(5,10)$	$\surd$	1
m3A510	$\mathfrak{m}_{3A}(5,10)$	$\surd$	∞
m2A610	$\mathfrak{m}_{2A}(6,10)$	$\surd$	1
m1A710	$\mathfrak{m}_{1A}(7,10)$	$\surd$	1
m1A211	$\mathfrak{m}_{1A}(2,11)$	$\surd$	1
m3A211	$\mathfrak{m}_{3A}(2,11)$	$\surd$	1
m5A211	$\mathfrak{m}_{5A}(2,11)$	$\surd$	1
m7A211	$\mathfrak{m}_{7A}(2,11)$	$\surd$	∞
m2A311	$\mathfrak{m}_{2A}(3,11)$	$\surd$	1
m4A311	$\mathfrak{m}_{4A}(3,11)$	$\surd$	1
m6A311	$\mathfrak{m}_{6A}(3,11)$	$\surd$	∞
m1A411	$\mathfrak{m}_{1A}(4,11)$	$\surd$	1

search	algebra	Jac	sol
m3A411	$\mathfrak{m}_{3A}(4,11)$	$\surd$	∞
m5A411	$\mathfrak{m}_{5A}(4,11)$	$\surd$	∞
m2A511	$\mathfrak{m}_{2A}(5,11)$	$\surd$	1
m4A511	$\mathfrak{m}_{4A}(5,11)$	$\surd$	∞
m1A611	$\mathfrak{m}_{1A}(6,11)$	$\surd$	1
m3A611	$\mathfrak{m}_{3A}(6,11)$	$\surd$	∞
m2A711	$\mathfrak{m}_{2A}(7,11)$	$\surd$	1
m1A811	$\mathfrak{m}_{1A}(8,11)$	$\surd$	1
m2A212	$\mathfrak{m}_{2A}(2,12)$	$\surd$	1
m4A212	$\mathfrak{m}_{4A}(2,12)$		0
m6A212	$\mathfrak{m}_{6A}(2,12)$		0
m8A212	$\mathfrak{m}_{8A}(2,12)$	$\surd$	2
m1A312	$\mathfrak{m}_{1A}(3,12)$	$\surd$	1
m3A312	$\mathfrak{m}_{3A}(3,12)$	$\surd$	∞
m5A312	$\mathfrak{m}_{5A}(3,12)$	$\surd$	∞
m7A312	$\mathfrak{m}_{7A}(3,12)$	$\surd$	∞
m2A412	$\mathfrak{m}_{2A}(4,12)$	$\surd$	1
m4A412	$\mathfrak{m}_{4A}(4,12)$	$\surd$	∞
m6A412	$\mathfrak{m}_{6A}(4,12)$	$\surd$	∞
m1A512	$\mathfrak{m}_{1A}(5,12)$	$\surd$	1
m3A512	$\mathfrak{m}_{3A}(5,12)$	$\surd$	∞
m5A512	$\mathfrak{m}_{5A}(5,12)$	$\surd$	∞
m2A612	$\mathfrak{m}_{2A}(6,12)$	$\surd$	1
m4A612	$\mathfrak{m}_{4A}(6,12)$	$\surd$	∞
m1A712	$\mathfrak{m}_{1A}(7,12)$	$\surd$	1
m3A712	$\mathfrak{m}_{3A}(7,12)$	$\surd$	∞
m2A812	$\mathfrak{m}_{2A}(8,12)$	$\surd$	1
m1A912	$\mathfrak{m}_{1A}(9,12)$	$\surd$	1
m1A213	$\mathfrak{m}_{1A}(2,13)$	$\surd$	1
m3A213	$\mathfrak{m}_{3A}(2,13)$	$\surd$	1
m9A213	$\mathfrak{m}_{9A}(2,13)$	$\surd$	2
m2A313	$\mathfrak{m}_{2A}(3,13)$	$\surd$	1
m4A313	$\mathfrak{m}_{4A}(3,13)$	$\surd$	1
m6A313	$\mathfrak{m}_{6A}(3,13)$	$\surd$	1
m8A313	$\mathfrak{m}_{8A}(3,13)$	$\surd$	∞
m1A413	$\mathfrak{m}_{1A}(4,13)$	$\surd$	1
m3A413	$\mathfrak{m}_{3A}(4,13)$	$\surd$	∞
m5A413	$\mathfrak{m}_{5A}(4,13)$	$\surd$	∞
m7A413	$\mathfrak{m}_{7A}(4,13)$	$\surd$	∞
m2A513	$\mathfrak{m}_{2A}(5,13)$	$\surd$	1
m4A513	$\mathfrak{m}_{4A}(5,13)$	$\surd$	∞
m6A513	$\mathfrak{m}_{6A}(5,13)$	$\surd$	∞

search	algebra	Jac	sol
m1A613	$\mathfrak{m}_{1A}(6,13)$	$\surd$	1
m3A613	$\mathfrak{m}_{3A}(6,13)$	$\surd$	∞
m5A613	$\mathfrak{m}_{5A}(6,13)$	$\surd$	∞
m2A713	$\mathfrak{m}_{2A}(7,13)$	$\surd$	1
m4A713	$\mathfrak{m}_{4A}(7,13)$	$\surd$	∞
m1A813	$\mathfrak{m}_{1A}(8,13)$	$\surd$	1
m3A813	$\mathfrak{m}_{3A}(8,13)$	$\surd$	∞
m2A913	$\mathfrak{m}_{2A}(9,13)$	$\surd$	1
m1A1013	$\mathfrak{m}_{1A}(10,13)$	$\surd$	1
m2A214	$\mathfrak{m}_{2A}(2,14)$	$\surd$	1
m4A214	$\mathfrak{m}_{4A}(2,14)$		0
m10A214	$\mathfrak{m}_{10A}(2,14)$	$\surd$	2
m1A314	$\mathfrak{m}_{1A}(3,14)$	$\surd$	1
m3A314	$\mathfrak{m}_{3A}(3,14)$	$\surd$	∞
m5A314	$\mathfrak{m}_{5A}(3,14)$	$\surd$	1
m7A314	$\mathfrak{m}_{7A}(3,14)$	$\surd$	1
m9A314	$\mathfrak{m}_{9A}(3,14)$	$\surd$	∞
m2A414	$\mathfrak{m}_{2A}(4,14)$	$\surd$	1
m4A414	$\mathfrak{m}_{4A}(4,14)$	$\surd$	∞
m6A414	$\mathfrak{m}_{6A}(4,14)$	$\surd$	∞
m8A414	$\mathfrak{m}_{8A}(4,14)$	$\surd$	∞
m1A514	$\mathfrak{m}_{1A}(5,14)$	$\surd$	1
m3A514	$\mathfrak{m}_{3A}(5,14)$	$\surd$	∞
m5A514	$\mathfrak{m}_{5A}(5,14)$	$\surd$	∞
m7A514	$\mathfrak{m}_{7A}(5,14)$	$\surd$	∞
m2A614	$\mathfrak{m}_{2A}(6,14)$	$\surd$	1
m4A614	$\mathfrak{m}_{4A}(6,14)$	$\surd$	∞
m6A614	$\mathfrak{m}_{6A}(6,14)$	$\surd$	∞
m1A714	$\mathfrak{m}_{1A}(7,14)$	$\surd$	1
m3A714	$\mathfrak{m}_{3A}(7,14)$	$\surd$	∞
m5A714	$\mathfrak{m}_{5A}(7,14)$	$\surd$	∞
m2A814	$\mathfrak{m}_{2A}(8,14)$	$\surd$	1
m4A814	$\mathfrak{m}_{4A}(8,14)$	$\surd$	∞
m1A914	$\mathfrak{m}_{1A}(9,14)$	$\surd$	1
m3A914	$\mathfrak{m}_{3A}(9,14)$	$\surd$	∞
m2A1014	$\mathfrak{m}_{2A}(10,14)$	$\surd$	1
m1A1114	$\mathfrak{m}_{1A}(11,14)$	$\surd$	1
m1A215	$\mathfrak{m}_{1A}(2,15)$	$\surd$	1
m3A215	$\mathfrak{m}_{3A}(2,15)$	$\surd$	1
m11A215	$\mathfrak{m}_{11A}(2,15)$	$\surd$	2
m2A315	$\mathfrak{m}_{2A}(3,15)$	$\surd$	1
m4A315	$\mathfrak{m}_{4A}(3,15)$	$\surd$	1

search	algebra	Jac	sol
m6A315	$\mathfrak{m}_{6A}(3,15)$		0
m8A315	$\mathfrak{m}_{8A}(3,15)$		0
m10A315	$\mathfrak{m}_{10A}(3,15)$	$\surd$	3
m1A415	$\mathfrak{m}_{1A}(4,15)$	$\surd$	1
m3A415	$\mathfrak{m}_{3A}(4,15)$	$\surd$	∞
m5A415	$\mathfrak{m}_{5A}(4,15)$	$\surd$	∞
m7A415	$\mathfrak{m}_{7A}(4,15)$	$\surd$	∞
m9A415	$\mathfrak{m}_{9A}(4,15)$	$\surd$	∞
m2A515	$\mathfrak{m}_{2A}(5,15)$	$\surd$	1
m4A515	$\mathfrak{m}_{4A}(5,15)$	$\surd$	∞
m6A515	$\mathfrak{m}_{6A}(5,15)$	$\surd$	∞
m8A515	$\mathfrak{m}_{8A}(5,15)$	$\surd$	∞
m1A615	$\mathfrak{m}_{1A}(6,15)$	$\surd$	1
m3A615	$\mathfrak{m}_{3A}(6,15)$	$\surd$	∞
m5A615	$\mathfrak{m}_{5A}(6,15)$	$\surd$	∞
m7A615	$\mathfrak{m}_{7A}(6,15)$	$\surd$	∞
m2A715	$\mathfrak{m}_{2A}(7,15)$	$\surd$	1
m4A715	$\mathfrak{m}_{4A}(7,15)$	$\surd$	∞
m6A715	$\mathfrak{m}_{6A}(7,15)$	$\surd$	∞
m1A815	$\mathfrak{m}_{1A}(8,15)$	$\surd$	1
m3A815	$\mathfrak{m}_{3A}(8,15)$	$\surd$	∞
m5A815	$\mathfrak{m}_{5A}(8,15)$	$\surd$	∞
m2A915	$\mathfrak{m}_{2A}(9,15)$	$\surd$	1
m4A915	$\mathfrak{m}_{4A}(9,15)$	$\surd$	∞
m1A1015	$\mathfrak{m}_{1A}(10,15)$	$\surd$	1
m3A1015	$\mathfrak{m}_{3A}(10,15)$	$\surd$	∞
m2A1115	$\mathfrak{m}_{2A}(11,15)$	$\surd$	1
m1A1215	$\mathfrak{m}_{1A}(12,15)$	$\surd$	1
m2B26	$\mathfrak{m}_{2B}(2,6)$	$\surd$	1
m2B28	$\mathfrak{m}_{2B}(2,8)$		0
m4B28	$\mathfrak{m}_{4B}(2,8)$	$\surd$	1
m3B38	$\mathfrak{m}_{3B}(3,8)$	$\surd$	1
m2B48	$\mathfrak{m}_{2B}(4,8)$	$\surd$	1
m2B210	$\mathfrak{m}_{2B}(2,10)$		0
m4B210	$\mathfrak{m}_{4B}(2,10)$		0
m6B210	$\mathfrak{m}_{6B}(2,10)$	$\surd$	2
m3B310	$\mathfrak{m}_{3B}(3,10)$	$\surd$	1
m5B310	$\mathfrak{m}_{5B}(3,10)$	$\surd$	∞
m2B410	$\mathfrak{m}_{2B}(4,10)$		0
m4B410	$\mathfrak{m}_{4B}(4,10)$	$\surd$	1
m3B510	$\mathfrak{m}_{3B}(5,10)$	$\surd$	1
m2B610	$\mathfrak{m}_{2B}(6,10)$	$\surd$	1

search	algebra	Jac	sol
m2B212	$\mathfrak{m}_{2B}(2,12)$		0
m4B212	$\mathfrak{m}_{4B}(2,12)$		0
m6B212	$\mathfrak{m}_{6B}(2,12)$		0
m8B212	$\mathfrak{m}_{8B}(2,12)$	$\surd$	1
m3B312	$\mathfrak{m}_{3B}(3,12)$		0
m5B312	$\mathfrak{m}_{5B}(3,12)$		0
m7B312	$\mathfrak{m}_{7B}(3,12)$	$\surd$	2
m2B412	$\mathfrak{m}_{2B}(4,12)$		0
m4B412	$\mathfrak{m}_{4B}(4,12)$	$\surd$	1
m6B412	$\mathfrak{m}_{6B}(4,12)$	$\surd$	∞
m3B512	$\mathfrak{m}_{3B}(5,12)$	$\surd$	1
m5B512	$\mathfrak{m}_{5B}(5,12)$	$\surd$	∞
m2B612	$\mathfrak{m}_{2B}(6,12)$		0
m4B612	$\mathfrak{m}_{4B}(6,12)$	$\surd$	1
m3B712	$\mathfrak{m}_{3B}(7,12)$	$\surd$	1
m2B812	$\mathfrak{m}_{2B}(8,12)$	$\surd$	1
m2B214	$\mathfrak{m}_{2B}(2,14)$		0
m4B214	$\mathfrak{m}_{4B}(2,14)$		0
m10B214	$\mathfrak{m}_{10B}(2,14)$		0
m3B314	$\mathfrak{m}_{3B}(3,14)$		0
m5B314	$\mathfrak{m}_{5B}(3,14)$		0
m7B314	$\mathfrak{m}_{7B}(3,14)$		0
m9B314	$\mathfrak{m}_{9B}(3,14)$	$\surd$	4
m2B414	$\mathfrak{m}_{2B}(4,14)$		0
m4B414	$\mathfrak{m}_{4B}(4,14)$	$\surd$	1
m6B414	$\mathfrak{m}_{6B}(4,14)$	$\surd$	1
m8B414	$\mathfrak{m}_{8B}(4,14)$	$\surd$	∞
m3B514	$\mathfrak{m}_{3B}(5,14)$		0
m5B514	$\mathfrak{m}_{5B}(5,14)$	$\surd$	1
m7B514	$\mathfrak{m}_{7B}(5,14)$	$\surd$	∞
m2B614	$\mathfrak{m}_{2B}(6,14)$		0
m4B614	$\mathfrak{m}_{4B}(6,14)$	$\surd$	1
m6B614	$\mathfrak{m}_{6B}(6,14)$	$\surd$	∞
m3B714	$\mathfrak{m}_{3B}(7,14)$	$\surd$	1
m5B714	$\mathfrak{m}_{5B}(7,14)$	$\surd$	∞
m2B814	$\mathfrak{m}_{2B}(8,14)$		0
m4B814	$\mathfrak{m}_{4B}(8,14)$	$\surd$	1
m3B914	$\mathfrak{m}_{3B}(9,14)$	$\surd$	1
m2B1014	$\mathfrak{m}_{2B}(10,14)$	$\surd$	1

Algebra details

$\mathfrak{m}_{1A}(2,5)$

m1A25 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_2,e_3]=e_5$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,6)$

m2A26 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_2,e_3]=e_5 \qquad [e_2,e_4]=e_6$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(3,6)$

m1A36 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_2,e_3]=e_6$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2,7)$

m1A27 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_2,e_5]=e_7$$
$$[e_3,e_4]=-e_7$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2,7)$

m3A27 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_2,e_3] &= e_5\\
[e_2,e_4] &= e_6 & [e_2,e_5] &= \alpha_{2,5}^{7}e_7\\
[e_3,e_4] &= \alpha_{3,4}^{7}e_7
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{7}-\alpha_{3,4}^{7}+1 \qquad = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,5}^{7} &\to x_1\\
\alpha_{3,4}^{7} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1-x_2+1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1+x_2-1=0$$

$\mathfrak{m}_{2A}(3,7)$

m2A37 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_2,e_3] &= e_6\\
[e_2,e_4] &= e_7
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(4,7)$

m1A47 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_2,e_3]=e_7$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,8)$

m2A28 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_2,e_5]=e_7 \qquad [e_2,e_6]=2e_8$$
$$[e_3,e_4]=-e_7 \qquad [e_3,e_5]=-e_8$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2,8)$

m4A28 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_2,e_3]=e_5 \qquad [e_2,e_4]=e_6$$
$$[e_2,e_5]=\alpha_{2,5}^{7}e_7 \qquad [e_2,e_6]=\alpha_{2,6}^{8}e_8$$
$$[e_3,e_4]=\alpha_{3,4}^{7}e_7 \qquad [e_3,e_5]=\alpha_{3,5}^{8}e_8$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^7 - \alpha_{3,4}^7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^7 - \alpha_{2,6}^8 - \alpha_{3,5}^8 &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^7 - \alpha_{3,5}^8 &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^8 \to x_1$$
$$\alpha_{2,6}^8 \to x_2$$
$$\alpha_{2,5}^7 \to x_3$$
$$\alpha_{3,4}^7 \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_4 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_2 + x_3 &&= 0\\
(e_1, e_3, e_4): &\quad -x_1 + x_4 &&= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 = 0$$
$$x_2 + 2x_4 - 1 = 0$$
$$x_3 + x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(3, 8)$

m1A38 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_2, e_5] &= e_8 & [e_3, e_4] &= -e_8
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3,8)$

m3A38 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_2, e_3] &= e_6 & [e_2, e_4] &= e_7 \\
[e_2, e_5] &= \alpha_{2,5}^8 e_8 & [e_3, e_4] &= \alpha_{3,4}^8 e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^8 - \alpha_{3,4}^8 + 1 = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,5}^8 &\to x_1 \\
\alpha_{3,4}^8 &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(4,8)$

m2A48 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_2, e_3] &= e_7 & [e_2, e_4] &= e_8
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(5, 8)$

m1A58 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_2, e_3] = e_8$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2, 9)$

m1A29 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_2, e_7] = e_9$$
$$[e_3, e_6] = -e_9 \qquad [e_4, e_5] = e_9$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2, 9)$

m3A29 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_2, e_5] = e_7$$
$$[e_2, e_6] = 2e_8 \qquad [e_2, e_7] = 0$$
$$[e_3, e_4] = -e_7 \qquad [e_3, e_5] = -e_8$$
$$[e_3, e_6] = 2e_9 \qquad [e_4, e_5] = -3e_9$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_5] = e_7 \\
&[e_2, e_6] = 2e_8 && [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^{9} e_9 && [e_4, e_5] = \alpha_{4,5}^{9} e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,6}^{9} &= 2 \\
\alpha_{4,5}^{9} &= -3 \\
\alpha_{2,7}^{9} &= 0
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,6}^{9} &\to x_1 \\
\alpha_{4,5}^{9} &\to x_2 \\
\alpha_{2,7}^{9} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_3 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_1 - x_2 - 1 &&= 0 \\
(e_2, e_3, e_4): &\quad -x_3 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 - 2 = 0$$
$$x_2 + 3 = 0$$
$$x_3 = 0$$

Solution 1:

$$x_1 = 2$$
$$x_2 = -3$$
$$x_3 = 0$$

$\mathfrak{m}_{5A}(2, 9)$

m5A29 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_5 \\
[e_2, e_4] &= e_6 & [e_2, e_5] &= \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] &= \alpha_{2,6}^{8} e_8 & [e_2, e_7] &= \alpha_{2,7}^{9} e_9 \\
[e_3, e_4] &= \alpha_{3,4}^{7} e_7 & [e_3, e_5] &= \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] &= \alpha_{3,6}^{9} e_9 & [e_4, e_5] &= \alpha_{4,5}^{9} e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &&= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &&= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &&= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &&= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &&= 0 \\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{8} \to x_1$$

$$\alpha_{3,5}^{8} \to x_2$$
$$\alpha_{2,5}^{7} \to x_3$$
$$\alpha_{2,7}^{9} \to x_4$$
$$\alpha_{3,6}^{9} \to x_5$$
$$\alpha_{4,5}^{9} \to x_6$$
$$\alpha_{3,4}^{7} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_3 - x_7 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_1 - x_2 + x_3 &&= 0\\
(e_1, e_3, e_4):&\quad -x_2 + x_7 &&= 0\\
(e_1, e_2, e_6):&\quad x_1 - x_4 - x_5 &&= 0\\
(e_1, e_3, e_5):&\quad x_2 - x_5 - x_6 &&= 0\\
(e_2, e_3, e_4):&\quad x_4 x_7 - x_5 + x_6 &&= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 1 nonlinear)

$$x_1 + 2x_7 - 1 = 0$$
$$x_2 - x_7 = 0$$
$$x_3 + x_7 - 1 = 0$$
$$x_4 - x_6 + 3x_7 - 1 = 0$$
$$x_5 + x_6 - x_7 = 0$$
$$x_6 x_7 + 2x_6 - 3x_7^2 = 0$$

$\mathfrak{m}_{2A}(3, 9)$

m2A39 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_5] &= e_8\\
[e_2, e_6] &= 2e_9 & [e_3, e_4] &= -e_8\\
[e_3, e_5] &= -e_9
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3,9)$

m4A39 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_6 \\
[e_2,e_4] &= e_7 & [e_2,e_5] &= \alpha_{2,5}^{8} e_8 \\
[e_2,e_6] &= \alpha_{2,6}^{9} e_9 & [e_3,e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3,e_5] &= \alpha_{3,5}^{9} e_9
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &&= 0 \\
(e_1,e_2,e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &&= 0 \\
(e_1,e_3,e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{3,5}^{9} &\to x_1 \\
\alpha_{2,6}^{9} &\to x_2 \\
\alpha_{2,5}^{8} &\to x_3 \\
\alpha_{3,4}^{8} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -x_3 - x_4 + 1 &&= 0 \\
(e_1,e_2,e_5): &\quad -x_1 - x_2 + x_3 &&= 0 \\
(e_1,e_3,e_4): &\quad -x_1 + x_4 &&= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 - x_4 &= 0 \\
x_2 + 2x_4 - 1 &= 0 \\
x_3 + x_4 - 1 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(4,9)$

m1A49 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_5] &= e_9 \\
[e_3,e_4] &= -e_9 & &
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4,9)$

m3A49 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_7 \\
[e_2,e_4] &= e_8 & [e_2,e_5] &= \alpha_{2,5}^{9} e_9 \\
[e_3,e_4] &= \alpha_{3,4}^{9} e_9 & &
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,5}^{9} &\to x_1 \\
\alpha_{3,4}^{9} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(5,9)$

m2A59 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_8 \\
[e_2,e_4] &= e_9
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(6,9)$

m1A69 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_9
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2,10)$

m2A210 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_2,e_7] &= e_9 & [e_2,e_8] &= 3e_{10} \\
[e_3,e_6] &= -e_9 & [e_3,e_7] &= -2e_{10} \\
[e_4,e_5] &= e_9 & [e_4,e_6] &= e_{10}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2, 10)$

m4A210 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_5] = e_7 && [e_2, e_6] = 2e_8 \\
&[e_2, e_7] = 0 && [e_2, e_8] = -5e_{10} \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = 2e_9 && [e_3, e_7] = 5e_{10} \\
&[e_4, e_5] = -3e_9 && [e_4, e_6] = -3e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_2, e_5] = e_7 && [e_2, e_6] = 2e_8 \\
&[e_2, e_7] = \alpha_{2,7}^{9} e_9 && [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^{9} e_9 && [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
&[e_4, e_5] = \alpha_{4,5}^{9} e_9 && [e_4, e_6] = \alpha_{4,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1, e_2, e_6): && -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 && = 0 \\
&(e_1, e_3, e_5): && -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 && = 0 \\
&(e_2, e_3, e_4): && -\alpha_{2,7}^{9} && = 0 \\
&(e_1, e_2, e_7): && \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} && = 0 \\
&(e_1, e_3, e_6): && \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} && = 0 \\
&(e_1, e_4, e_5): && \alpha_{4,5}^{9} - \alpha_{4,6}^{10} && = 0 \\
&(e_2, e_3, e_5): && -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} && = 0
\end{aligned}
$$

Solution 1:

$$\alpha^{9}_{2,7} = 0$$
$$\alpha^{10}_{3,7} = 5$$
$$\alpha^{10}_{2,8} = -5$$
$$\alpha^{9}_{3,6} = 2$$
$$\alpha^{9}_{4,5} = -3$$
$$\alpha^{10}_{4,6} = -3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{9}_{2,7} \to x_1$$
$$\alpha^{10}_{3,7} \to x_2$$
$$\alpha^{10}_{2,8} \to x_3$$
$$\alpha^{9}_{3,6} \to x_4$$
$$\alpha^{9}_{4,5} \to x_5$$
$$\alpha^{10}_{4,6} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_1 - x_4 + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -x_4 - x_5 - 1 &&= 0\\
(e_2, e_3, e_4):&\quad -x_1 &&= 0\\
(e_1, e_2, e_7):&\quad x_1 - x_2 - x_3 &&= 0\\
(e_1, e_3, e_6):&\quad -x_2 + x_4 - x_6 &&= 0\\
(e_1, e_4, e_5):&\quad x_5 - x_6 &&= 0\\
(e_2, e_3, e_5):&\quad -x_2 - x_3 &&= 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 = 0$$
$$x_2 - 5 = 0$$
$$x_3 + 5 = 0$$
$$x_4 - 2 = 0$$

$$x_5 + 3 = 0$$
$$x_6 + 3 = 0$$

Solution 1:

$$x_1 = 0$$
$$x_2 = 5$$
$$x_3 = -5$$
$$x_4 = 2$$
$$x_5 = -3$$
$$x_6 = -3$$

$\mathfrak{m}_{6A}(2, 10)$

m6A210 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_2, e_3] = e_5 & [e_2, e_4] = e_6 \\
[e_2, e_5] = \alpha_{2,5}^{7} e_7 & [e_2, e_6] = \alpha_{2,6}^{8} e_8 \\
[e_2, e_7] = \alpha_{2,7}^{9} e_9 & [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_4, e_5] = \alpha_{4,5}^{9} e_9 & [e_4, e_6] = \alpha_{4,6}^{10} e_{10}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10} + \alpha_{2,8}^{10}\alpha_{3,5}^{8} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{8} \to x_1$$

$$\alpha_{3,5}^{8} \to x_2$$

$$\alpha_{2,5}^{7} \to x_3$$

$$\alpha_{2,7}^{9} \to x_4$$

$$\alpha_{3,7}^{10} \to x_5$$

$$\alpha_{2,8}^{10} \to x_6$$

$$\alpha_{3,6}^{9} \to x_7$$

$$\alpha_{4,5}^{9} \to x_8$$

$$\alpha_{4,6}^{10} \to x_9$$

$$\alpha_{3,4}^{7} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{10}-x_3+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_1-x_2+x_3 &&=0\\
(e_1,e_3,e_4):&\quad x_{10}-x_2 &&=0\\
(e_1,e_2,e_6):&\quad x_1-x_4-x_7 &&=0\\
(e_1,e_3,e_5):&\quad x_2-x_7-x_8 &&=0\\
(e_2,e_3,e_4):&\quad x_{10}x_4-x_7+x_8 &&=0\\
(e_1,e_2,e_7):&\quad x_4-x_5-x_6 &&=0\\
(e_1,e_3,e_6):&\quad -x_5+x_7-x_9 &&=0\\
(e_1,e_4,e_5):&\quad x_8-x_9 &&=0\\
(e_2,e_3,e_5):&\quad x_2x_6-x_3x_5 &&=0
\end{aligned}
$$

Groebner basis (10 variables, 8 linear, 1 nonlinear)

$$x_1+2x_{10}-1=0$$
$$-x_{10}+x_2=0$$
$$x_{10}+x_3-1=0$$
$$3x_{10}+x_4-x_9-1=0$$
$$-x_{10}+x_5+2x_9=0$$
$$4x_{10}+x_6-3x_9-1=0$$
$$-x_{10}+x_7+x_9=0$$
$$x_8-x_9=0$$
$$-3x_{10}^2+x_{10}x_9+2x_9=0$$

$\mathfrak{m}_{1A}(3,10)$

m1A310 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_2,e_7]&=e_{10} & [e_3,e_6]&=-e_{10}\\
[e_4,e_5]&=e_{10}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3,10)$

m3A310 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_2,e_5] &= e_8 & [e_2,e_6] &= 2e_9 \\
[e_2,e_7] &= \alpha_{2,7}^{10} e_{10} & [e_3,e_4] &= -e_8 \\
[e_3,e_5] &= -e_9 & [e_3,e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_4,e_5] &= \alpha_{4,5}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0 \\
(e_1,e_3,e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,7}^{10} &\to x_1 \\
\alpha_{3,6}^{10} &\to x_2 \\
\alpha_{4,5}^{10} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_6): &\quad -x_1 - x_2 + 2 &&= 0 \\
(e_1,e_3,e_5): &\quad -x_2 - x_3 - 1 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 - x_3 - 3 &= 0 \\
x_2 + x_3 + 1 &= 0
\end{aligned}
$$

$\mathfrak{m}_{5A}(3,10)$

m5A310 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_1,e_9]=e_{10}\\
&[e_2,e_3]=e_6 && [e_2,e_4]=e_7\\
&[e_2,e_5]=\alpha_{2,5}^{8}e_8 && [e_2,e_6]=\alpha_{2,6}^{9}e_9\\
&[e_2,e_7]=\alpha_{2,7}^{10}e_{10} && [e_3,e_4]=\alpha_{3,4}^{8}e_8\\
&[e_3,e_5]=\alpha_{3,5}^{9}e_9 && [e_3,e_6]=\alpha_{3,6}^{10}e_{10}\\
&[e_4,e_5]=\alpha_{4,5}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1,e_2,e_4): && -\alpha_{2,5}^{8}-\alpha_{3,4}^{8}+1 &&=0\\
&(e_1,e_2,e_5): && \alpha_{2,5}^{8}-\alpha_{2,6}^{9}-\alpha_{3,5}^{9} &&=0\\
&(e_1,e_3,e_4): && \alpha_{3,4}^{8}-\alpha_{3,5}^{9} &&=0\\
&(e_1,e_2,e_6): && \alpha_{2,6}^{9}-\alpha_{2,7}^{10}-\alpha_{3,6}^{10} &&=0\\
&(e_1,e_3,e_5): && \alpha_{3,5}^{9}-\alpha_{3,6}^{10}-\alpha_{4,5}^{10} &&=0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^{8}&\to x_1\\
\alpha_{2,7}^{10}&\to x_2\\
\alpha_{2,6}^{9}&\to x_3\\
\alpha_{2,5}^{8}&\to x_4\\
\alpha_{4,5}^{10}&\to x_5\\
\alpha_{3,6}^{10}&\to x_6\\
\alpha_{3,5}^{9}&\to x_7
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_4 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_3 + x_4 - x_7 &= 0\\
(e_1, e_3, e_4): &\quad x_1 - x_7 &= 0\\
(e_1, e_2, e_6): &\quad -x_2 + x_3 - x_6 &= 0\\
(e_1, e_3, e_5): &\quad -x_5 - x_6 + x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - x_7 = 0$$

$$x_2 + x_6 + 2x_7 - 1 = 0$$

$$x_3 + 2x_7 - 1 = 0$$

$$x_4 + x_7 - 1 = 0$$

$$x_5 + x_6 - x_7 = 0$$

$\mathfrak{m}_{2A}(4, 10)$

m2A410 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_2, e_5] &= e_9 & [e_2, e_6] &= 2e_{10}\\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(4, 10)$

m4A410 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_2, e_3] &= e_7 & [e_2, e_4] &= e_8 \\
[e_2, e_5] &= \alpha_{2,5}^{9} e_9 & [e_2, e_6] &= \alpha_{2,6}^{10} e_{10} \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,5}^{9} \to x_1$$

$$\alpha_{3,4}^{9} \to x_2$$

$$\alpha_{2,6}^{10} \to x_3$$

$$\alpha_{3,5}^{10} \to x_4$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_2 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad x_1 - x_3 - x_4 &= 0 \\
(e_1, e_3, e_4): &\quad x_2 - x_4 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + x_4 - 1 = 0$$

$$x_2 - x_4 = 0$$

$$x_3 + 2x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(5,10)$

m1A510 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_2,e_5] &= e_{10} & [e_3,e_4] &= -e_{10}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(5,10)$

m3A510 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_2,e_3] &= e_8 & [e_2,e_4] &= e_9\\
[e_2,e_5] &= \alpha_{2,5}^{10}e_{10} & [e_3,e_4] &= \alpha_{3,4}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{10}-\alpha_{3,4}^{10}+1 = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^{10} &\to x_1\\
\alpha_{2,5}^{10} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1,e_2,e_4): \quad -x_1-x_2+1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(6, 10)$

m2A610 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_2, e_3] &= e_9 & [e_2, e_4] &= e_{10} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(7, 10)$

m1A710 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_2, e_3] &= e_{10} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2, 11)$

m1A211 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_2, e_9] &= e_{11} \\ [e_3, e_8] &= -e_{11} & [e_4, e_7] &= e_{11} \\ [e_5, e_6] &= -e_{11} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2,11)$

m3A211 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_7] &= e_9 \\
[e_2,e_8] &= 3e_{10} & [e_2,e_9] &= 0 \\
[e_3,e_6] &= -e_9 & [e_3,e_7] &= -2e_{10} \\
[e_3,e_8] &= 3e_{11} & [e_4,e_5] &= e_9 \\
[e_4,e_6] &= e_{10} & [e_4,e_7] &= -5e_{11} \\
[e_5,e_6] &= 6e_{11}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_7] &= e_9 \\
[e_2,e_8] &= 3e_{10} & [e_2,e_9] &= \alpha_{2,9}^{11}e_{11} \\
[e_3,e_6] &= -e_9 & [e_3,e_7] &= -2e_{10} \\
[e_3,e_8] &= \alpha_{3,8}^{11}e_{11} & [e_4,e_5] &= e_9 \\
[e_4,e_6] &= e_{10} & [e_4,e_7] &= \alpha_{4,7}^{11}e_{11} \\
[e_5,e_6] &= \alpha_{5,6}^{11}e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8):&\quad -\alpha_{2,9}^{11}-\alpha_{3,8}^{11}+3 &&= 0 \\
(e_1,e_3,e_7):&\quad -\alpha_{3,8}^{11}-\alpha_{4,7}^{11}-2 &&= 0 \\
(e_1,e_4,e_6):&\quad -\alpha_{4,7}^{11}-\alpha_{5,6}^{11}+1 &&= 0 \\
(e_2,e_3,e_6):&\quad -\alpha_{2,9}^{11} &&= 0 \\
(e_2,e_4,e_5):&\quad \alpha_{2,9}^{11} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{11}_{2,9} = 0$$
$$\alpha^{11}_{3,8} = 3$$
$$\alpha^{11}_{4,7} = -5$$
$$\alpha^{11}_{5,6} = 6$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{2,9} \to x_1$$
$$\alpha^{11}_{3,8} \to x_2$$
$$\alpha^{11}_{4,7} \to x_3$$
$$\alpha^{11}_{5,6} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8):&\quad -x_1 - x_2 + 3 &= 0\\
(e_1, e_3, e_7):&\quad -x_2 - x_3 - 2 &= 0\\
(e_1, e_4, e_6):&\quad -x_3 - x_4 + 1 &= 0\\
(e_2, e_3, e_6):&\quad -x_1 &= 0\\
(e_2, e_4, e_5):&\quad x_1 &= 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 = 0$$
$$x_2 - 3 = 0$$
$$x_3 + 5 = 0$$
$$x_4 - 6 = 0$$

Solution 1:

$$x_1 = 0$$
$$x_2 = 3$$
$$x_3 = -5$$
$$x_4 = 6$$

$\mathfrak{m}_{5A}(2,11)$

m5A211 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_2,e_5]=e_7$$
$$[e_2,e_6]=2e_8 \qquad [e_2,e_7]=0$$
$$[e_2,e_8]=-5e_{10} \qquad [e_2,e_9]=-\frac{5e_{11}}{2}$$
$$[e_3,e_4]=-e_7 \qquad [e_3,e_5]=-e_8$$
$$[e_3,e_6]=2e_9 \qquad [e_3,e_7]=5e_{10}$$
$$[e_3,e_8]=-\frac{5e_{11}}{2} \qquad [e_4,e_5]=-3e_9$$
$$[e_4,e_6]=-3e_{10} \qquad [e_4,e_7]=\frac{15e_{11}}{2}$$
$$[e_5,e_6]=-\frac{21e_{11}}{2}$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_2,e_5]=e_7$$
$$[e_2,e_6]=2e_8 \qquad [e_2,e_7]=\alpha_{2,7}^{9}e_9$$
$$[e_2,e_8]=\alpha_{2,8}^{10}e_{10} \qquad [e_2,e_9]=\alpha_{2,9}^{11}e_{11}$$
$$[e_3,e_4]=-e_7 \qquad [e_3,e_5]=-e_8$$
$$[e_3,e_6]=\alpha_{3,6}^{9}e_9 \qquad [e_3,e_7]=\alpha_{3,7}^{10}e_{10}$$
$$[e_3,e_8]=\alpha_{3,8}^{11}e_{11} \qquad [e_4,e_5]=\alpha_{4,5}^{9}e_9$$
$$[e_4,e_6]=\alpha_{4,6}^{10}e_{10} \qquad [e_4,e_7]=\alpha_{4,7}^{11}e_{11}$$
$$[e_5,e_6]=\alpha_{5,6}^{11}e_{11}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,7}^{9} &= 0\\
\alpha_{3,7}^{10} &= 5\\
\alpha_{3,8}^{11} &= -5/2\\
\alpha_{2,8}^{10} &= -5\\
\alpha_{3,6}^{9} &= 2\\
\alpha_{4,5}^{9} &= -3\\
\alpha_{4,7}^{11} &= 15/2\\
\alpha_{4,6}^{10} &= -3\\
\alpha_{5,6}^{11} &= -21/2\\
\alpha_{2,9}^{11} &= -5/2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,7}^{9} &\to x_1\\
\alpha_{3,7}^{10} &\to x_2\\
\alpha_{3,8}^{11} &\to x_3
\end{aligned}
$$

$$\alpha_{2,8}^{10} \to x_4$$

$$\alpha_{3,6}^{9} \to x_5$$

$$\alpha_{4,5}^{9} \to x_6$$

$$\alpha_{4,7}^{11} \to x_7$$

$$\alpha_{4,6}^{10} \to x_8$$

$$\alpha_{5,6}^{11} \to x_9$$

$$\alpha_{2,9}^{11} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_5 + 2 && = 0\\
(e_1, e_3, e_5): &\quad -x_5 - x_6 - 1 && = 0\\
(e_2, e_3, e_4): &\quad -x_1 && = 0\\
(e_1, e_2, e_7): &\quad x_1 - x_2 - x_4 && = 0\\
(e_1, e_3, e_6): &\quad -x_2 + x_5 - x_8 && = 0\\
(e_1, e_4, e_5): &\quad x_6 - x_8 && = 0\\
(e_2, e_3, e_5): &\quad -x_2 - x_4 && = 0\\
(e_1, e_2, e_8): &\quad -x_{10} - x_3 + x_4 && = 0\\
(e_1, e_3, e_7): &\quad x_2 - x_3 - x_7 && = 0\\
(e_1, e_4, e_6): &\quad -x_7 + x_8 - x_9 && = 0\\
(e_2, e_3, e_6): &\quad x_{10}x_5 - 2x_3 && = 0\\
(e_2, e_4, e_5): &\quad x_{10}x_6 - x_7 && = 0
\end{aligned}$$

Groebner basis (10 variables, 10 linear, 0 nonlinear)

$$x_1 = 0$$

$$x_2 - 5 = 0$$

$$x_3 + \frac{5}{2} = 0$$

$$x_4 + 5 = 0$$

$$x_5 - 2 = 0$$

$$x_6 + 3 = 0$$

$$x_7 - \frac{15}{2} = 0$$

$$x_8 + 3 = 0$$

$$x_9 + \frac{21}{2} = 0$$

$$x_{10} + \frac{5}{2} = 0$$

Solution 1:

$$x_1 = 0$$

$$x_2 = 5$$

$$x_3 = -5/2$$

$$x_4 = -5$$

$$x_5 = 2$$

$$x_6 = -3$$

$$x_7 = 15/2$$

$$x_8 = -3$$

$$x_9 = -21/2$$

$$x_1 0 = -5/2$$

$\mathfrak{m}_{7A}(2, 11)$

m7A211 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_5 \\
[e_2, e_4] &= e_6 & [e_2, e_5] &= \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] &= \alpha_{2,6}^{8} e_8 & [e_2, e_7] &= \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] &= \alpha_{2,8}^{10} e_{10} & [e_2, e_9] &= \alpha_{2,9}^{11} e_{11} \\
[e_3, e_4] &= \alpha_{3,4}^{7} e_7 & [e_3, e_5] &= \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] &= \alpha_{3,6}^{9} e_9 & [e_3, e_7] &= \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] &= \alpha_{4,6}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{11} e_{11} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10} + \alpha_{2,8}^{10}\alpha_{3,5}^{8} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11} + \alpha_{2,9}^{11}\alpha_{3,6}^{9} - \alpha_{5,6}^{11} &= 0\\
(e_2, e_4, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11} + \alpha_{2,9}^{11}\alpha_{4,5}^{9} + \alpha_{5,6}^{11} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{8} \to x_1$$

$$\alpha_{3,5}^{8} \to x_2$$

$$\alpha_{2,5}^{7} \to x_3$$

$$\alpha_{2,7}^{9} \to x_4$$

$$\alpha_{3,7}^{10} \to x_5$$

$$\alpha_{2,8}^{10} \to x_6$$

$$\alpha_{3,6}^{9} \to x_7$$

$$\alpha_{4,5}^{9} \to x_8$$

$$\alpha_{3,8}^{11} \to x_9$$

$$\alpha_{4,6}^{10} \to x_{10}$$

$$\alpha_{4,7}^{11} \to x_{11}$$

$$\alpha_{5,6}^{11} \to x_{12}$$
$$\alpha_{3,4}^{7} \to x_{13}$$
$$\alpha_{2,9}^{11} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{13} - x_3 + 1 && = 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_2 + x_3 && = 0\\
(e_1, e_3, e_4): &\quad x_{13} - x_2 && = 0\\
(e_1, e_2, e_6): &\quad x_1 - x_4 - x_7 && = 0\\
(e_1, e_3, e_5): &\quad x_2 - x_7 - x_8 && = 0\\
(e_2, e_3, e_4): &\quad x_{13}x_4 - x_7 + x_8 && = 0\\
(e_1, e_2, e_7): &\quad x_4 - x_5 - x_6 && = 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_5 + x_7 && = 0\\
(e_1, e_4, e_5): &\quad -x_{10} + x_8 && = 0\\
(e_2, e_3, e_5): &\quad x_2x_6 - x_3x_5 && = 0\\
(e_1, e_2, e_8): &\quad -x_{14} + x_6 - x_9 && = 0\\
(e_1, e_3, e_7): &\quad -x_{11} + x_5 - x_9 && = 0\\
(e_1, e_4, e_6): &\quad x_{10} - x_{11} - x_{12} && = 0\\
(e_2, e_3, e_6): &\quad -x_1x_9 - x_{12} + x_{14}x_7 && = 0\\
(e_2, e_4, e_5): &\quad -x_{11}x_3 + x_{12} + x_{14}x_8 && = 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 3 nonlinear)

$$x_1 + 2x_{13} - 1 = 0$$
$$-x_{13} + x_2 = 0$$
$$x_{13} + x_3 - 1 = 0$$
$$-\frac{x_{12}}{6} + \frac{13x_{13}}{6} - \frac{x_{14}}{6} + x_4 - \frac{5}{6} = 0$$
$$\frac{x_{12}}{3} + \frac{2x_{13}}{3} + \frac{x_{14}}{3} + x_5 - \frac{1}{3} = 0$$
$$-\frac{x_{12}}{2} + \frac{3x_{13}}{2} - \frac{x_{14}}{2} + x_6 - \frac{1}{2} = 0$$
$$\frac{x_{12}}{6} - \frac{x_{13}}{6} + \frac{x_{14}}{6} + x_7 - \frac{1}{6} = 0$$
$$-\frac{x_{12}}{6} - \frac{5x_{13}}{6} - \frac{x_{14}}{6} + x_8 + \frac{1}{6} = 0$$

$$-\frac{x_{12}}{2} + \frac{3x_{13}}{2} + \frac{x_{14}}{2} + x_9 - \frac{1}{2} = 0$$
$$x_{10} - \frac{x_{12}}{6} - \frac{5x_{13}}{6} - \frac{x_{14}}{6} + \frac{1}{6} = 0$$
$$x_{11} + \frac{5x_{12}}{6} - \frac{5x_{13}}{6} - \frac{x_{14}}{6} + \frac{1}{6} = 0$$
$$x_{12}x_{13} + 2x_{12} - 13x_{13}^2 + x_{13}x_{14} + 9x_{13} + 2x_{14} - 2 = 0$$
$$x_{12}x_{14} + 21x_{12} - 60x_{13}^2 + 11x_{13}x_{14} + 39x_{13} + x_{14}^2 + 8x_{14} - 9 = 0$$
$$x_{13}^3 - \frac{2x_{13}^2x_{14}}{5} - \frac{16x_{13}^2}{5} + 2x_{13} + \frac{2x_{14}}{5} - \frac{2}{5} = 0$$

$\mathfrak{m}_{2A}(3, 11)$

m2A311 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_{10} \\
[e_2, e_8] &= 3e_{11} & [e_3, e_6] &= -e_{10} \\
[e_3, e_7] &= -2e_{11} & [e_4, e_5] &= e_{10} \\
[e_4, e_6] &= e_{11} & &
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3, 11)$

m4A311 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_2,e_5]=e_8$$
$$[e_2,e_6]=2e_9 \qquad [e_2,e_7]=\frac{5e_{10}}{3}$$
$$[e_2,e_8]=0 \qquad [e_3,e_4]=-e_8$$
$$[e_3,e_5]=-e_9 \qquad [e_3,e_6]=\frac{e_{10}}{3}$$
$$[e_3,e_7]=\frac{5e_{11}}{3} \qquad [e_4,e_5]=-\frac{4e_{10}}{3}$$
$$[e_4,e_6]=-\frac{4e_{11}}{3}$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_2,e_5]=e_8$$
$$[e_2,e_6]=2e_9 \qquad [e_2,e_7]=\alpha_{2,7}^{10}e_{10}$$
$$[e_2,e_8]=\alpha_{2,8}^{11}e_{11} \qquad [e_3,e_4]=-e_8$$
$$[e_3,e_5]=-e_9 \qquad [e_3,e_6]=\alpha_{3,6}^{10}e_{10}$$
$$[e_3,e_7]=\alpha_{3,7}^{11}e_{11} \qquad [e_4,e_5]=\alpha_{4,5}^{10}e_{10}$$
$$[e_4,e_6]=\alpha_{4,6}^{11}e_{11}$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_6): \quad -\alpha_{2,7}^{10}-\alpha_{3,6}^{10}+2 \qquad =0$$
$$(e_1,e_3,e_5): \quad -\alpha_{3,6}^{10}-\alpha_{4,5}^{10}-1 \qquad =0$$
$$(e_1,e_2,e_7): \quad \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} \qquad =0$$
$$(e_1,e_3,e_6): \quad \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} \qquad =0$$
$$(e_1,e_4,e_5): \quad \alpha_{4,5}^{10}-\alpha_{4,6}^{11} \qquad =0$$
$$(e_2,e_3,e_4): \quad -\alpha_{2,8}^{11} \qquad =0$$

Solution 1:

$$\alpha^{10}_{2,7} = 5/3$$
$$\alpha^{11}_{3,7} = 5/3$$
$$\alpha^{10}_{3,6} = 1/3$$
$$\alpha^{11}_{2,8} = 0$$
$$\alpha^{10}_{4,5} = -4/3$$
$$\alpha^{11}_{4,6} = -4/3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{10}_{2,7} \to x_1$$
$$\alpha^{11}_{3,7} \to x_2$$
$$\alpha^{10}_{3,6} \to x_3$$
$$\alpha^{11}_{2,8} \to x_4$$
$$\alpha^{10}_{4,5} \to x_5$$
$$\alpha^{11}_{4,6} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6) &: \quad -x_1 - x_3 + 2 && = 0 \\
(e_1, e_3, e_5) &: \quad -x_3 - x_5 - 1 && = 0 \\
(e_1, e_2, e_7) &: \quad x_1 - x_2 - x_4 && = 0 \\
(e_1, e_3, e_6) &: \quad -x_2 + x_3 - x_6 && = 0 \\
(e_1, e_4, e_5) &: \quad x_5 - x_6 && = 0 \\
(e_2, e_3, e_4) &: \quad -x_4 && = 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 - \frac{5}{3} = 0$$
$$x_2 - \frac{5}{3} = 0$$
$$x_3 - \frac{1}{3} = 0$$
$$x_4 = 0$$

$$x_5 + \frac{4}{3} = 0$$

$$x_6 + \frac{4}{3} = 0$$

Solution 1:

$$x_1 = 5/3$$

$$x_2 = 5/3$$

$$x_3 = 1/3$$

$$x_4 = 0$$

$$x_5 = -4/3$$

$$x_6 = -4/3$$

$\mathfrak{m}_{6A}(3, 11)$

m6A311 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \alpha_{2,5}^{8} e_8$$
$$[e_2, e_6] = \alpha_{2,6}^{9} e_9 \qquad [e_2, e_7] = \alpha_{2,7}^{10} e_{10}$$
$$[e_2, e_8] = \alpha_{2,8}^{11} e_{11} \qquad [e_3, e_4] = \alpha_{3,4}^{8} e_8$$
$$[e_3, e_5] = \alpha_{3,5}^{9} e_9 \qquad [e_3, e_6] = \alpha_{3,6}^{10} e_{10}$$
$$[e_3, e_7] = \alpha_{3,7}^{11} e_{11} \qquad [e_4, e_5] = \alpha_{4,5}^{10} e_{10}$$
$$[e_4, e_6] = \alpha_{4,6}^{11} e_{11}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^{8} \to x_1$$

$$\alpha_{2,7}^{10} \to x_2$$

$$\alpha_{2,6}^{9} \to x_3$$

$$\alpha_{2,5}^{8} \to x_4$$

$$\alpha_{4,5}^{10} \to x_5$$

$$\alpha_{3,7}^{11} \to x_6$$

$$\alpha_{3,6}^{10} \to x_7$$

$$\alpha_{2,8}^{11} \to x_8$$

$$\alpha_{3,5}^{9} \to x_9$$

$$\alpha_{4,6}^{11} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_1 - x_4 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_3 + x_4 - x_9 &&= 0\\
(e_1, e_3, e_4): &\quad x_1 - x_9 &&= 0\\
(e_1, e_2, e_6): &\quad -x_2 + x_3 - x_7 &&= 0\\
(e_1, e_3, e_5): &\quad -x_5 - x_7 + x_9 &&= 0\\
(e_1, e_2, e_7): &\quad x_2 - x_6 - x_8 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_6 + x_7 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{10} + x_5 &&= 0\\
(e_2, e_3, e_4): &\quad x_1 x_8 + x_{10} - x_6 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 8 linear, 1 nonlinear)

$$x_1 - x_9 = 0$$
$$-x_{10} + x_2 + 3x_9 - 1 = 0$$
$$x_3 + 2x_9 - 1 = 0$$
$$x_4 + x_9 - 1 = 0$$
$$-x_{10} + x_5 = 0$$
$$2x_{10} + x_6 - x_9 = 0$$
$$x_{10} + x_7 - x_9 = 0$$
$$-3x_{10} + x_8 + 4x_9 - 1 = 0$$
$$-\frac{3x_{10}x_9}{4} - \frac{3x_{10}}{4} + x_9^2 = 0$$

$\mathfrak{m}_{1A}(4, 11)$

m1A411 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_{11}\\
[e_3, e_6] &= -e_{11} & [e_4, e_5] &= e_{11}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4, 11)$

m3A411 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_9 \\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{4,5}^{11} &\to x_1 \\
\alpha_{3,6}^{11} &\to x_2 \\
\alpha_{2,7}^{11} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_3 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_1 - x_2 - 1 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 - x_3 + 3 &= 0 \\
x_2 + x_3 - 2 &= 0
\end{aligned}
$$

$\mathfrak{m}_{5A}(4, 11)$

m5A411 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_7 \\
[e_2, e_4] &= e_8 & [e_2, e_5] &= \alpha_{2,5}^{9} e_9 \\
[e_2, e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,6}^{10} &\to x_1 \\
\alpha_{2,5}^{9} &\to x_2 \\
\alpha_{3,4}^{9} &\to x_3 \\
\alpha_{4,5}^{11} &\to x_4 \\
\alpha_{2,7}^{11} &\to x_5 \\
\alpha_{3,5}^{10} &\to x_6 \\
\alpha_{3,6}^{11} &\to x_7
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_3 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_1 + x_2 - x_6 &= 0\\
(e_1, e_3, e_4): &\quad x_3 - x_6 &= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_5 - x_7 &= 0\\
(e_1, e_3, e_5): &\quad -x_4 + x_6 - x_7 &= 0
\end{aligned}
$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 + 2x_6 - 1 = 0$$

$$x_2 + x_6 - 1 = 0$$

$$x_3 - x_6 = 0$$

$$x_4 - x_6 + x_7 = 0$$

$$x_5 + 2x_6 + x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(5, 11)$

m2A511 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{10}\\
[e_2, e_6] &= 2e_{11} & [e_3, e_4] &= -e_{10}\\
[e_3, e_5] &= -e_{11}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(5, 11)$

m4A511 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_8 \\
[e_2, e_4] &= e_9 & [e_2, e_5] &= \alpha_{2,5}^{10} e_{10} \\
[e_2, e_6] &= \alpha_{2,6}^{11} e_{11} & [e_3, e_4] &= \alpha_{3,4}^{10} e_{10} \\
[e_3, e_5] &= \alpha_{3,5}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,6}^{11} &\to x_1 \\
\alpha_{3,4}^{10} &\to x_2 \\
\alpha_{2,5}^{10} &\to x_3 \\
\alpha_{3,5}^{11} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_3 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad -x_1 + x_3 - x_4 &= 0 \\
(e_1, e_3, e_4): &\quad x_2 - x_4 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 + 2x_4 - 1 &= 0 \\
x_2 - x_4 &= 0 \\
x_3 + x_4 - 1 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(6, 11)$

m1A611 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{11} \\
[e_3, e_4] &= -e_{11}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(6, 11)$

m3A611 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_9 \\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= \alpha_{2,5}^{11} e_{11} \\
[e_3, e_4] &= \alpha_{3,4}^{11} e_{11}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 \qquad = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,5}^{11} &\to x_1 \\
\alpha_{3,4}^{11} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(7, 11)$

m2A711 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_{10}$$
$$[e_2, e_4] = e_{11}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(8, 11)$

m1A811 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_{11}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2, 12)$

m2A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_9] &= e_{11} & [e_2, e_{10}] &= 4e_{12} \\
[e_3, e_8] &= -e_{11} & [e_3, e_9] &= -3e_{12} \\
[e_4, e_7] &= e_{11} & [e_4, e_8] &= 2e_{12} \\
[e_5, e_6] &= -e_{11} & [e_5, e_7] &= -e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2, 12)$

m4A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_7] &= e_9 & [e_2, e_8] &= 3e_{10} \\
[e_2, e_9] &= \alpha_{2,9}^{11} e_{11} & [e_2, e_{10}] &= \alpha_{2,10}^{12} e_{12} \\
[e_3, e_6] &= -e_9 & [e_3, e_7] &= -2e_{10} \\
[e_3, e_8] &= \alpha_{3,8}^{11} e_{11} & [e_3, e_9] &= \alpha_{3,9}^{12} e_{12} \\
[e_4, e_5] &= e_9 & [e_4, e_6] &= e_{10} \\
[e_4, e_7] &= \alpha_{4,7}^{11} e_{11} & [e_4, e_8] &= \alpha_{4,8}^{12} e_{12} \\
[e_5, e_6] &= \alpha_{5,6}^{11} e_{11} & [e_5, e_7] &= \alpha_{5,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{11} - \alpha_{3,8}^{11} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{11} - \alpha_{4,7}^{11} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{11} - \alpha_{5,6}^{11} + 1 &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,9}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad -2\alpha_{2,10}^{12} - \alpha_{3,9}^{12} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,9}^{12} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,9}^{12} \to x_1$$
$$\alpha_{4,8}^{12} \to x_2$$
$$\alpha_{3,8}^{11} \to x_3$$
$$\alpha_{4,7}^{11} \to x_4$$
$$\alpha_{5,6}^{11} \to x_5$$
$$\alpha_{2,9}^{11} \to x_6$$
$$\alpha_{5,7}^{12} \to x_7$$
$$\alpha_{2,10}^{12} \to x_8$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - x_6 + 3 && = 0\\
(e_1, e_3, e_7): &\quad -x_3 - x_4 - 2 && = 0\\
(e_1, e_4, e_6): &\quad -x_4 - x_5 + 1 && = 0\\
(e_2, e_3, e_6): &\quad -x_6 && = 0\\
(e_2, e_4, e_5): &\quad x_6 && = 0\\
(e_1, e_2, e_9): &\quad -x_1 + x_6 - x_8 && = 0\\
(e_1, e_3, e_8): &\quad -x_1 - x_2 + x_3 && = 0\\
(e_1, e_4, e_7): &\quad -x_2 + x_4 - x_7 && = 0\\
(e_1, e_5, e_6): &\quad x_5 - x_7 && = 0\\
(e_2, e_3, e_7): &\quad -x_1 - 2x_8 && = 0\\
(e_2, e_4, e_6): &\quad x_8 && = 0\\
(e_3, e_4, e_5): &\quad x_1 && = 0
\end{aligned}
$$

Groebner basis (8 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6A}(2, 12)$

m6A212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_2, e_5] = e_7 && [e_2, e_6] = 2e_8 \\
&[e_2, e_7] = \alpha_{2,7}^{9} e_9 && [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
&[e_2, e_9] = \alpha_{2,9}^{11} e_{11} && [e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} \\
&[e_3, e_4] = -e_7 && [e_3, e_5] = -e_8 \\
&[e_3, e_6] = \alpha_{3,6}^{9} e_9 && [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
&[e_3, e_8] = \alpha_{3,8}^{11} e_{11} && [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
&[e_4, e_5] = \alpha_{4,5}^{9} e_9 && [e_4, e_6] = \alpha_{4,6}^{10} e_{10} \\
&[e_4, e_7] = \alpha_{4,7}^{11} e_{11} && [e_4, e_8] = \alpha_{4,8}^{12} e_{12} \\
&[e_5, e_6] = \alpha_{5,6}^{11} e_{11} && [e_5, e_7] = \alpha_{5,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10} - \alpha_{2,7}^{9}\alpha_{3,9}^{12} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10} - 2\alpha_{4,8}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,9}^{12}\alpha_{4,5}^{9} + \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,9}^{12} \to x_1$$

$$\alpha_{4,8}^{12} \to x_2$$

$$\alpha_{2,7}^{9} \to x_3$$

$$\alpha_{3,7}^{10} \to x_4$$

$$\alpha_{3,8}^{11} \to x_5$$

$$\alpha_{2,8}^{10} \to x_6$$

$$\alpha_{3,6}^{9} \to x_7$$

$$\alpha_{4,5}^{9} \to x_8$$

$$\alpha_{4,7}^{11} \to x_9$$

$$\alpha_{4,6}^{10} \to x_{10}$$

$$\alpha_{5,6}^{11} \to x_{11}$$

$$\alpha_{2,9}^{11} \to x_{12}$$

$$\alpha_{5,7}^{12} \to x_{13}$$

$$\alpha_{2,10}^{12} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_6): &\quad -x_3 - x_7 + 2 &&= 0\\
(e_1,e_3,e_5): &\quad -x_7 - x_8 - 1 &&= 0\\
(e_2,e_3,e_4): &\quad -x_3 &&= 0\\
(e_1,e_2,e_7): &\quad x_3 - x_4 - x_6 &&= 0\\
(e_1,e_3,e_6): &\quad -x_{10} - x_4 + x_7 &&= 0\\
(e_1,e_4,e_5): &\quad -x_{10} + x_8 &&= 0\\
(e_2,e_3,e_5): &\quad -x_4 - x_6 &&= 0\\
(e_1,e_2,e_8): &\quad -x_{12} - x_5 + x_6 &&= 0\\
(e_1,e_3,e_7): &\quad x_4 - x_5 - x_9 &&= 0\\
(e_1,e_4,e_6): &\quad x_{10} - x_{11} - x_9 &&= 0\\
(e_2,e_3,e_6): &\quad x_{12}x_7 - 2x_5 &&= 0\\
(e_2,e_4,e_5): &\quad x_{12}x_8 - x_9 &&= 0\\
(e_1,e_2,e_9): &\quad -x_1 + x_{12} - x_{14} &&= 0\\
(e_1,e_3,e_8): &\quad -x_1 - x_2 + x_5 &&= 0\\
(e_1,e_4,e_7): &\quad -x_{13} - x_2 + x_9 &&= 0\\
(e_1,e_5,e_6): &\quad x_{11} - x_{13} &&= 0\\
(e_2,e_3,e_7): &\quad -x_1x_3 + x_{14}x_4 &&= 0\\
(e_2,e_4,e_6): &\quad x_{10}x_{14} - 2x_2 &&= 0\\
(e_3,e_4,e_5): &\quad x_1x_8 - x_{13} + x_2 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{8A}(2, 12)$

m8A212 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_3] = e_5 \qquad [e_2, e_4] = e_6$$
$$[e_2, e_5] = \frac{9e_7}{10} \qquad [e_2, e_6] = \frac{4e_8}{5}$$
$$[e_2, e_7] = \frac{5e_9}{7} \qquad [e_2, e_8] = \frac{9e_{10}}{14}$$
$$[e_2, e_9] = \frac{7e_{11}}{12} \qquad [e_2, e_{10}] = \frac{8e_{12}}{15}$$
$$[e_3, e_4] = \frac{e_7}{10} \qquad [e_3, e_5] = \frac{e_8}{10}$$
$$[e_3, e_6] = \frac{3e_9}{35} \qquad [e_3, e_7] = \frac{e_{10}}{14}$$
$$[e_3, e_8] = \frac{5e_{11}}{84} \qquad [e_3, e_9] = \frac{e_{12}}{20}$$
$$[e_4, e_5] = \frac{e_9}{70} \qquad [e_4, e_6] = \frac{e_{10}}{70}$$
$$[e_4, e_7] = \frac{e_{11}}{84} \qquad [e_4, e_8] = \frac{e_{12}}{105}$$
$$[e_5, e_6] = \frac{e_{11}}{420} \qquad [e_5, e_7] = \frac{e_{12}}{420}$$

Solution 2

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_2, e_3] = e_5 & [e_2, e_4] = e_6 \\
[e_2, e_5] = e_7 & [e_2, e_6] = e_8 \\
[e_2, e_7] = e_9 & [e_2, e_8] = e_{10} \\
[e_2, e_9] = e_{11} & [e_2, e_{10}] = e_{12} \\
[e_3, e_4] = 0 & [e_3, e_5] = 0 \\
[e_3, e_6] = 0 & [e_3, e_7] = 0 \\
[e_3, e_8] = 0 & [e_3, e_9] = 0 \\
[e_4, e_5] = 0 & [e_4, e_6] = 0 \\
[e_4, e_7] = 0 & [e_4, e_8] = 0 \\
[e_5, e_6] = 0 & [e_5, e_7] = 0
\end{array}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_2, e_3] = e_5 & [e_2, e_4] = e_6 \\
[e_2, e_5] = \alpha_{2,5}^{7} e_7 & [e_2, e_6] = \alpha_{2,6}^{8} e_8 \\
[e_2, e_7] = \alpha_{2,7}^{9} e_9 & [e_2, e_8] = \alpha_{2,8}^{10} e_{10} \\
[e_2, e_9] = \alpha_{2,9}^{11} e_{11} & [e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] = \alpha_{3,8}^{11} e_{11} & [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
[e_4, e_5] = \alpha_{4,5}^{9} e_9 & [e_4, e_6] = \alpha_{4,6}^{10} e_{10} \\
[e_4, e_7] = \alpha_{4,7}^{11} e_{11} & [e_4, e_8] = \alpha_{4,8}^{12} e_{12} \\
[e_5, e_6] = \alpha_{5,6}^{11} e_{11} & [e_5, e_7] = \alpha_{5,7}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10} + \alpha_{2,8}^{10}\alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11} + \alpha_{2,9}^{11}\alpha_{3,6}^{9} - \alpha_{5,6}^{11} &&= 0\\
(e_2, e_4, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11} + \alpha_{2,9}^{11}\alpha_{4,5}^{9} + \alpha_{5,6}^{11} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{12} + \alpha_{2,9}^{11} - \alpha_{3,9}^{12} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{11} - \alpha_{3,9}^{12} - \alpha_{4,8}^{12} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{11} - \alpha_{4,8}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{11} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10} - \alpha_{2,7}^{9}\alpha_{3,9}^{12} - \alpha_{5,7}^{12} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10} - \alpha_{2,6}^{8}\alpha_{4,8}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,4}^{7}\alpha_{5,7}^{12} - \alpha_{3,5}^{8}\alpha_{4,8}^{12} + \alpha_{3,9}^{12}\alpha_{4,5}^{9} &&= 0
\end{aligned}
$$

Solution 1:

$$\begin{aligned}
\alpha^{8}_{2,6} &= 4/5 \\
\alpha^{12}_{3,9} &= 1/20 \\
\alpha^{8}_{3,5} &= 1/10 \\
\alpha^{12}_{4,8} &= 1/105 \\
\alpha^{7}_{2,5} &= 9/10 \\
\alpha^{9}_{2,7} &= 5/7 \\
\alpha^{10}_{3,7} &= 1/14 \\
\alpha^{10}_{2,8} &= 9/14 \\
\alpha^{9}_{3,6} &= 3/35 \\
\alpha^{9}_{4,5} &= 1/70 \\
\alpha^{11}_{3,8} &= 5/84 \\
\alpha^{10}_{4,6} &= 1/70 \\
\alpha^{11}_{4,7} &= 1/84 \\
\alpha^{11}_{5,6} &= 1/420 \\
\alpha^{7}_{3,4} &= 1/10 \\
\alpha^{11}_{2,9} &= 7/12 \\
\alpha^{12}_{5,7} &= 1/420 \\
\alpha^{12}_{2,10} &= 8/15
\end{aligned}$$

Solution 2:

$$\alpha_{2,6}^{8} = 1$$
$$\alpha_{3,9}^{12} = 0$$
$$\alpha_{3,5}^{8} = 0$$
$$\alpha_{4,8}^{12} = 0$$
$$\alpha_{2,5}^{7} = 1$$
$$\alpha_{2,7}^{9} = 1$$
$$\alpha_{3,7}^{10} = 0$$
$$\alpha_{2,8}^{10} = 1$$
$$\alpha_{3,6}^{9} = 0$$
$$\alpha_{4,5}^{9} = 0$$
$$\alpha_{3,8}^{11} = 0$$
$$\alpha_{4,6}^{10} = 0$$
$$\alpha_{4,7}^{11} = 0$$
$$\alpha_{5,6}^{11} = 0$$
$$\alpha_{3,4}^{7} = 0$$
$$\alpha_{2,9}^{11} = 1$$
$$\alpha_{5,7}^{12} = 0$$
$$\alpha_{2,10}^{12} = 1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{8} \to x_1$$
$$\alpha_{3,9}^{12} \to x_2$$
$$\alpha_{3,5}^{8} \to x_3$$
$$\alpha_{4,8}^{12} \to x_4$$
$$\alpha_{2,5}^{7} \to x_5$$
$$\alpha_{2,7}^{9} \to x_6$$
$$\alpha_{3,7}^{10} \to x_7$$
$$\alpha_{2,8}^{10} \to x_8$$
$$\alpha_{3,6}^{9} \to x_9$$

$$\alpha^{9}_{4,5} \to x_{10}$$
$$\alpha^{11}_{3,8} \to x_{11}$$
$$\alpha^{10}_{4,6} \to x_{12}$$
$$\alpha^{11}_{4,7} \to x_{13}$$
$$\alpha^{11}_{5,6} \to x_{14}$$
$$\alpha^{7}_{3,4} \to x_{15}$$
$$\alpha^{11}_{2,9} \to x_{16}$$
$$\alpha^{12}_{5,7} \to x_{17}$$
$$\alpha^{12}_{2,10} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{15}-x_5+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_1-x_3+x_5 &&=0\\
(e_1,e_3,e_4):&\quad x_{15}-x_3 &&=0\\
(e_1,e_2,e_6):&\quad x_1-x_6-x_9 &&=0\\
(e_1,e_3,e_5):&\quad -x_{10}+x_3-x_9 &&=0\\
(e_2,e_3,e_4):&\quad x_{10}+x_{15}x_6-x_9 &&=0\\
(e_1,e_2,e_7):&\quad x_6-x_7-x_8 &&=0\\
(e_1,e_3,e_6):&\quad -x_{12}-x_7+x_9 &&=0\\
(e_1,e_4,e_5):&\quad x_{10}-x_{12} &&=0\\
(e_2,e_3,e_5):&\quad x_3x_8-x_5x_7 &&=0\\
(e_1,e_2,e_8):&\quad -x_{11}-x_{16}+x_8 &&=0\\
(e_1,e_3,e_7):&\quad -x_{11}-x_{13}+x_7 &&=0\\
(e_1,e_4,e_6):&\quad x_{12}-x_{13}-x_{14} &&=0\\
(e_2,e_3,e_6):&\quad -x_1x_{11}-x_{14}+x_{16}x_9 &&=0\\
(e_2,e_4,e_5):&\quad x_{10}x_{16}-x_{13}x_5+x_{14} &&=0\\
(e_1,e_2,e_9):&\quad x_{16}-x_{18}-x_2 &&=0\\
(e_1,e_3,e_8):&\quad x_{11}-x_2-x_4 &&=0\\
(e_1,e_4,e_7):&\quad x_{13}-x_{17}-x_4 &&=0\\
(e_1,e_5,e_6):&\quad x_{14}-x_{17} &&=0\\
(e_2,e_3,e_7):&\quad -x_{17}+x_{18}x_7-x_2x_6 &&=0\\
(e_2,e_4,e_6):&\quad -x_1x_4+x_{12}x_{18} &&=0\\
(e_3,e_4,e_5):&\quad x_{10}x_2+x_{15}x_{17}-x_3x_4 &&=0
\end{aligned}$$

Groebner basis (18 variables, 0 linear, 18 nonlinear)

$$x_1 - \frac{3114395x_{18}^5}{29042496} + \frac{6555889x_{18}^4}{10890936} - \frac{61909453x_{18}^3}{43563744} + \frac{751421x_{18}^2}{403368} - \frac{145272173x_{18}}{87127488} - \frac{2930909}{10890936} = 0$$

$$-\frac{2915995x_{18}^5}{58084992} + \frac{3049927x_{18}^4}{10890936} - \frac{57387209x_{18}^3}{87127488} + \frac{120333x_{18}^2}{134456} - \frac{89992789x_{18}}{174254976} + x_2 + \frac{1095599}{21781872} = 0$$

$$\frac{3114395x_{18}^5}{58084992} - \frac{6555889x_{18}^4}{21781872} + \frac{61909453x_{18}^3}{87127488} - \frac{751421x_{18}^2}{806736} + \frac{145272173x_{18}}{174254976} + x_3 - \frac{7960027}{21781872} = 0$$

$$\frac{509065x_{18}^5}{135531648} - \frac{969203x_{18}^4}{50824368} + \frac{8577167x_{18}^3}{203297472} - \frac{516037x_{18}^2}{5647152} + \frac{46224847x_{18}}{406594944} + x_4 - \frac{2499761}{50824368} = 0$$

$$-\frac{3114395x_{18}^5}{58084992} + \frac{6555889x_{18}^4}{21781872} - \frac{61909453x_{18}^3}{87127488} + \frac{751421x_{18}^2}{806736} - \frac{145272173x_{18}}{174254976} + x_5 - \frac{13821845}{21781872} = 0$$

$$-\frac{1029865x_{18}^5}{8470728} + \frac{17330359x_{18}^4}{25412184} - \frac{40896115x_{18}^3}{25412184} + \frac{5981945x_{18}^2}{2823576} - \frac{25783109x_{18}}{12706092} + x_6 - \frac{128515}{3176523} = 0$$

$$-\frac{10136785x_{18}^5}{406594944} + \frac{21491807x_{18}^4}{152473104} - \frac{203817335x_{18}^3}{609892416} + \frac{2371955x_{18}^2}{5647152} - \frac{134169127x_{18}}{1219784832} + x_7 - \frac{14007823}{152473104} = 0$$

$$-\frac{39296735x_{18}^5}{406594944} + \frac{82490347x_{18}^4}{152473104} - \frac{777689425x_{18}^3}{609892416} + \frac{9591935x_{18}^2}{5647152} - \frac{2341009337x_{18}}{1219784832} + x_8 + \frac{7839103}{152473104} = 0$$

$$\frac{2915995x_{18}^5}{203297472} - \frac{3049927x_{18}^4}{38118276} + \frac{57387209x_{18}^3}{304946208} - \frac{120333x_{18}^2}{470596} + \frac{220684021x_{18}}{609892416} + x_9 - \frac{17432003}{76236552} = 0$$

$$x_{10} + \frac{5322925x_{18}^5}{135531648} - \frac{11230505x_{18}^4}{50824368} + \frac{106197251x_{18}^3}{203297472} - \frac{3815951x_{18}^2}{5647152} + \frac{191845723x_{18}}{406594944} - \frac{6952061}{50824368} = 0$$

$$x_{11} - \frac{9442385x_{18}^5}{203297472} + \frac{39791369x_{18}^4}{152473104} - \frac{187989481x_{18}^3}{304946208} + \frac{4537949x_{18}^2}{5647152} - \frac{245637491x_{18}}{609892416} + \frac{84955}{76236552} = 0$$

$$x_{12} + \frac{5322925x_{18}^5}{135531648} - \frac{11230505x_{18}^4}{50824368} + \frac{106197251x_{18}^3}{203297472} - \frac{3815951x_{18}^2}{5647152} + \frac{191845723x_{18}}{406594944} - \frac{6952061}{50824368} = 0$$

$$x_{13} + \frac{2915995x_{18}^5}{135531648} - \frac{3049927x_{18}^4}{25412184} + \frac{57387209x_{18}^3}{203297472} - \frac{360999x_{18}^2}{941192} + \frac{119035285x_{18}}{406594944} - \frac{4725911}{50824368} = 0$$

$$x_{14} + \frac{401155x_{18}^5}{22588608} - \frac{1710217x_{18}^4}{16941456} + \frac{2711669x_{18}^3}{11294304} - \frac{1649957x_{18}^2}{5647152} + \frac{12135073x_{18}}{67765824} - \frac{41225}{941192} = 0$$

$$x_{15} + \frac{3114395x_{18}^5}{58084992} - \frac{6555889x_{18}^4}{21781872} + \frac{61909453x_{18}^3}{87127488} - \frac{751421x_{18}^2}{806736} + \frac{145272173x_{18}}{174254976} - \frac{7960027}{21781872} = 0$$

$$x_{16} - \frac{2915995x_{18}^5}{58084992} + \frac{3049927x_{18}^4}{10890936} - \frac{57387209x_{18}^3}{87127488} + \frac{120333x_{18}^2}{134456} - \frac{264247765x_{18}}{174254976} + \frac{1095599}{21781872} = 0$$

$$x_{17} + \frac{401155x_{18}^5}{22588608} - \frac{1710217x_{18}^4}{16941456} + \frac{2711669x_{18}^3}{11294304} - \frac{1649957x_{18}^2}{5647152} + \frac{12135073x_{18}}{67765824} - \frac{41225}{941192} = 0$$

$$x_{18}^6 - \frac{83x_{18}^5}{15} + \frac{38x_{18}^4}{3} - \frac{46x_{18}^3}{3} + \frac{31x_{18}^2}{3} - \frac{11x_{18}}{3} + \frac{8}{15} = 0$$

Solution 1:

$$x_1 = 4/5$$

$$x_2 = 1/20$$
$$x_3 = 1/10$$
$$x_4 = 1/105$$
$$x_5 = 9/10$$
$$x_6 = 5/7$$
$$x_7 = 1/14$$
$$x_8 = 9/14$$
$$x_9 = 3/35$$
$$x_10 = 1/70$$
$$x_11 = 5/84$$
$$x_12 = 1/70$$
$$x_13 = 1/84$$
$$x_14 = 1/420$$
$$x_15 = 1/10$$
$$x_16 = 7/12$$
$$x_17 = 1/420$$
$$x_18 = 8/15$$

Solution 2:

$$x_1 = 1$$
$$x_2 = 0$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 1$$
$$x_6 = 1$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = 0$$
$$x_10 = 0$$
$$x_11 = 0$$
$$x_12 = 0$$
$$x_13 = 0$$
$$x_14 = 0$$
$$x_15 = 0$$
$$x_16 = 1$$
$$x_17 = 0$$
$$x_18 = 1$$

$\mathfrak{m}_{1A}(3, 12)$

m1A312 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_9] &= e_{12} & [e_3, e_8] &= -e_{12} \\
[e_4, e_7] &= e_{12} & [e_5, e_6] &= -e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3, 12)$

m3A312 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_7] &= e_{10} & [e_2, e_8] &= 3e_{11} \\
[e_2, e_9] &= \alpha_{2,9}^{12} e_{12} & [e_3, e_6] &= -e_{10} \\
[e_3, e_7] &= -2e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12} \\
[e_4, e_5] &= e_{10} & [e_4, e_6] &= e_{11} \\
[e_4, e_7] &= \alpha_{4,7}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &&= 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &&= 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:

Change variables

$$\alpha_{3,8}^{12} \to x_1$$
$$\alpha_{2,9}^{12} \to x_2$$
$$\alpha_{4,7}^{12} \to x_3$$
$$\alpha_{5,6}^{12} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): \quad & -x_1 - x_2 + 3 & = 0 \\
(e_1, e_3, e_7): \quad & -x_1 - x_3 - 2 & = 0 \\
(e_1, e_4, e_6): \quad & -x_3 - x_4 + 1 & = 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 + 3 = 0$$
$$x_2 + x_4 - 6 = 0$$
$$x_3 + x_4 - 1 = 0$$

$\mathfrak{m}_{5A}(3, 12)$

m5A312 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_5] &= e_8 & [e_2, e_6] &= 2e_9 \\
[e_2, e_7] &= \alpha_{2,7}^{10} e_{10} & [e_2, e_8] &= \alpha_{2,8}^{11} e_{11} \\
[e_2, e_9] &= \alpha_{2,9}^{12} e_{12} & [e_3, e_4] &= -e_8 \\
[e_3, e_5] &= -e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{10} e_{10} & [e_4, e_6] &= \alpha_{4,6}^{11} e_{11} \\
[e_4, e_7] &= \alpha_{4,7}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{gathered}
\alpha_{2,9}^{12} \to x_1\\
\alpha_{2,7}^{10} \to x_2\\
\alpha_{3,8}^{12} \to x_3\\
\alpha_{3,7}^{11} \to x_4\\
\alpha_{3,6}^{10} \to x_5\\
\alpha_{4,7}^{12} \to x_6\\
\alpha_{2,8}^{11} \to x_7\\
\alpha_{5,6}^{12} \to x_8\\
\alpha_{4,5}^{10} \to x_9\\
\alpha_{4,6}^{11} \to x_{10}
\end{gathered}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_5 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_5 - x_9 - 1 &= 0\\
(e_1, e_2, e_7): &\quad x_2 - x_4 - x_7 &= 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_4 + x_5 &= 0\\
(e_1, e_4, e_5): &\quad -x_{10} + x_9 &= 0\\
(e_2, e_3, e_4): &\quad -x_7 &= 0\\
(e_1, e_2, e_8): &\quad -x_1 - x_3 + x_7 &= 0\\
(e_1, e_3, e_7): &\quad -x_3 + x_4 - x_6 &= 0\\
(e_1, e_4, e_6): &\quad x_{10} - x_6 - x_8 &= 0\\
(e_2, e_3, e_5): &\quad -x_1 - x_3 &= 0
\end{aligned}$$

Groebner basis (10 variables, 9 linear, 0 nonlinear)

$$x_1 + x_8 + 3 = 0$$
$$x_2 - \frac{5}{3} = 0$$
$$x_3 - x_8 - 3 = 0$$
$$x_4 - \frac{5}{3} = 0$$
$$x_5 - \frac{1}{3} = 0$$
$$x_6 + x_8 + \frac{4}{3} = 0$$
$$x_7 = 0$$
$$x_9 + \frac{4}{3} = 0$$
$$x_{10} + \frac{4}{3} = 0$$

$\mathfrak{m}_{7A}(3, 12)$

m7A312 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_6 & [e_2, e_4] &= e_7 \\
[e_2, e_5] &= \alpha_{2,5}^{8} e_8 & [e_2, e_6] &= \alpha_{2,6}^{9} e_9 \\
[e_2, e_7] &= \alpha_{2,7}^{10} e_{10} & [e_2, e_8] &= \alpha_{2,8}^{11} e_{11} \\
[e_2, e_9] &= \alpha_{2,9}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] &= \alpha_{3,5}^{9} e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{10} e_{10} & [e_4, e_6] &= \alpha_{4,6}^{11} e_{11} \\
[e_4, e_7] &= \alpha_{4,7}^{12} e_{12} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0 \\
(e_2, e_3, e_4): &\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &= 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &= 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &= 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{12} + \alpha_{2,9}^{12}\alpha_{3,5}^{9} + \alpha_{5,6}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{12} \to x_1$$

$$\alpha_{3,4}^{8} \to x_2$$
$$\alpha_{2,7}^{10} \to x_3$$
$$\alpha_{2,6}^{9} \to x_4$$
$$\alpha_{3,8}^{12} \to x_5$$
$$\alpha_{2,5}^{8} \to x_6$$
$$\alpha_{4,5}^{10} \to x_7$$
$$\alpha_{3,7}^{11} \to x_8$$
$$\alpha_{3,6}^{10} \to x_9$$
$$\alpha_{4,7}^{12} \to x_{10}$$
$$\alpha_{2,8}^{11} \to x_{11}$$
$$\alpha_{5,6}^{12} \to x_{12}$$
$$\alpha_{3,5}^{9} \to x_{13}$$
$$\alpha_{4,6}^{11} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4): &\quad -x_2 - x_6 + 1 &&= 0\\
(e_1,e_2,e_5): &\quad -x_{13} - x_4 + x_6 &&= 0\\
(e_1,e_3,e_4): &\quad -x_{13} + x_2 &&= 0\\
(e_1,e_2,e_6): &\quad -x_3 + x_4 - x_9 &&= 0\\
(e_1,e_3,e_5): &\quad x_{13} - x_7 - x_9 &&= 0\\
(e_1,e_2,e_7): &\quad -x_{11} + x_3 - x_8 &&= 0\\
(e_1,e_3,e_6): &\quad -x_{14} - x_8 + x_9 &&= 0\\
(e_1,e_4,e_5): &\quad -x_{14} + x_7 &&= 0\\
(e_2,e_3,e_4): &\quad x_{11}x_2 + x_{14} - x_8 &&= 0\\
(e_1,e_2,e_8): &\quad -x_1 + x_{11} - x_5 &&= 0\\
(e_1,e_3,e_7): &\quad -x_{10} - x_5 + x_8 &&= 0\\
(e_1,e_4,e_6): &\quad -x_{10} - x_{12} + x_{14} &&= 0\\
(e_2,e_3,e_5): &\quad x_1x_{13} + x_{12} - x_5x_6 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 1 nonlinear)

$$x_1 + x_{12} + 5x_{13} - 6x_{14} - 1 = 0$$
$$-x_{13} + x_2 = 0$$

$$3x_{13} - x_{14} + x_3 - 1 = 0$$
$$2x_{13} + x_4 - 1 = 0$$
$$-x_{12} - x_{13} + 3x_{14} + x_5 = 0$$
$$x_{13} + x_6 - 1 = 0$$
$$-x_{14} + x_7 = 0$$
$$-x_{13} + 2x_{14} + x_8 = 0$$
$$-x_{13} + x_{14} + x_9 = 0$$
$$x_{10} + x_{12} - x_{14} = 0$$
$$x_{11} + 4x_{13} - 3x_{14} - 1 = 0$$
$$x_{13}^2 - \frac{3x_{13}x_{14}}{4} - \frac{3x_{14}}{4} = 0$$

$\mathfrak{m}_{2A}(4,12)$

m2A412 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\ [e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\ [e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\ [e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\ [e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\ [e_2,e_7] &= e_{11} & [e_2,e_8] &= 3e_{12} \\ [e_3,e_6] &= -e_{11} & [e_3,e_7] &= -2e_{12} \\ [e_4,e_5] &= e_{11} & [e_4,e_6] &= e_{12} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(4,12)$

m4A412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_5] &= e_9 & [e_2, e_6] &= 2e_{10} \\
[e_2, e_7] &= \alpha_{2,7}^{11} e_{11} & [e_2, e_8] &= \alpha_{2,8}^{12} e_{12} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{11} e_{11} & [e_4, e_6] &= \alpha_{4,6}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,5}^{11} &\to x_1 \\
\alpha_{2,7}^{11} &\to x_2 \\
\alpha_{2,8}^{12} &\to x_3 \\
\alpha_{3,7}^{12} &\to x_4 \\
\alpha_{3,6}^{11} &\to x_5 \\
\alpha_{4,6}^{12} &\to x_6
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_5 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_1 - x_5 - 1 &= 0\\
(e_1, e_2, e_7): &\quad x_2 - x_3 - x_4 &= 0\\
(e_1, e_3, e_6): &\quad -x_4 + x_5 - x_6 &= 0\\
(e_1, e_4, e_5): &\quad x_1 - x_6 &= 0
\end{aligned}
$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$x_1 - x_6 = 0$$

$$x_2 - x_6 - 3 = 0$$

$$x_3 - 3x_6 - 4 = 0$$

$$x_4 + 2x_6 + 1 = 0$$

$$x_5 + x_6 + 1 = 0$$

$\mathfrak{m}_{6A}(4, 12)$

m6A412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_2, e_3] &= e_7 & [e_2, e_4] &= e_8\\
[e_2, e_5] &= \alpha_{2,5}^{9} e_9 & [e_2, e_6] &= \alpha_{2,6}^{10} e_{10}\\
[e_2, e_7] &= \alpha_{2,7}^{11} e_{11} & [e_2, e_8] &= \alpha_{2,8}^{12} e_{12}\\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10}\\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12}\\
[e_4, e_5] &= \alpha_{4,5}^{11} e_{11} & [e_4, e_6] &= \alpha_{4,6}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0\\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0\\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0\\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha_{2,6}^{10} &\to x_1\\
\alpha_{2,5}^{9} &\to x_2\\
\alpha_{3,4}^{9} &\to x_3\\
\alpha_{4,5}^{11} &\to x_4\\
\alpha_{2,7}^{11} &\to x_5\\
\alpha_{2,8}^{12} &\to x_6\\
\alpha_{3,7}^{12} &\to x_7\\
\alpha_{3,5}^{10} &\to x_8\\
\alpha_{3,6}^{11} &\to x_9\\
\alpha_{4,6}^{12} &\to x_{10}
\end{aligned}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_2 - x_3 + 1 &= 0\\
(e_1, e_2, e_5):&\quad -x_1 + x_2 - x_8 &= 0\\
(e_1, e_3, e_4):&\quad x_3 - x_8 &= 0\\
(e_1, e_2, e_6):&\quad x_1 - x_5 - x_9 &= 0\\
(e_1, e_3, e_5):&\quad -x_4 + x_8 - x_9 &= 0\\
(e_1, e_2, e_7):&\quad x_5 - x_6 - x_7 &= 0\\
(e_1, e_3, e_6):&\quad -x_{10} - x_7 + x_9 &= 0\\
(e_1, e_4, e_5):&\quad -x_{10} + x_4 &= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + 2x_{10} + 2x_9 - 1 = 0$$
$$x_{10} + x_2 + x_9 - 1 = 0$$
$$-x_{10} + x_3 - x_9 = 0$$
$$-x_{10} + x_4 = 0$$
$$2x_{10} + x_5 + 3x_9 - 1 = 0$$
$$x_{10} + x_6 + 4x_9 - 1 = 0$$
$$x_{10} + x_7 - x_9 = 0$$
$$-x_{10} + x_8 - x_9 = 0$$

$\mathfrak{m}_{1A}(5, 12)$

m1A512 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_7] = e_{12} \qquad [e_3, e_6] = -e_{12}$$
$$[e_4, e_5] = e_{12}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(5, 12)$

m3A512 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_2, e_5] = e_{10} \qquad [e_2, e_6] = 2e_{11}$$
$$[e_2, e_7] = \alpha_{2,7}^{12} e_{12} \qquad [e_3, e_4] = -e_{10}$$
$$[e_3, e_5] = -e_{11} \qquad [e_3, e_6] = \alpha_{3,6}^{12} e_{12}$$
$$[e_4, e_5] = \alpha_{4,5}^{12} e_{12}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): \quad & -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 && = 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{12} \to x_1$$

$$\alpha_{4,5}^{12} \to x_2$$

$$\alpha_{3,6}^{12} \to x_3$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): \quad & -x_1 - x_3 + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -x_2 - x_3 - 1 && = 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 - 2 = 0$$

$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(5, 12)$

m5A512 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_8 & [e_2, e_4] &= e_9 \\
[e_2, e_5] &= \alpha_{2,5}^{10} e_{10} & [e_2, e_6] &= \alpha_{2,6}^{11} e_{11} \\
[e_2, e_7] &= \alpha_{2,7}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{10} e_{10} \\
[e_3, e_5] &= \alpha_{3,5}^{11} e_{11} & [e_3, e_6] &= \alpha_{3,6}^{12} e_{12} \\
[e_4, e_5] &= \alpha_{4,5}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{12} \to x_1$$
$$\alpha_{3,5}^{11} \to x_2$$
$$\alpha_{2,5}^{10} \to x_3$$
$$\alpha_{3,6}^{12} \to x_4$$
$$\alpha_{2,7}^{12} \to x_5$$
$$\alpha_{3,4}^{10} \to x_6$$
$$\alpha_{2,6}^{11} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_6 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_2 + x_3 - x_7 &= 0\\
(e_1, e_3, e_4): &\quad -x_2 + x_6 &= 0\\
(e_1, e_2, e_6): &\quad -x_4 - x_5 + x_7 &= 0\\
(e_1, e_3, e_5): &\quad -x_1 + x_2 - x_4 &= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - x_5 + \frac{3x_7}{2} - \frac{1}{2} = 0$$
$$x_2 + \frac{x_7}{2} - \frac{1}{2} = 0$$
$$x_3 - \frac{x_7}{2} - \frac{1}{2} = 0$$
$$x_4 + x_5 - x_7 = 0$$
$$x_6 + \frac{x_7}{2} - \frac{1}{2} = 0$$

$\mathfrak{m}_{2A}(6,12)$

m2A612 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_2,e_5]&=e_{11} & [e_2,e_6]&=2e_{12}\\
[e_3,e_4]&=-e_{11} & [e_3,e_5]&=-e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(6,12)$

m4A612 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_2,e_3]&=e_9 & [e_2,e_4]&=e_{10}\\
[e_2,e_5]&=\alpha_{2,5}^{11}e_{11} & [e_2,e_6]&=\alpha_{2,6}^{12}e_{12}\\
[e_3,e_4]&=\alpha_{3,4}^{11}e_{11} & [e_3,e_5]&=\alpha_{3,5}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha_{2,5}^{11}-\alpha_{3,4}^{11}+1 &&=0\\
(e_1,e_2,e_5): &\quad \alpha_{2,5}^{11}-\alpha_{2,6}^{12}-\alpha_{3,5}^{12} &&=0\\
(e_1,e_3,e_4): &\quad \alpha_{3,4}^{11}-\alpha_{3,5}^{12} &&=0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{12}\to x_1$$

$$\alpha_{2,5}^{11} \to x_2$$

$$\alpha_{3,5}^{12} \to x_3$$

$$\alpha_{3,4}^{11} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_4 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad -x_1 + x_2 - x_3 &= 0 \\
(e_1, e_3, e_4): &\quad -x_3 + x_4 &= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 + 2x_4 - 1 = 0$$

$$x_2 + x_4 - 1 = 0$$

$$x_3 - x_4 = 0$$

$\mathfrak{m}_{1A}(7, 12)$

m1A712 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_5] &= e_{12} & [e_3, e_4] &= -e_{12}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(7, 12)$

m3A712 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{10} & [e_2, e_4] &= e_{11} \\
[e_2, e_5] &= \alpha_{2,5}^{12} e_{12} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{3,4}^{12} &\to x_1 \\
\alpha_{2,5}^{12} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(8, 12)$

m2A812 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{11} & [e_2, e_4] &= e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(9, 12)$

m1A912 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_2, e_3] &= e_{12}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2, 13)$

m1A213 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_{11}] &= e_{13} \\
[e_3, e_{10}] &= -e_{13} & [e_4, e_9] &= e_{13} \\
[e_5, e_8] &= -e_{13} & [e_6, e_7] &= e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2, 13)$

m3A213 (this line included for string searching purposes)

Solution 1

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_9] = e_{11} \\
[e_2, e_{10}] = 4e_{12} & [e_2, e_{11}] = 0 \\
[e_3, e_8] = -e_{11} & [e_3, e_9] = -3e_{12} \\
[e_3, e_{10}] = 4e_{13} & [e_4, e_7] = e_{11} \\
[e_4, e_8] = 2e_{12} & [e_4, e_9] = -7e_{13} \\
[e_5, e_6] = -e_{11} & [e_5, e_7] = -e_{12} \\
[e_5, e_8] = 9e_{13} & [e_6, e_7] = -10e_{13}
\end{array}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_9] = e_{11} \\
[e_2, e_{10}] = 4e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_3, e_8] = -e_{11} & [e_3, e_9] = -3e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_4, e_7] = e_{11} \\
[e_4, e_8] = 2e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_5, e_6] = -e_{11} & [e_5, e_7] = -e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13}
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}):& \quad -\alpha^{13}_{2,11} - \alpha^{13}_{3,10} + 4 &= 0\\
(e_1, e_3, e_9):& \quad -\alpha^{13}_{3,10} - \alpha^{13}_{4,9} - 3 &= 0\\
(e_1, e_4, e_8):& \quad -\alpha^{13}_{4,9} - \alpha^{13}_{5,8} + 2 &= 0\\
(e_1, e_5, e_7):& \quad -\alpha^{13}_{5,8} - \alpha^{13}_{6,7} - 1 &= 0\\
(e_2, e_3, e_8):& \quad -\alpha^{13}_{2,11} &= 0\\
(e_2, e_4, e_7):& \quad \alpha^{13}_{2,11} &= 0\\
(e_2, e_5, e_6):& \quad -\alpha^{13}_{2,11} &= 0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha^{13}_{2,11} &= 0\\
\alpha^{13}_{5,8} &= 9\\
\alpha^{13}_{3,10} &= 4\\
\alpha^{13}_{6,7} &= -10\\
\alpha^{13}_{4,9} &= -7
\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha^{13}_{2,11} &\to x_1\\
\alpha^{13}_{5,8} &\to x_2\\
\alpha^{13}_{3,10} &\to x_3\\
\alpha^{13}_{6,7} &\to x_4\\
\alpha^{13}_{4,9} &\to x_5
\end{aligned}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}):& \quad -x_1 - x_3 + 4 &= 0\\
(e_1, e_3, e_9):& \quad -x_3 - x_5 - 3 &= 0\\
(e_1, e_4, e_8):& \quad -x_2 - x_5 + 2 &= 0\\
(e_1, e_5, e_7):& \quad -x_2 - x_4 - 1 &= 0\\
(e_2, e_3, e_8):& \quad -x_1 &= 0\\
(e_2, e_4, e_7):& \quad x_1 &= 0\\
(e_2, e_5, e_6):& \quad -x_1 &= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 = 0$$
$$x_2 - 9 = 0$$
$$x_3 - 4 = 0$$
$$x_4 + 10 = 0$$
$$x_5 + 7 = 0$$

Solution 1:

$$x_1 = 0$$
$$x_2 = 9$$
$$x_3 = 4$$
$$x_4 = -10$$
$$x_5 = -7$$

$\mathfrak{m}_{9A}(2, 13)$

m9A213 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7$$
$$[e_2, e_6] = e_8 \qquad [e_2, e_7] = e_9$$
$$[e_2, e_8] = e_{10} \qquad [e_2, e_9] = e_{11}$$
$$[e_2, e_{10}] = e_{12} \qquad [e_2, e_{11}] = e_{13}$$
$$[e_3, e_4] = 0 \qquad [e_3, e_5] = 0$$
$$[e_3, e_6] = 0 \qquad [e_3, e_7] = 0$$
$$[e_3, e_8] = 0 \qquad [e_3, e_9] = 0$$
$$[e_3, e_{10}] = 0 \qquad [e_4, e_5] = 0$$
$$[e_4, e_6] = 0 \qquad [e_4, e_7] = 0$$
$$[e_4, e_8] = 0 \qquad [e_4, e_9] = 0$$
$$[e_5, e_6] = 0 \qquad [e_5, e_7] = 0$$
$$[e_5, e_8] = 0 \qquad [e_6, e_7] = 0$$

Solution 2

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = \frac{9e_7}{10}$$
$$[e_2, e_6] = \frac{4e_8}{5} \qquad [e_2, e_7] = \frac{5e_9}{7}$$
$$[e_2, e_8] = \frac{9e_{10}}{14} \qquad [e_2, e_9] = \frac{7e_{11}}{12}$$
$$[e_2, e_{10}] = \frac{8e_{12}}{15} \qquad [e_2, e_{11}] = \frac{27e_{13}}{55}$$
$$[e_3, e_4] = \frac{e_7}{10} \qquad [e_3, e_5] = \frac{e_8}{10}$$
$$[e_3, e_6] = \frac{3e_9}{35} \qquad [e_3, e_7] = \frac{e_{10}}{14}$$
$$[e_3, e_8] = \frac{5e_{11}}{84} \qquad [e_3, e_9] = \frac{e_{12}}{20}$$
$$[e_3, e_{10}] = \frac{7e_{13}}{165} \qquad [e_4, e_5] = \frac{e_9}{70}$$
$$[e_4, e_6] = \frac{e_{10}}{70} \qquad [e_4, e_7] = \frac{e_{11}}{84}$$
$$[e_4, e_8] = \frac{e_{12}}{105} \qquad [e_4, e_9] = \frac{e_{13}}{132}$$
$$[e_5, e_6] = \frac{e_{11}}{420} \qquad [e_5, e_7] = \frac{e_{12}}{420}$$
$$[e_5, e_8] = \frac{3e_{13}}{1540} \qquad [e_6, e_7] = \frac{e_{13}}{2310}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] = \alpha_{3,8}^{11} e_{11} & [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_8] = \alpha_{4,8}^{12} e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_5, e_6] = \alpha_{5,6}^{11} e_{11} & [e_5, e_7] = \alpha_{5,7}^{12} e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{7}-\alpha_{3,4}^{7}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{7}-\alpha_{2,6}^{8}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{7}-\alpha_{3,5}^{8} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{8}-\alpha_{2,7}^{9}-\alpha_{3,6}^{9} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{8}-\alpha_{3,6}^{9}-\alpha_{4,5}^{9} &&=0\\
(e_2,e_3,e_4):&\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7}-\alpha_{3,6}^{9}+\alpha_{4,5}^{9} &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{9}-\alpha_{2,8}^{10}-\alpha_{3,7}^{10} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{9}-\alpha_{3,7}^{10}-\alpha_{4,6}^{10} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{9}-\alpha_{4,6}^{10} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10}+\alpha_{2,8}^{10}\alpha_{3,5}^{8} &&=0\\
(e_1,e_2,e_8):&\quad \alpha_{2,8}^{10}-\alpha_{2,9}^{11}-\alpha_{3,8}^{11} &&=0\\
(e_1,e_3,e_7):&\quad \alpha_{3,7}^{10}-\alpha_{3,8}^{11}-\alpha_{4,7}^{11} &&=0\\
(e_1,e_4,e_6):&\quad \alpha_{4,6}^{10}-\alpha_{4,7}^{11}-\alpha_{5,6}^{11} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11}+\alpha_{2,9}^{11}\alpha_{3,6}^{9}-\alpha_{5,6}^{11} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11}+\alpha_{2,9}^{11}\alpha_{4,5}^{9}+\alpha_{5,6}^{11} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha_{2,10}^{12}+\alpha_{2,9}^{11}-\alpha_{3,9}^{12} &&=0\\
(e_1,e_3,e_8):&\quad \alpha_{3,8}^{11}-\alpha_{3,9}^{12}-\alpha_{4,8}^{12} &&=0\\
(e_1,e_4,e_7):&\quad \alpha_{4,7}^{11}-\alpha_{4,8}^{12}-\alpha_{5,7}^{12} &&=0\\
(e_1,e_5,e_6):&\quad \alpha_{5,6}^{11}-\alpha_{5,7}^{12} &&=0\\
(e_2,e_3,e_7):&\quad \alpha_{2,10}^{12}\alpha_{3,7}^{10}-\alpha_{2,7}^{9}\alpha_{3,9}^{12}-\alpha_{5,7}^{12} &&=0\\
(e_2,e_4,e_6):&\quad \alpha_{2,10}^{12}\alpha_{4,6}^{10}-\alpha_{2,6}^{8}\alpha_{4,8}^{12} &&=0\\
(e_3,e_4,e_5):&\quad \alpha_{3,4}^{7}\alpha_{5,7}^{12}-\alpha_{3,5}^{8}\alpha_{4,8}^{12}+\alpha_{3,9}^{12}\alpha_{4,5}^{9} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha_{2,10}^{12}-\alpha_{2,11}^{13}-\alpha_{3,10}^{13} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{13}+\alpha_{3,9}^{12}-\alpha_{4,9}^{13} &&=0\\
(e_1,e_4,e_8):&\quad \alpha_{4,8}^{12}-\alpha_{4,9}^{13}-\alpha_{5,8}^{13} &&=0\\
(e_1,e_5,e_7):&\quad \alpha_{5,7}^{12}-\alpha_{5,8}^{13}-\alpha_{6,7}^{13} &&=0\\
(e_2,e_3,e_8):&\quad \alpha_{2,11}^{13}\alpha_{3,8}^{11}-\alpha_{2,8}^{10}\alpha_{3,10}^{13}-\alpha_{5,8}^{13} &&=0\\
(e_2,e_4,e_7):&\quad \alpha_{2,11}^{13}\alpha_{4,7}^{11}-\alpha_{2,7}^{9}\alpha_{4,9}^{13}-\alpha_{6,7}^{13} &&=0\\
(e_2,e_5,e_6):&\quad \alpha_{2,11}^{13}\alpha_{5,6}^{11}+\alpha_{2,5}^{7}\alpha_{6,7}^{13}-\alpha_{2,6}^{8}\alpha_{5,8}^{13} &&=0\\
(e_3,e_4,e_6):&\quad \alpha_{3,10}^{13}\alpha_{4,6}^{10}+\alpha_{3,4}^{7}\alpha_{6,7}^{13}-\alpha_{3,6}^{9}\alpha_{4,9}^{13} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha_{3,8}^{11} = 0$$
$$\alpha_{4,7}^{11} = 0$$
$$\alpha_{6,7}^{13} = 0$$
$$\alpha_{5,7}^{12} = 0$$
$$\alpha_{2,6}^{8} = 1$$
$$\alpha_{3,9}^{12} = 0$$
$$\alpha_{4,8}^{12} = 0$$
$$\alpha_{2,5}^{7} = 1$$
$$\alpha_{2,7}^{9} = 1$$
$$\alpha_{3,7}^{10} = 0$$
$$\alpha_{2,8}^{10} = 1$$
$$\alpha_{4,5}^{9} = 0$$
$$\alpha_{3,10}^{13} = 0$$
$$\alpha_{5,8}^{13} = 0$$
$$\alpha_{3,5}^{8} = 0$$
$$\alpha_{5,6}^{11} = 0$$
$$\alpha_{4,6}^{10} = 0$$
$$\alpha_{2,9}^{11} = 1$$
$$\alpha_{2,11}^{13} = 1$$
$$\alpha_{4,9}^{13} = 0$$
$$\alpha_{3,6}^{9} = 0$$
$$\alpha_{3,4}^{7} = 0$$
$$\alpha_{2,10}^{12} = 1$$

Solution 2:

$$\alpha^{11}_{3,8} = 5/84$$
$$\alpha^{11}_{4,7} = 1/84$$
$$\alpha^{13}_{6,7} = 1/2310$$
$$\alpha^{12}_{5,7} = 1/420$$
$$\alpha^{8}_{2,6} = 4/5$$
$$\alpha^{12}_{3,9} = 1/20$$
$$\alpha^{12}_{4,8} = 1/105$$
$$\alpha^{7}_{2,5} = 9/10$$
$$\alpha^{9}_{2,7} = 5/7$$
$$\alpha^{10}_{3,7} = 1/14$$
$$\alpha^{10}_{2,8} = 9/14$$
$$\alpha^{9}_{4,5} = 1/70$$
$$\alpha^{13}_{3,10} = 7/165$$
$$\alpha^{13}_{5,8} = 3/1540$$
$$\alpha^{8}_{3,5} = 1/10$$
$$\alpha^{11}_{5,6} = 1/420$$
$$\alpha^{10}_{4,6} = 1/70$$
$$\alpha^{11}_{2,9} = 7/12$$
$$\alpha^{13}_{2,11} = 27/55$$
$$\alpha^{13}_{4,9} = 1/132$$
$$\alpha^{9}_{3,6} = 3/35$$
$$\alpha^{7}_{3,4} = 1/10$$
$$\alpha^{12}_{2,10} = 8/15$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{3,8} \to x_1$$
$$\alpha^{11}_{4,7} \to x_2$$
$$\alpha^{13}_{6,7} \to x_3$$
$$\alpha^{12}_{5,7} \to x_4$$
$$\alpha^{8}_{2,6} \to x_5$$

$$\alpha^{12}_{3,9} \to x_6$$

$$\alpha^{12}_{4,8} \to x_7$$

$$\alpha^{7}_{2,5} \to x_8$$

$$\alpha^{9}_{2,7} \to x_9$$

$$\alpha^{10}_{3,7} \to x_{10}$$

$$\alpha^{10}_{2,8} \to x_{11}$$

$$\alpha^{9}_{4,5} \to x_{12}$$

$$\alpha^{13}_{3,10} \to x_{13}$$

$$\alpha^{13}_{5,8} \to x_{14}$$

$$\alpha^{8}_{3,5} \to x_{15}$$

$$\alpha^{11}_{5,6} \to x_{16}$$

$$\alpha^{10}_{4,6} \to x_{17}$$

$$\alpha^{11}_{2,9} \to x_{18}$$

$$\alpha^{13}_{2,11} \to x_{19}$$

$$\alpha^{13}_{4,9} \to x_{20}$$

$$\alpha^{9}_{3,6} \to x_{21}$$

$$\alpha^{7}_{3,4} \to x_{22}$$

$$\alpha^{12}_{2,10} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -x_{22}-x_8+1 &&=0\\
(e_1,e_2,e_5): &\quad -x_{15}-x_5+x_8 &&=0\\
(e_1,e_3,e_4): &\quad -x_{15}+x_{22} &&=0\\
(e_1,e_2,e_6): &\quad -x_{21}+x_5-x_9 &&=0\\
(e_1,e_3,e_5): &\quad -x_{12}+x_{15}-x_{21} &&=0\\
(e_2,e_3,e_4): &\quad x_{12}-x_{21}+x_{22}x_9 &&=0\\
(e_1,e_2,e_7): &\quad -x_{10}-x_{11}+x_9 &&=0\\
(e_1,e_3,e_6): &\quad -x_{10}-x_{17}+x_{21} &&=0\\
(e_1,e_4,e_5): &\quad x_{12}-x_{17} &&=0\\
(e_2,e_3,e_5): &\quad -x_{10}x_8+x_{11}x_{15} &&=0\\
(e_1,e_2,e_8): &\quad -x_1+x_{11}-x_{18} &&=0\\
(e_1,e_3,e_7): &\quad -x_1+x_{10}-x_2 &&=0\\
(e_1,e_4,e_6): &\quad -x_{16}+x_{17}-x_2 &&=0\\
(e_2,e_3,e_6): &\quad -x_1x_5-x_{16}+x_{18}x_{21} &&=0\\
(e_2,e_4,e_5): &\quad x_{12}x_{18}+x_{16}-x_2x_8 &&=0\\
(e_1,e_2,e_9): &\quad x_{18}-x_{23}-x_6 &&=0\\
(e_1,e_3,e_8): &\quad x_1-x_6-x_7 &&=0\\
(e_1,e_4,e_7): &\quad x_2-x_4-x_7 &&=0\\
(e_1,e_5,e_6): &\quad x_{16}-x_4 &&=0\\
(e_2,e_3,e_7): &\quad x_{10}x_{23}-x_4-x_6x_9 &&=0\\
(e_2,e_4,e_6): &\quad x_{17}x_{23}-x_5x_7 &&=0\\
(e_3,e_4,e_5): &\quad x_{12}x_6-x_{15}x_7+x_{22}x_4 &&=0\\
(e_1,e_2,e_{10}): &\quad -x_{13}-x_{19}+x_{23} &&=0\\
(e_1,e_3,e_9): &\quad -x_{13}-x_{20}+x_6 &&=0\\
(e_1,e_4,e_8): &\quad -x_{14}-x_{20}+x_7 &&=0\\
(e_1,e_5,e_7): &\quad -x_{14}-x_3+x_4 &&=0\\
(e_2,e_3,e_8): &\quad x_1x_{19}-x_{11}x_{13}-x_{14} &&=0\\
(e_2,e_4,e_7): &\quad x_{19}x_2-x_{20}x_9-x_3 &&=0\\
(e_2,e_5,e_6): &\quad -x_{14}x_5+x_{16}x_{19}+x_3x_8 &&=0\\
(e_3,e_4,e_6): &\quad x_{13}x_{17}-x_{20}x_{21}+x_{22}x_3 &&=0
\end{aligned}
$$

Groebner basis (23 variables, 0 linear, 23 nonlinear)

$$x_1-\frac{9442385x_{23}^5}{203297472}+\frac{39791369x_{23}^4}{152473104}-\frac{187989481x_{23}^3}{304946208}+\frac{4537949x_{23}^2}{5647152}-\frac{245637491x_{23}}{609892416}+\frac{84955}{76236552}=0$$

$$x_2+\frac{2915995x_{23}^5}{135531648}-\frac{3049927x_{23}^4}{25412184}+\frac{57387209x_{23}^3}{203297472}-\frac{360999x_{23}^2}{941192}+\frac{119035285x_{23}}{406594944}-\frac{4725911}{50824368}=0$$

$$\frac{2668075x_{23}^5}{372712032}-\frac{11622955x_{23}^4}{279534024}+\frac{52962515x_{23}^3}{559068048}-\frac{1100615x_{23}^2}{10353112}+\frac{65903905x_{23}}{1118136096}+x_3-\frac{1809365}{139767012}=0$$

$$\frac{401155x_{23}^5}{22588608}-\frac{1710217x_{23}^4}{16941456}+\frac{2711669x_{23}^3}{11294304}-\frac{1649957x_{23}^2}{5647152}+\frac{12135073x_{23}}{67765824}+x_4-\frac{41225}{941192}=0$$

$$-\frac{3114395x_{23}^5}{29042496}+\frac{6555889x_{23}^4}{10890936}-\frac{61909453x_{23}^3}{43563744}+\frac{751421x_{23}^2}{403368}-\frac{145272173x_{23}}{87127488}+x_5-\frac{2930909}{10890936}=0$$

$$-\frac{2915995x_{23}^5}{58084992}+\frac{3049927x_{23}^4}{10890936}-\frac{57387209x_{23}^3}{87127488}+\frac{120333x_{23}^2}{134456}-\frac{89992789x_{23}}{174254976}+x_6+\frac{1095599}{21781872}=0$$

$$\frac{509065x_{23}^5}{135531648}-\frac{969203x_{23}^4}{50824368}+\frac{8577167x_{23}^3}{203297472}-\frac{516037x_{23}^2}{5647152}+\frac{46224847x_{23}}{406594944}+x_7-\frac{2499761}{50824368}=0$$

$$-\frac{3114395x_{23}^5}{58084992}+\frac{6555889x_{23}^4}{21781872}-\frac{61909453x_{23}^3}{87127488}+\frac{751421x_{23}^2}{806736}-\frac{145272173x_{23}}{174254976}+x_8-\frac{13821845}{21781872}=0$$

$$-\frac{1029865x_{23}^5}{8470728}+\frac{17330359x_{23}^4}{25412184}-\frac{40896115x_{23}^3}{25412184}+\frac{5981945x_{23}^2}{2823576}-\frac{25783109x_{23}}{12706092}+x_9-\frac{128515}{3176523}=0$$

$$x_{10}-\frac{10136785x_{23}^5}{406594944}+\frac{21491807x_{23}^4}{152473104}-\frac{203817335x_{23}^3}{609892416}+\frac{2371955x_{23}^2}{5647152}-\frac{134169127x_{23}}{1219784832}-\frac{14007823}{152473104}=0$$

$$x_{11}-\frac{39296735x_{23}^5}{406594944}+\frac{82490347x_{23}^4}{152473104}-\frac{777689425x_{23}^3}{609892416}+\frac{9591935x_{23}^2}{5647152}-\frac{2341009337x_{23}}{1219784832}+\frac{7839103}{152473104}=0$$

$$x_{12}+\frac{5322925x_{23}^5}{135531648}-\frac{11230505x_{23}^4}{50824368}+\frac{106197251x_{23}^3}{203297472}-\frac{3815951x_{23}^2}{5647152}+\frac{191845723x_{23}}{406594944}-\frac{6952061}{50824368}=0$$

$$x_{13}-\frac{1978765x_{23}^5}{45638208}+\frac{512881x_{23}^4}{2139291}-\frac{38027351x_{23}^3}{68457312}+\frac{63424x_{23}^2}{79233}-\frac{69826195x_{23}}{136914624}+\frac{1174517}{17114328}=0$$

$$x_{14}+\frac{7901965x_{23}^5}{745424064}-\frac{33191251x_{23}^4}{559068048}+\frac{162530201x_{23}^3}{1118136096}-\frac{11545837x_{23}^2}{62118672}+\frac{268649599x_{23}}{2236272192}-\frac{8625095}{279534024}=0$$

$$x_{15}+\frac{3114395x_{23}^5}{58084992}-\frac{6555889x_{23}^4}{21781872}+\frac{61909453x_{23}^3}{87127488}-\frac{751421x_{23}^2}{806736}+\frac{145272173x_{23}}{174254976}-\frac{7960027}{21781872}=0$$

$$x_{16}+\frac{401155x_{23}^5}{22588608}-\frac{1710217x_{23}^4}{16941456}+\frac{2711669x_{23}^3}{11294304}-\frac{1649957x_{23}^2}{5647152}+\frac{12135073x_{23}}{67765824}-\frac{41225}{941192}=0$$

$$x_{17}+\frac{5322925x_{23}^5}{135531648}-\frac{11230505x_{23}^4}{50824368}+\frac{106197251x_{23}^3}{203297472}-\frac{3815951x_{23}^2}{5647152}+\frac{191845723x_{23}}{406594944}-\frac{6952061}{50824368}=0$$

$$x_{18}-\frac{2915995x_{23}^5}{58084992}+\frac{3049927x_{23}^4}{10890936}-\frac{57387209x_{23}^3}{87127488}+\frac{120333x_{23}^2}{134456}-\frac{264247765x_{23}}{174254976}+\frac{1095599}{21781872}=0$$

$$x_{19}+\frac{1978765x_{23}^5}{45638208}-\frac{512881x_{23}^4}{2139291}+\frac{38027351x_{23}^3}{68457312}-\frac{63424x_{23}^2}{79233}-\frac{67088429x_{23}}{136914624}-\frac{1174517}{17114328}=0$$

$$x_{20}-\frac{485915x_{23}^5}{70992768}+\frac{536429x_{23}^4}{13311144}-\frac{10986265x_{23}^3}{106489152}+\frac{419245x_{23}^2}{4437048}-\frac{1372661x_{23}}{212978304}-\frac{487961}{26622288}=0$$

$$x_{21}+\frac{2915995x_{23}^5}{203297472}-\frac{3049927x_{23}^4}{38118276}+\frac{57387209x_{23}^3}{304946208}-\frac{120333x_{23}^2}{470596}+\frac{220684021x_{23}}{609892416}-\frac{17432003}{76236552}=0$$

$$x_{22}+\frac{3114395x_{23}^5}{58084992}-\frac{6555889x_{23}^4}{21781872}+\frac{61909453x_{23}^3}{87127488}-\frac{751421x_{23}^2}{806736}+\frac{145272173x_{23}}{174254976}-\frac{7960027}{21781872}=0$$

$$x_{23}^6-\frac{83x_{23}^5}{15}+\frac{38x_{23}^4}{3}-\frac{46x_{23}^3}{3}+\frac{31x_{23}^2}{3}-\frac{11x_{23}}{3}+\frac{8}{15}=0$$

Solution 1:

$$x_1 = 0$$
$$x_2 = 0$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 1$$
$$x_6 = 0$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = 1$$
$$x_1 0 = 0$$
$$x_1 1 = 1$$
$$x_1 2 = 0$$
$$x_1 3 = 0$$
$$x_1 4 = 0$$
$$x_1 5 = 0$$
$$x_1 6 = 0$$
$$x_1 7 = 0$$
$$x_1 8 = 1$$
$$x_1 9 = 1$$
$$x_2 0 = 0$$
$$x_2 1 = 0$$
$$x_2 2 = 0$$
$$x_2 3 = 1$$

Solution 2:

$$x_1 = 5/84$$
$$x_2 = 1/84$$
$$x_3 = 1/2310$$
$$x_4 = 1/420$$

$$x_5 = 4/5$$
$$x_6 = 1/20$$
$$x_7 = 1/105$$
$$x_8 = 9/10$$
$$x_9 = 5/7$$
$$x_10 = 1/14$$
$$x_11 = 9/14$$
$$x_12 = 1/70$$
$$x_13 = 7/165$$
$$x_14 = 3/1540$$
$$x_15 = 1/10$$
$$x_16 = 1/420$$
$$x_17 = 1/70$$
$$x_18 = 7/12$$
$$x_19 = 27/55$$
$$x_20 = 1/132$$
$$x_21 = 3/35$$
$$x_22 = 1/10$$
$$x_23 = 8/15$$

$\mathfrak{m}_{2A}(3, 13)$

m2A313 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_9] = e_{12}$$
$$[e_2, e_{10}] = 4e_{13} \qquad [e_3, e_8] = -e_{12}$$
$$[e_3, e_9] = -3e_{13} \qquad [e_4, e_7] = e_{12}$$
$$[e_4, e_8] = 2e_{13} \qquad [e_5, e_6] = -e_{12}$$
$$[e_5, e_7] = -e_{13}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3,13)$

m4A313 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_7]=e_{10}$$
$$[e_2,e_8]=3e_{11} \qquad [e_2,e_9]=\frac{7e_{12}}{2}$$
$$[e_2,e_{10}]=0 \qquad [e_3,e_6]=-e_{10}$$
$$[e_3,e_7]=-2e_{11} \qquad [e_3,e_8]=-\frac{e_{12}}{2}$$
$$[e_3,e_9]=\frac{7e_{13}}{2} \qquad [e_4,e_5]=e_{10}$$
$$[e_4,e_6]=e_{11} \qquad [e_4,e_7]=-\frac{3e_{12}}{2}$$
$$[e_4,e_8]=-4e_{13} \qquad [e_5,e_6]=\frac{5e_{12}}{2}$$
$$[e_5,e_7]=\frac{5e_{13}}{2}$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_7]=e_{10}$$
$$[e_2,e_8]=3e_{11} \qquad [e_2,e_9]=\alpha_{2,9}^{12}e_{12}$$
$$[e_2,e_{10}]=\alpha_{2,10}^{13}e_{13} \qquad [e_3,e_6]=-e_{10}$$
$$[e_3,e_7]=-2e_{11} \qquad [e_3,e_8]=\alpha_{3,8}^{12}e_{12}$$
$$[e_3,e_9]=\alpha_{3,9}^{13}e_{13} \qquad [e_4,e_5]=e_{10}$$
$$[e_4,e_6]=e_{11} \qquad [e_4,e_7]=\alpha_{4,7}^{12}e_{12}$$
$$[e_4,e_8]=\alpha_{4,8}^{13}e_{13} \qquad [e_5,e_6]=\alpha_{5,6}^{12}e_{12}$$
$$[e_5,e_7]=\alpha_{5,7}^{13}e_{13}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,10}^{13} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{2,9}^{12} &= 7/2\\
\alpha_{5,7}^{13} &= 5/2\\
\alpha_{3,8}^{12} &= -1/2\\
\alpha_{4,8}^{13} &= -4\\
\alpha_{2,10}^{13} &= 0\\
\alpha_{4,7}^{12} &= -3/2\\
\alpha_{5,6}^{12} &= 5/2\\
\alpha_{3,9}^{13} &= 7/2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,9}^{12} &\to x_1\\
\alpha_{5,7}^{13} &\to x_2\\
\alpha_{3,8}^{12} &\to x_3\\
\alpha_{4,8}^{13} &\to x_4\\
\alpha_{2,10}^{13} &\to x_5\\
\alpha_{4,7}^{12} &\to x_6\\
\alpha_{5,6}^{12} &\to x_7
\end{aligned}
$$

$$\alpha_{3,9}^{13} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_3 + 3 &= 0\\
(e_1, e_3, e_7): &\quad -x_3 - x_6 - 2 &= 0\\
(e_1, e_4, e_6): &\quad -x_6 - x_7 + 1 &= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_5 - x_8 &= 0\\
(e_1, e_3, e_8): &\quad x_3 - x_4 - x_8 &= 0\\
(e_1, e_4, e_7): &\quad -x_2 - x_4 + x_6 &= 0\\
(e_1, e_5, e_6): &\quad -x_2 + x_7 &= 0\\
(e_2, e_3, e_6): &\quad -x_5 &= 0\\
(e_2, e_4, e_5): &\quad x_5 &= 0
\end{aligned}$$

Groebner basis (8 variables, 8 linear, 0 nonlinear)

$$x_1 - \frac{7}{2} = 0$$

$$x_2 - \frac{5}{2} = 0$$

$$x_3 + \frac{1}{2} = 0$$

$$x_4 + 4 = 0$$

$$x_5 = 0$$

$$x_6 + \frac{3}{2} = 0$$

$$x_7 - \frac{5}{2} = 0$$

$$x_8 - \frac{7}{2} = 0$$

Solution 1:

$$x_1 = 7/2$$

$$x_2 = 5/2$$

$$x_3 = -1/2$$

$$x_4 = -4$$

$$x_5 = 0$$

$$x_6 = -3/2$$

$$x_7 = 5/2$$

$$x_8 = 7/2$$

$\mathfrak{m}_{6A}(3,13)$

m6A313 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_5]=e_8$$
$$[e_2,e_6]=2e_9 \qquad [e_2,e_7]=\frac{5e_{10}}{3}$$
$$[e_2,e_8]=0 \qquad [e_2,e_9]=-\frac{49e_{12}}{33}$$
$$[e_2,e_{10}]=-\frac{14e_{13}}{11} \qquad [e_3,e_4]=-e_8$$
$$[e_3,e_5]=-e_9 \qquad [e_3,e_6]=\frac{e_{10}}{3}$$
$$[e_3,e_7]=\frac{5e_{11}}{3} \qquad [e_3,e_8]=\frac{49e_{12}}{33}$$
$$[e_3,e_9]=-\frac{7e_{13}}{33} \qquad [e_4,e_5]=-\frac{4e_{10}}{3}$$
$$[e_4,e_6]=-\frac{4e_{11}}{3} \qquad [e_4,e_7]=\frac{2e_{12}}{11}$$
$$[e_4,e_8]=\frac{56e_{13}}{33} \qquad [e_5,e_6]=-\frac{50e_{12}}{33}$$
$$[e_5,e_7]=-\frac{50e_{13}}{33}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_5] = e_8 \\
[e_2, e_6] = 2e_9 & [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] = \alpha_{2,8}^{11} e_{11} & [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} & [e_3, e_4] = -e_8 \\
[e_3, e_5] = -e_9 & [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] = \alpha_{3,7}^{11} e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] = \alpha_{4,6}^{11} e_{11} & [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] = \alpha_{4,8}^{13} e_{13} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_6):&\quad -\alpha_{2,7}^{10}-\alpha_{3,6}^{10}+2 &&=0\\
(e_1,e_3,e_5):&\quad -\alpha_{3,6}^{10}-\alpha_{4,5}^{10}-1 &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{10}-\alpha_{4,6}^{11} &&=0\\
(e_2,e_3,e_4):&\quad -\alpha_{2,8}^{11} &&=0\\
(e_1,e_2,e_8):&\quad \alpha_{2,8}^{11}-\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_3,e_7):&\quad \alpha_{3,7}^{11}-\alpha_{3,8}^{12}-\alpha_{4,7}^{12} &&=0\\
(e_1,e_4,e_6):&\quad \alpha_{4,6}^{11}-\alpha_{4,7}^{12}-\alpha_{5,6}^{12} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha_{2,10}^{13}+\alpha_{2,9}^{12}-\alpha_{3,9}^{13} &&=0\\
(e_1,e_3,e_8):&\quad \alpha_{3,8}^{12}-\alpha_{3,9}^{13}-\alpha_{4,8}^{13} &&=0\\
(e_1,e_4,e_7):&\quad \alpha_{4,7}^{12}-\alpha_{4,8}^{13}-\alpha_{5,7}^{13} &&=0\\
(e_1,e_5,e_6):&\quad \alpha_{5,6}^{12}-\alpha_{5,7}^{13} &&=0\\
(e_2,e_3,e_6):&\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10}-2\alpha_{3,9}^{13} &&=0\\
(e_2,e_4,e_5):&\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10}-\alpha_{4,8}^{13} &&=0
\end{aligned}$$

Solution 1:

$$\alpha^{12}_{2,9} = -49/33$$
$$\alpha^{13}_{5,7} = -50/33$$
$$\alpha^{10}_{2,7} = 5/3$$
$$\alpha^{12}_{3,8} = 49/33$$
$$\alpha^{13}_{4,8} = 56/33$$
$$\alpha^{11}_{3,7} = 5/3$$
$$\alpha^{13}_{2,10} = -14/11$$
$$\alpha^{10}_{3,6} = 1/3$$
$$\alpha^{12}_{4,7} = 2/11$$
$$\alpha^{11}_{2,8} = 0$$
$$\alpha^{12}_{5,6} = -50/33$$
$$\alpha^{10}_{4,5} = -4/3$$
$$\alpha^{13}_{3,9} = -7/33$$
$$\alpha^{11}_{4,6} = -4/3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{2,9} \to x_1$$
$$\alpha^{13}_{5,7} \to x_2$$
$$\alpha^{10}_{2,7} \to x_3$$
$$\alpha^{12}_{3,8} \to x_4$$
$$\alpha^{13}_{4,8} \to x_5$$
$$\alpha^{11}_{3,7} \to x_6$$
$$\alpha^{13}_{2,10} \to x_7$$
$$\alpha^{10}_{3,6} \to x_8$$
$$\alpha^{12}_{4,7} \to x_9$$
$$\alpha^{11}_{2,8} \to x_{10}$$
$$\alpha^{12}_{5,6} \to x_{11}$$
$$\alpha^{10}_{4,5} \to x_{12}$$
$$\alpha^{13}_{3,9} \to x_{13}$$

$$\alpha_{4,6}^{11} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_3 - x_8 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_{12} - x_8 - 1 &= 0\\
(e_1, e_2, e_7): &\quad -x_{10} + x_3 - x_6 &= 0\\
(e_1, e_3, e_6): &\quad -x_{14} - x_6 + x_8 &= 0\\
(e_1, e_4, e_5): &\quad x_{12} - x_{14} &= 0\\
(e_2, e_3, e_4): &\quad -x_{10} &= 0\\
(e_1, e_2, e_8): &\quad -x_1 + x_{10} - x_4 &= 0\\
(e_1, e_3, e_7): &\quad -x_4 + x_6 - x_9 &= 0\\
(e_1, e_4, e_6): &\quad -x_{11} + x_{14} - x_9 &= 0\\
(e_2, e_3, e_5): &\quad -x_1 - x_4 &= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_{13} - x_7 &= 0\\
(e_1, e_3, e_8): &\quad -x_{13} + x_4 - x_5 &= 0\\
(e_1, e_4, e_7): &\quad -x_2 - x_5 + x_9 &= 0\\
(e_1, e_5, e_6): &\quad x_{11} - x_2 &= 0\\
(e_2, e_3, e_6): &\quad -2x_{13} + x_7 x_8 &= 0\\
(e_2, e_4, e_5): &\quad x_{12} x_7 - x_5 &= 0
\end{aligned}$$

Groebner basis (14 variables, 14 linear, 0 nonlinear)

$$x_1 + \frac{49}{33} = 0$$

$$x_2 + \frac{50}{33} = 0$$

$$x_3 - \frac{5}{3} = 0$$

$$x_4 - \frac{49}{33} = 0$$

$$x_5 - \frac{56}{33} = 0$$

$$x_6 - \frac{5}{3} = 0$$

$$x_7 + \frac{14}{11} = 0$$

$$x_8 - \frac{1}{3} = 0$$

$$x_9 - \frac{2}{11} = 0$$
$$x_{10} = 0$$
$$x_{11} + \frac{50}{33} = 0$$
$$x_{12} + \frac{4}{3} = 0$$
$$x_{13} + \frac{7}{33} = 0$$
$$x_{14} + \frac{4}{3} = 0$$

Solution 1:

$$x_1 = -49/33$$
$$x_2 = -50/33$$
$$x_3 = 5/3$$
$$x_4 = 49/33$$
$$x_5 = 56/33$$
$$x_6 = 5/3$$
$$x_7 = -14/11$$
$$x_8 = 1/3$$
$$x_9 = 2/11$$
$$x_10 = 0$$
$$x_11 = -50/33$$
$$x_12 = -4/3$$
$$x_13 = -7/33$$
$$x_14 = -4/3$$

$\mathfrak{m}_{8A}(3,13)$

m8A313 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_3]=e_6$$
$$[e_2,e_4]=e_7 \qquad [e_2,e_5]=\alpha_{2,5}^{8}e_8$$
$$[e_2,e_6]=\alpha_{2,6}^{9}e_9 \qquad [e_2,e_7]=\alpha_{2,7}^{10}e_{10}$$
$$[e_2,e_8]=\alpha_{2,8}^{11}e_{11} \qquad [e_2,e_9]=\alpha_{2,9}^{12}e_{12}$$
$$[e_2,e_{10}]=\alpha_{2,10}^{13}e_{13} \qquad [e_3,e_4]=\alpha_{3,4}^{8}e_8$$
$$[e_3,e_5]=\alpha_{3,5}^{9}e_9 \qquad [e_3,e_6]=\alpha_{3,6}^{10}e_{10}$$
$$[e_3,e_7]=\alpha_{3,7}^{11}e_{11} \qquad [e_3,e_8]=\alpha_{3,8}^{12}e_{12}$$
$$[e_3,e_9]=\alpha_{3,9}^{13}e_{13} \qquad [e_4,e_5]=\alpha_{4,5}^{10}e_{10}$$
$$[e_4,e_6]=\alpha_{4,6}^{11}e_{11} \qquad [e_4,e_7]=\alpha_{4,7}^{12}e_{12}$$
$$[e_4,e_8]=\alpha_{4,8}^{13}e_{13} \qquad [e_5,e_6]=\alpha_{5,6}^{12}e_{12}$$
$$[e_5,e_7]=\alpha_{5,7}^{13}e_{13}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{12} + \alpha_{2,9}^{12}\alpha_{3,5}^{9} + \alpha_{5,6}^{12} &= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - \alpha_{2,6}^{9}\alpha_{3,9}^{13} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{2,5}^{8}\alpha_{4,8}^{13} + \alpha_{5,7}^{13} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{12} \to x_1$$

$$\alpha_{3,4}^{8} \to x_2$$

$$\alpha_{5,7}^{13} \to x_3$$

$$\alpha_{2,7}^{10} \to x_4$$

$$\alpha_{2,6}^{9} \to x_5$$

$$\alpha_{3,8}^{12} \to x_6$$

$$\alpha_{4,8}^{13} \to x_7$$

$$\alpha_{2,5}^{8} \to x_8$$

$$\alpha_{4,5}^{10} \to x_9$$

$$\alpha_{3,7}^{11} \to x_{10}$$

$$\alpha_{2,10}^{13} \to x_{11}$$

$$\alpha_{3,6}^{10} \to x_{12}$$

$$\alpha_{4,7}^{12} \to x_{13}$$

$$\alpha_{2,8}^{11} \to x_{14}$$

$$\alpha_{5,6}^{12} \to x_{15}$$

$$\alpha_{3,5}^{9} \to x_{16}$$

$$\alpha_{3,9}^{13} \to x_{17}$$

$$\alpha_{4,6}^{11} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_8 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{16} - x_5 + x_8 &&= 0\\
(e_1, e_3, e_4): &\quad -x_{16} + x_2 &&= 0\\
(e_1, e_2, e_6): &\quad -x_{12} - x_4 + x_5 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{12} + x_{16} - x_9 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{10} - x_{14} + x_4 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} + x_{12} - x_{18} &&= 0\\
(e_1, e_4, e_5): &\quad -x_{18} + x_9 &&= 0\\
(e_2, e_3, e_4): &\quad -x_{10} + x_{14}x_2 + x_{18} &&= 0\\
(e_1, e_2, e_8): &\quad -x_1 + x_{14} - x_6 &&= 0\\
(e_1, e_3, e_7): &\quad x_{10} - x_{13} - x_6 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{13} - x_{15} + x_{18} &&= 0\\
(e_2, e_3, e_5): &\quad x_1x_{16} + x_{15} - x_6x_8 &&= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_{11} - x_{17} &&= 0\\
(e_1, e_3, e_8): &\quad -x_{17} + x_6 - x_7 &&= 0\\
(e_1, e_4, e_7): &\quad x_{13} - x_3 - x_7 &&= 0\\
(e_1, e_5, e_6): &\quad x_{15} - x_3 &&= 0\\
(e_2, e_3, e_6): &\quad x_{11}x_{12} - x_{17}x_5 &&= 0\\
(e_2, e_4, e_5): &\quad x_{11}x_9 + x_3 - x_7x_8 &&= 0
\end{aligned}$$

Groebner basis (18 variables, 15 linear, 4 nonlinear)

$$x_1 + \frac{14x_{16}}{3} + \frac{x_{17}}{3} - \frac{14x_{18}}{3} - 1 = 0$$
$$-x_{16} + x_2 = 0$$
$$\frac{x_{16}}{3} - \frac{x_{17}}{3} - \frac{4x_{18}}{3} + x_3 = 0$$
$$3x_{16} - x_{18} + x_4 - 1 = 0$$
$$2x_{16} + x_5 - 1 = 0$$
$$-\frac{2x_{16}}{3} - \frac{x_{17}}{3} + \frac{5x_{18}}{3} + x_6 = 0$$
$$-\frac{2x_{16}}{3} + \frac{2x_{17}}{3} + \frac{5x_{18}}{3} + x_7 = 0$$
$$x_{16} + x_8 - 1 = 0$$
$$-x_{18} + x_9 = 0$$
$$x_{10} - x_{16} + 2x_{18} = 0$$
$$x_{11} + \frac{14x_{16}}{3} + \frac{4x_{17}}{3} - \frac{14x_{18}}{3} - 1 = 0$$
$$x_{12} - x_{16} + x_{18} = 0$$
$$x_{13} - \frac{x_{16}}{3} + \frac{x_{17}}{3} + \frac{x_{18}}{3} = 0$$
$$x_{14} + 4x_{16} - 3x_{18} - 1 = 0$$
$$x_{15} + \frac{x_{16}}{3} - \frac{x_{17}}{3} - \frac{4x_{18}}{3} = 0$$
$$x_{16}^2 - \frac{3x_{16}x_{18}}{4} - \frac{3x_{18}}{4} = 0$$
$$x_{16}x_{17} + \frac{35x_{16}x_{18}}{4} + \frac{3x_{16}}{2} + 2x_{17}x_{18} - \frac{3x_{17}}{2} - 7x_{18}^2 - \frac{27x_{18}}{4} = 0$$
$$x_{16}x_{18}^2 - \frac{27x_{16}x_{18}}{196} - \frac{9x_{16}}{98} + \frac{11x_{17}x_{18}^2}{49} - \frac{9x_{17}x_{18}}{28} + \frac{9x_{17}}{98} - \frac{11x_{18}^3}{14} - \frac{117x_{18}^2}{196} + \frac{18x_{18}}{49} = 0$$
$$x_{17}^2x_{18}^2 - \frac{63x_{17}^2x_{18}}{44} + \frac{9x_{17}^2}{22} - \frac{161x_{17}x_{18}^3}{44} - \frac{1017x_{17}x_{18}^2}{176} + \frac{261x_{17}x_{18}}{88} + \frac{49x_{18}^4}{88} - \frac{483x_{18}^3}{176} + \frac{585x_{18}^2}{176} - \frac{27x_{18}}{88} = 0$$

$\mathfrak{m}_{1A}(4,13)$

m1A413 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_9]=e_{13}$$
$$[e_3,e_8]=-e_{13} \qquad [e_4,e_7]=e_{13}$$
$$[e_5,e_6]=-e_{13}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4,13)$

m3A413 (this line included for string searching purposes)
Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_7]=e_{11}$$
$$[e_2,e_8]=3e_{12} \qquad [e_2,e_9]=\alpha_{2,9}^{13}e_{13}$$
$$[e_3,e_6]=-e_{11} \qquad [e_3,e_7]=-2e_{12}$$
$$[e_3,e_8]=\alpha_{3,8}^{13}e_{13} \qquad [e_4,e_5]=e_{11}$$
$$[e_4,e_6]=e_{12} \qquad [e_4,e_7]=\alpha_{4,7}^{13}e_{13}$$
$$[e_5,e_6]=\alpha_{5,6}^{13}e_{13}$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_8): \quad -\alpha_{2,9}^{13}-\alpha_{3,8}^{13}+3 = 0$$
$$(e_1,e_3,e_7): \quad -\alpha_{3,8}^{13}-\alpha_{4,7}^{13}-2 = 0$$
$$(e_1,e_4,e_6): \quad -\alpha_{4,7}^{13}-\alpha_{5,6}^{13}+1 = 0$$

Infinite number of solutions.
How the solution(s) were or were not found:

Change variables

$$\alpha_{5,6}^{13} \to x_1$$
$$\alpha_{4,7}^{13} \to x_2$$
$$\alpha_{2,9}^{13} \to x_3$$
$$\alpha_{3,8}^{13} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - x_4 + 3 &= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_4 - 2 &= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_2 + 1 &= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 - 3 = 0$$
$$x_2 + x_4 + 2 = 0$$
$$x_3 + x_4 - 3 = 0$$

$\mathfrak{m}_{5A}(4, 13)$

m5A413 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_9\\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11}\\
[e_2, e_8] &= \alpha_{2,8}^{12} e_{12} & [e_2, e_9] &= \alpha_{2,9}^{13} e_{13}\\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10}\\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12}\\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11}\\
[e_4, e_6] &= \alpha_{4,6}^{12} e_{12} & [e_4, e_7] &= \alpha_{4,7}^{13} e_{13}\\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,9}^{13} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{11} \to x_1$$

$$\alpha_{2,7}^{11} \to x_2$$

$$\alpha_{2,8}^{12} \to x_3$$

$$\alpha_{5,6}^{13} \to x_4$$

$$\alpha_{3,7}^{12} \to x_5$$

$$\alpha_{3,8}^{13} \to x_6$$

$$\alpha_{2,9}^{13} \to x_7$$

$$\alpha_{3,6}^{11} \to x_8$$

$$\alpha_{4,7}^{13} \to x_9$$

$$\alpha_{4,6}^{12} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_8 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_1 - x_8 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad x_2 - x_3 - x_5 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_5 + x_8 &&= 0\\
(e_1, e_4, e_5): &\quad x_1 - x_{10} &&= 0\\
(e_1, e_2, e_8): &\quad x_3 - x_6 - x_7 &&= 0\\
(e_1, e_3, e_7): &\quad x_5 - x_6 - x_9 &&= 0\\
(e_1, e_4, e_6): &\quad x_{10} - x_4 - x_9 &&= 0\\
(e_2, e_3, e_4): &\quad -x_7 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 9 linear, 0 nonlinear)

$$x_1 - x_{10} = 0$$

$$-x_{10} + x_2 - 3 = 0$$

$$-3x_{10} + x_3 - 4 = 0$$

$$-6x_{10} + x_4 - 5 = 0$$

$$2x_{10} + x_5 + 1 = 0$$

$$-3x_{10} + x_6 - 4 = 0$$

$$x_7 = 0$$

$$x_{10} + x_8 + 1 = 0$$

$$5x_{10} + x_9 + 5 = 0$$

$\mathfrak{m}_{7A}(4, 13)$

m7A413 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_7 \\
[e_2, e_4] &= e_8 & [e_2, e_5] &= \alpha_{2,5}^{9} e_9 \\
[e_2, e_6] &= \alpha_{2,6}^{10} e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_2, e_8] &= \alpha_{2,8}^{12} e_{12} & [e_2, e_9] &= \alpha_{2,9}^{13} e_{13} \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11} \\
[e_4, e_6] &= \alpha_{4,6}^{12} e_{12} & [e_4, e_7] &= \alpha_{4,7}^{13} e_{13} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &= 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &= 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &= 0 \\
(e_2, e_3, e_4): &\quad \alpha_{2,9}^{13}\alpha_{3,4}^{9} - \alpha_{3,8}^{13} + \alpha_{4,7}^{13} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{10} \to x_1$$

$$\alpha^{9}_{2,5} \to x_2$$
$$\alpha^{9}_{3,4} \to x_3$$
$$\alpha^{11}_{4,5} \to x_4$$
$$\alpha^{11}_{2,7} \to x_5$$
$$\alpha^{12}_{2,8} \to x_6$$
$$\alpha^{13}_{5,6} \to x_7$$
$$\alpha^{12}_{3,7} \to x_8$$
$$\alpha^{10}_{3,5} \to x_9$$
$$\alpha^{13}_{3,8} \to x_{10}$$
$$\alpha^{13}_{2,9} \to x_{11}$$
$$\alpha^{11}_{3,6} \to x_{12}$$
$$\alpha^{13}_{4,7} \to x_{13}$$
$$\alpha^{12}_{4,6} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_2 - x_3 + 1 && = 0\\
(e_1, e_2, e_5):&\quad -x_1 + x_2 - x_9 && = 0\\
(e_1, e_3, e_4):&\quad x_3 - x_9 && = 0\\
(e_1, e_2, e_6):&\quad x_1 - x_{12} - x_5 && = 0\\
(e_1, e_3, e_5):&\quad -x_{12} - x_4 + x_9 && = 0\\
(e_1, e_2, e_7):&\quad x_5 - x_6 - x_8 && = 0\\
(e_1, e_3, e_6):&\quad x_{12} - x_{14} - x_8 && = 0\\
(e_1, e_4, e_5):&\quad -x_{14} + x_4 && = 0\\
(e_1, e_2, e_8):&\quad -x_{10} - x_{11} + x_6 && = 0\\
(e_1, e_3, e_7):&\quad -x_{10} - x_{13} + x_8 && = 0\\
(e_1, e_4, e_6):&\quad -x_{13} + x_{14} - x_7 && = 0\\
(e_2, e_3, e_4):&\quad -x_{10} + x_{11}x_3 + x_{13} && = 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 1 nonlinear)

$$x_1 + 2x_{12} + 2x_{14} - 1 = 0$$
$$x_{12} + x_{14} + x_2 - 1 = 0$$
$$-x_{12} - x_{14} + x_3 = 0$$

$$-x_{14} + x_4 = 0$$
$$3x_{12} + 2x_{14} + x_5 - 1 = 0$$
$$4x_{12} + x_{14} + x_6 - 1 = 0$$
$$x_{13} - x_{14} + x_7 = 0$$
$$-x_{12} + x_{14} + x_8 = 0$$
$$-x_{12} - x_{14} + x_9 = 0$$
$$x_{10} - x_{12} + x_{13} + x_{14} = 0$$
$$x_{11} + 5x_{12} - x_{13} - 1 = 0$$
$$x_{12}^2 - \frac{x_{12}x_{13}}{5} + x_{12}x_{14} - \frac{x_{13}x_{14}}{5} - \frac{2x_{13}}{5} - \frac{2x_{14}}{5} = 0$$

$\mathfrak{m}_{2A}(5, 13)$

m2A513 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{12} \\
[e_2, e_8] &= 3e_{13} & [e_3, e_6] &= -e_{12} \\
[e_3, e_7] &= -2e_{13} & [e_4, e_5] &= e_{12} \\
[e_4, e_6] &= e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(5, 13)$

m4A513 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{10} \\
[e_2, e_6] &= 2e_{11} & [e_2, e_7] &= \alpha_{2,7}^{12} e_{12} \\
[e_2, e_8] &= \alpha_{2,8}^{13} e_{13} & [e_3, e_4] &= -e_{10} \\
[e_3, e_5] &= -e_{11} & [e_3, e_6] &= \alpha_{3,6}^{12} e_{12} \\
[e_3, e_7] &= \alpha_{3,7}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{12} e_{12} \\
[e_4, e_6] &= \alpha_{4,6}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,6}^{13} &\to x_1 \\
\alpha_{4,5}^{12} &\to x_2 \\
\alpha_{2,8}^{13} &\to x_3 \\
\alpha_{3,7}^{13} &\to x_4 \\
\alpha_{2,7}^{12} &\to x_5 \\
\alpha_{3,6}^{12} &\to x_6
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -x_5-x_6+2 &&=0\\
(e_1,e_3,e_5):&\quad -x_2-x_6-1 &&=0\\
(e_1,e_2,e_7):&\quad -x_3-x_4+x_5 &&=0\\
(e_1,e_3,e_6):&\quad -x_1-x_4+x_6 &&=0\\
(e_1,e_4,e_5):&\quad -x_1+x_2 &&=0
\end{aligned}
$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$x_1+x_6+1=0$$
$$x_2+x_6+1=0$$
$$x_3+3x_6-1=0$$
$$x_4-2x_6-1=0$$
$$x_5+x_6-2=0$$

$\mathfrak{m}_{6A}(5,13)$

m6A513 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2]&=e_3 & [e_1,e_3]&=e_4\\
[e_1,e_4]&=e_5 & [e_1,e_5]&=e_6\\
[e_1,e_6]&=e_7 & [e_1,e_7]&=e_8\\
[e_1,e_8]&=e_9 & [e_1,e_9]&=e_{10}\\
[e_1,e_{10}]&=e_{11} & [e_1,e_{11}]&=e_{12}\\
[e_1,e_{12}]&=e_{13} & [e_2,e_3]&=e_8\\
[e_2,e_4]&=e_9 & [e_2,e_5]&=\alpha_{2,5}^{10}e_{10}\\
[e_2,e_6]&=\alpha_{2,6}^{11}e_{11} & [e_2,e_7]&=\alpha_{2,7}^{12}e_{12}\\
[e_2,e_8]&=\alpha_{2,8}^{13}e_{13} & [e_3,e_4]&=\alpha_{3,4}^{10}e_{10}\\
[e_3,e_5]&=\alpha_{3,5}^{11}e_{11} & [e_3,e_6]&=\alpha_{3,6}^{12}e_{12}\\
[e_3,e_7]&=\alpha_{3,7}^{13}e_{13} & [e_4,e_5]&=\alpha_{4,5}^{12}e_{12}\\
[e_4,e_6]&=\alpha_{4,6}^{13}e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &= 0\\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &= 0\\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &= 0\\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha_{4,6}^{13} &\to x_1\\
\alpha_{4,5}^{12} &\to x_2\\
\alpha_{3,5}^{11} &\to x_3\\
\alpha_{2,8}^{13} &\to x_4\\
\alpha_{2,5}^{10} &\to x_5\\
\alpha_{3,7}^{13} &\to x_6\\
\alpha_{3,6}^{12} &\to x_7\\
\alpha_{2,7}^{12} &\to x_8\\
\alpha_{3,4}^{10} &\to x_9\\
\alpha_{2,6}^{11} &\to x_{10}
\end{aligned}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_5 - x_9 + 1 &= 0\\
(e_1, e_2, e_5):&\quad -x_{10} - x_3 + x_5 &= 0\\
(e_1, e_3, e_4):&\quad -x_3 + x_9 &= 0\\
(e_1, e_2, e_6):&\quad x_{10} - x_7 - x_8 &= 0\\
(e_1, e_3, e_5):&\quad -x_2 + x_3 - x_7 &= 0\\
(e_1, e_2, e_7):&\quad -x_4 - x_6 + x_8 &= 0\\
(e_1, e_3, e_6):&\quad -x_1 - x_6 + x_7 &= 0\\
(e_1, e_4, e_5):&\quad -x_1 + x_2 &= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + \frac{3x_{10}}{2} - x_8 - \frac{1}{2} = 0$$
$$\frac{3x_{10}}{2} + x_2 - x_8 - \frac{1}{2} = 0$$
$$\frac{x_{10}}{2} + x_3 - \frac{1}{2} = 0$$
$$\frac{5x_{10}}{2} + x_4 - 3x_8 - \frac{1}{2} = 0$$
$$-\frac{x_{10}}{2} + x_5 - \frac{1}{2} = 0$$
$$-\frac{5x_{10}}{2} + x_6 + 2x_8 + \frac{1}{2} = 0$$
$$-x_{10} + x_7 + x_8 = 0$$
$$\frac{x_{10}}{2} + x_9 - \frac{1}{2} = 0$$

$\mathfrak{m}_{1A}(6, 13)$

m1A613 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_7] = e_{13}$$
$$[e_3, e_6] = -e_{13} \qquad [e_4, e_5] = e_{13}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(6, 13)$

m3A613 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{11} \\
[e_2, e_6] &= 2e_{12} & [e_2, e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_3, e_4] &= -e_{11} & [e_3, e_5] &= -e_{12} \\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_4, e_5] &= \alpha_{4,5}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{13} - \alpha_{3,6}^{13} + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{13} - \alpha_{4,5}^{13} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{13} \to x_1$$

$$\alpha_{2,7}^{13} \to x_2$$

$$\alpha_{3,6}^{13} \to x_3$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_3 + 2 &&= 0 \\
(e_1, e_3, e_5): &\quad -x_1 - x_3 - 1 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 + x_3 + 1 = 0$$

$$x_2 + x_3 - 2 = 0$$

$\mathfrak{m}_{5A}(6,13)$

m5A613 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_2,e_3] &= e_9 \\
[e_2,e_4] &= e_{10} & [e_2,e_5] &= \alpha_{2,5}^{11} e_{11} \\
[e_2,e_6] &= \alpha_{2,6}^{12} e_{12} & [e_2,e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_3,e_4] &= \alpha_{3,4}^{11} e_{11} & [e_3,e_5] &= \alpha_{3,5}^{12} e_{12} \\
[e_3,e_6] &= \alpha_{3,6}^{13} e_{13} & [e_4,e_5] &= \alpha_{4,5}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &= 0 \\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &= 0 \\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &= 0 \\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{12} - \alpha_{2,7}^{13} - \alpha_{3,6}^{13} &= 0 \\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{12} - \alpha_{3,6}^{13} - \alpha_{4,5}^{13} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,6}^{12} &\to x_1 \\
\alpha_{4,5}^{13} &\to x_2 \\
\alpha_{3,6}^{13} &\to x_3 \\
\alpha_{2,7}^{13} &\to x_4 \\
\alpha_{3,4}^{11} &\to x_5 \\
\alpha_{2,5}^{11} &\to x_6 \\
\alpha_{3,5}^{12} &\to x_7
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_5 - x_6 + 1 &= 0\\
(e_1, e_2, e_5):&\quad -x_1 + x_6 - x_7 &= 0\\
(e_1, e_3, e_4):&\quad x_5 - x_7 &= 0\\
(e_1, e_2, e_6):&\quad x_1 - x_3 - x_4 &= 0\\
(e_1, e_3, e_5):&\quad -x_2 - x_3 + x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 + 2x_7 - 1 = 0$$

$$x_2 - x_4 - 3x_7 + 1 = 0$$

$$x_3 + x_4 + 2x_7 - 1 = 0$$

$$x_5 - x_7 = 0$$

$$x_6 + x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(7, 13)$

m2A713 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{12}\\
[e_2, e_6] &= 2e_{13} & [e_3, e_4] &= -e_{12}\\
[e_3, e_5] &= -e_{13}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(7, 13)$

m4A713 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_5] &= \alpha_{2,5}^{12} e_{12} \\
[e_2, e_6] &= \alpha_{2,6}^{13} e_{13} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12} \\
[e_3, e_5] &= \alpha_{3,5}^{13} e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{12} - \alpha_{3,5}^{13} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{2,6}^{13} &\to x_1 \\
\alpha_{3,4}^{12} &\to x_2 \\
\alpha_{2,5}^{12} &\to x_3 \\
\alpha_{3,5}^{13} &\to x_4
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_3 + 1 &= 0 \\
(e_1, e_2, e_5): &\quad -x_1 + x_3 - x_4 &= 0 \\
(e_1, e_3, e_4): &\quad x_2 - x_4 &= 0
\end{aligned}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 + 2x_4 - 1 &= 0 \\
x_2 - x_4 &= 0 \\
x_3 + x_4 - 1 &= 0
\end{aligned}
$$

$\mathfrak{m}_{1A}(8,13)$

m1A813 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_2,e_5] &= e_{13}\\
[e_3,e_4] &= -e_{13}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(8,13)$

m3A813 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_2,e_3] &= e_{11}\\
[e_2,e_4] &= e_{12} & [e_2,e_5] &= \alpha_{2,5}^{13}e_{13}\\
[e_3,e_4] &= \alpha_{3,4}^{13}e_{13}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_4): \quad -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,5}^{13} &\to x_1\\
\alpha_{3,4}^{13} &\to x_2
\end{aligned}
$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(9, 13)$

m2A913 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_{12}$$
$$[e_2, e_4] = e_{13}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(10, 13)$

m1A1013 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_{13}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2A}(2, 14)$

m2A214 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_{11}] &= e_{13} & [e_2, e_{12}] &= 5e_{14} \\
[e_3, e_{10}] &= -e_{13} & [e_3, e_{11}] &= -4e_{14} \\
[e_4, e_9] &= e_{13} & [e_4, e_{10}] &= 3e_{14} \\
[e_5, e_8] &= -e_{13} & [e_5, e_9] &= -2e_{14} \\
[e_6, e_7] &= e_{13} & [e_6, e_8] &= e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(2, 14)$

m4A214 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_9] &= e_{11} & [e_2, e_{10}] &= 4e_{12} \\
[e_2, e_{11}] &= \alpha_{2,11}^{13} e_{13} & [e_2, e_{12}] &= \alpha_{2,12}^{14} e_{14} \\
[e_3, e_8] &= -e_{11} & [e_3, e_9] &= -3e_{12} \\
[e_3, e_{10}] &= \alpha_{3,10}^{13} e_{13} & [e_3, e_{11}] &= \alpha_{3,11}^{14} e_{14} \\
[e_4, e_7] &= e_{11} & [e_4, e_8] &= 2e_{12} \\
[e_4, e_9] &= \alpha_{4,9}^{13} e_{13} & [e_4, e_{10}] &= \alpha_{4,10}^{14} e_{14} \\
[e_5, e_6] &= -e_{11} & [e_5, e_7] &= -e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{13} e_{13} & [e_5, e_9] &= \alpha_{5,9}^{14} e_{14} \\
[e_6, e_7] &= \alpha_{6,7}^{13} e_{13} & [e_6, e_8] &= \alpha_{6,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}):\quad & -\alpha_{2,11}^{13} - \alpha_{3,10}^{13} + 4 &&= 0\\
(e_1, e_3, e_9):\quad & -\alpha_{3,10}^{13} - \alpha_{4,9}^{13} - 3 &&= 0\\
(e_1, e_4, e_8):\quad & -\alpha_{4,9}^{13} - \alpha_{5,8}^{13} + 2 &&= 0\\
(e_1, e_5, e_7):\quad & -\alpha_{5,8}^{13} - \alpha_{6,7}^{13} - 1 &&= 0\\
(e_2, e_3, e_8):\quad & -\alpha_{2,11}^{13} &&= 0\\
(e_2, e_4, e_7):\quad & \alpha_{2,11}^{13} &&= 0\\
(e_2, e_5, e_6):\quad & -\alpha_{2,11}^{13} &&= 0\\
(e_1, e_2, e_{11}):\quad & \alpha_{2,11}^{13} - \alpha_{2,12}^{14} - \alpha_{3,11}^{14} &&= 0\\
(e_1, e_3, e_{10}):\quad & \alpha_{3,10}^{13} - \alpha_{3,11}^{14} - \alpha_{4,10}^{14} &&= 0\\
(e_1, e_4, e_9):\quad & -\alpha_{4,10}^{14} + \alpha_{4,9}^{13} - \alpha_{5,9}^{14} &&= 0\\
(e_1, e_5, e_8):\quad & \alpha_{5,8}^{13} - \alpha_{5,9}^{14} - \alpha_{6,8}^{14} &&= 0\\
(e_1, e_6, e_7):\quad & \alpha_{6,7}^{13} - \alpha_{6,8}^{14} &&= 0\\
(e_2, e_3, e_9):\quad & -3\alpha_{2,12}^{14} - \alpha_{3,11}^{14} &&= 0\\
(e_2, e_4, e_8):\quad & 2\alpha_{2,12}^{14} &&= 0\\
(e_2, e_5, e_7):\quad & -\alpha_{2,12}^{14} &&= 0\\
(e_3, e_4, e_7):\quad & \alpha_{3,11}^{14} &&= 0\\
(e_3, e_5, e_6):\quad & -\alpha_{3,11}^{14} &&= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{5,9}^{14} \to x_1$$

$$\alpha_{2,11}^{13} \to x_2$$

$$\alpha_{6,8}^{14} \to x_3$$

$$\alpha_{5,8}^{13} \to x_4$$

$$\alpha_{4,10}^{14} \to x_5$$

$$\alpha_{3,10}^{13} \to x_6$$

$$\alpha_{2,12}^{14} \to x_7$$

$$\alpha_{3,11}^{14} \to x_8$$

$$\alpha_{6,7}^{13} \to x_9$$

$$\alpha_{4,9}^{13} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_2 - x_6 + 4 && = 0 \\
(e_1, e_3, e_9): &\quad -x_{10} - x_6 - 3 && = 0 \\
(e_1, e_4, e_8): &\quad -x_{10} - x_4 + 2 && = 0 \\
(e_1, e_5, e_7): &\quad -x_4 - x_9 - 1 && = 0 \\
(e_2, e_3, e_8): &\quad -x_2 && = 0 \\
(e_2, e_4, e_7): &\quad x_2 && = 0 \\
(e_2, e_5, e_6): &\quad -x_2 && = 0 \\
(e_1, e_2, e_{11}): &\quad x_2 - x_7 - x_8 && = 0 \\
(e_1, e_3, e_{10}): &\quad -x_5 + x_6 - x_8 && = 0 \\
(e_1, e_4, e_9): &\quad -x_1 + x_{10} - x_5 && = 0 \\
(e_1, e_5, e_8): &\quad -x_1 - x_3 + x_4 && = 0 \\
(e_1, e_6, e_7): &\quad -x_3 + x_9 && = 0 \\
(e_2, e_3, e_9): &\quad -3x_7 - x_8 && = 0 \\
(e_2, e_4, e_8): &\quad 2x_7 && = 0 \\
(e_2, e_5, e_7): &\quad -x_7 && = 0 \\
(e_3, e_4, e_7): &\quad x_8 && = 0 \\
(e_3, e_5, e_6): &\quad -x_8 && = 0
\end{aligned}
$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{10A}(2, 14)$

m10A214 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_2, e_3] = e_5 && [e_2, e_4] = e_6 \\
&[e_2, e_5] = e_7 && [e_2, e_6] = e_8 \\
&[e_2, e_7] = e_9 && [e_2, e_8] = e_{10} \\
&[e_2, e_9] = e_{11} && [e_2, e_{10}] = e_{12} \\
&[e_2, e_{11}] = e_{13} && [e_2, e_{12}] = e_{14} \\
&[e_3, e_4] = 0 && [e_3, e_5] = 0 \\
&[e_3, e_6] = 0 && [e_3, e_7] = 0 \\
&[e_3, e_8] = 0 && [e_3, e_9] = 0 \\
&[e_3, e_{10}] = 0 && [e_3, e_{11}] = 0 \\
&[e_4, e_5] = 0 && [e_4, e_6] = 0 \\
&[e_4, e_7] = 0 && [e_4, e_8] = 0 \\
&[e_4, e_9] = 0 && [e_4, e_{10}] = 0 \\
&[e_5, e_6] = 0 && [e_5, e_7] = 0 \\
&[e_5, e_8] = 0 && [e_5, e_9] = 0 \\
&[e_6, e_7] = 0 && [e_6, e_8] = 0
\end{aligned}
$$

Solution 2

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4\\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6\\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8\\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10}\\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12}\\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14}\\
&[e_2, e_3] = e_5 && [e_2, e_4] = e_6\\
&[e_2, e_5] = \frac{9e_7}{10} && [e_2, e_6] = \frac{4e_8}{5}\\
&[e_2, e_7] = \frac{5e_9}{7} && [e_2, e_8] = \frac{9e_{10}}{14}\\
&[e_2, e_9] = \frac{7e_{11}}{12} && [e_2, e_{10}] = \frac{8e_{12}}{15}\\
&[e_2, e_{11}] = \frac{27e_{13}}{55} && [e_2, e_{12}] = \frac{5e_{14}}{11}\\
&[e_3, e_4] = \frac{e_7}{10} && [e_3, e_5] = \frac{e_8}{10}\\
&[e_3, e_6] = \frac{3e_9}{35} && [e_3, e_7] = \frac{e_{10}}{14}\\
&[e_3, e_8] = \frac{5e_{11}}{84} && [e_3, e_9] = \frac{e_{12}}{20}\\
&[e_3, e_{10}] = \frac{7e_{13}}{165} && [e_3, e_{11}] = \frac{2e_{14}}{55}\\
&[e_4, e_5] = \frac{e_9}{70} && [e_4, e_6] = \frac{e_{10}}{70}\\
&[e_4, e_7] = \frac{e_{11}}{84} && [e_4, e_8] = \frac{e_{12}}{105}\\
&[e_4, e_9] = \frac{e_{13}}{132} && [e_4, e_{10}] = \frac{e_{14}}{165}\\
&[e_5, e_6] = \frac{e_{11}}{420} && [e_5, e_7] = \frac{e_{12}}{420}\\
&[e_5, e_8] = \frac{3e_{13}}{1540} && [e_5, e_9] = \frac{e_{14}}{660}\\
&[e_6, e_7] = \frac{e_{13}}{2310} && [e_6, e_8] = \frac{e_{14}}{2310}
\end{aligned}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_1,e_{13}]=e_{14} \\
[e_2,e_3]=e_5 & [e_2,e_4]=e_6 \\
[e_2,e_5]=\alpha_{2,5}^{7}e_7 & [e_2,e_6]=\alpha_{2,6}^{8}e_8 \\
[e_2,e_7]=\alpha_{2,7}^{9}e_9 & [e_2,e_8]=\alpha_{2,8}^{10}e_{10} \\
[e_2,e_9]=\alpha_{2,9}^{11}e_{11} & [e_2,e_{10}]=\alpha_{2,10}^{12}e_{12} \\
[e_2,e_{11}]=\alpha_{2,11}^{13}e_{13} & [e_2,e_{12}]=\alpha_{2,12}^{14}e_{14} \\
[e_3,e_4]=\alpha_{3,4}^{7}e_7 & [e_3,e_5]=\alpha_{3,5}^{8}e_8 \\
[e_3,e_6]=\alpha_{3,6}^{9}e_9 & [e_3,e_7]=\alpha_{3,7}^{10}e_{10} \\
[e_3,e_8]=\alpha_{3,8}^{11}e_{11} & [e_3,e_9]=\alpha_{3,9}^{12}e_{12} \\
[e_3,e_{10}]=\alpha_{3,10}^{13}e_{13} & [e_3,e_{11}]=\alpha_{3,11}^{14}e_{14} \\
[e_4,e_5]=\alpha_{4,5}^{9}e_9 & [e_4,e_6]=\alpha_{4,6}^{10}e_{10} \\
[e_4,e_7]=\alpha_{4,7}^{11}e_{11} & [e_4,e_8]=\alpha_{4,8}^{12}e_{12} \\
[e_4,e_9]=\alpha_{4,9}^{13}e_{13} & [e_4,e_{10}]=\alpha_{4,10}^{14}e_{14} \\
[e_5,e_6]=\alpha_{5,6}^{11}e_{11} & [e_5,e_7]=\alpha_{5,7}^{12}e_{12} \\
[e_5,e_8]=\alpha_{5,8}^{13}e_{13} & [e_5,e_9]=\alpha_{5,9}^{14}e_{14} \\
[e_6,e_7]=\alpha_{6,7}^{13}e_{13} & [e_6,e_8]=\alpha_{6,8}^{14}e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{7}_{2,5}-\alpha^{7}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{7}_{2,5}-\alpha^{8}_{2,6}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{7}_{3,4}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{8}_{2,6}-\alpha^{9}_{2,7}-\alpha^{9}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{8}_{3,5}-\alpha^{9}_{3,6}-\alpha^{9}_{4,5} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{9}_{2,7}\alpha^{7}_{3,4}-\alpha^{9}_{3,6}+\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{9}_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{9}_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{9}_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha^{8}_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^{9}_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^{9}_{4,5}+\alpha^{11}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{12}_{2,10}+\alpha^{11}_{2,9}-\alpha^{12}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{11}_{3,8}-\alpha^{12}_{3,9}-\alpha^{12}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{11}_{4,7}-\alpha^{12}_{4,8}-\alpha^{12}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{11}_{5,6}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_3,e_7):&\quad \alpha^{12}_{2,10}\alpha^{10}_{3,7}-\alpha^{9}_{2,7}\alpha^{12}_{3,9}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_4,e_6):&\quad \alpha^{12}_{2,10}\alpha^{10}_{4,6}-\alpha^{8}_{2,6}\alpha^{12}_{4,8} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^{7}_{3,4}\alpha^{12}_{5,7}-\alpha^{8}_{3,5}\alpha^{12}_{4,8}+\alpha^{12}_{3,9}\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha^{12}_{2,10}-\alpha^{13}_{2,11}-\alpha^{13}_{3,10} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{13}_{3,10}+\alpha^{12}_{3,9}-\alpha^{13}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad \alpha^{12}_{4,8}-\alpha^{13}_{4,9}-\alpha^{13}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad \alpha^{12}_{5,7}-\alpha^{13}_{5,8}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_3,e_8):&\quad \alpha^{13}_{2,11}\alpha^{11}_{3,8}-\alpha^{10}_{2,8}\alpha^{13}_{3,10}-\alpha^{13}_{5,8} &&=0\\
(e_2,e_4,e_7):&\quad \alpha^{13}_{2,11}\alpha^{11}_{4,7}-\alpha^{9}_{2,7}\alpha^{13}_{4,9}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_5,e_6):&\quad \alpha^{13}_{2,11}\alpha^{11}_{5,6}+\alpha^{7}_{2,5}\alpha^{13}_{6,7}-\alpha^{8}_{2,6}\alpha^{13}_{5,8} &&=0\\
(e_3,e_4,e_6):&\quad \alpha^{13}_{3,10}\alpha^{10}_{4,6}+\alpha^{7}_{3,4}\alpha^{13}_{6,7}-\alpha^{9}_{3,6}\alpha^{13}_{4,9} &&=0\\
(e_1,e_2,e_{11}):&\quad \alpha^{13}_{2,11}-\alpha^{14}_{2,12}-\alpha^{14}_{3,11} &&=0\\
(e_1,e_3,e_{10}):&\quad \alpha^{13}_{3,10}-\alpha^{14}_{3,11}-\alpha^{14}_{4,10} &&=0\\
(e_1,e_4,e_9):&\quad -\alpha^{14}_{4,10}+\alpha^{13}_{4,9}-\alpha^{14}_{5,9} &&=0\\
(e_1,e_5,e_8):&\quad \alpha^{13}_{5,8}-\alpha^{14}_{5,9}-\alpha^{14}_{6,8} &&=0\\
(e_1,e_6,e_7):&\quad \alpha^{13}_{6,7}-\alpha^{14}_{6,8} &&=0\\
(e_2,e_3,e_9):&\quad \alpha^{14}_{2,12}\alpha^{12}_{3,9}-\alpha^{11}_{2,9}\alpha^{14}_{3,11}-\alpha^{14}_{5,9} &&=0\\
(e_2,e_4,e_8):&\quad \alpha^{14}_{2,12}\alpha^{12}_{4,8}-\alpha^{10}_{2,8}\alpha^{14}_{4,10}-\alpha^{14}_{6,8} &&=0\\
(e_2,e_5,e_7):&\quad \alpha^{14}_{2,12}\alpha^{12}_{5,7}-\alpha^{9}_{2,7}\alpha^{14}_{5,9} &&=0\\
(e_3,e_4,e_7):&\quad \alpha^{14}_{3,11}\alpha^{11}_{4,7}-\alpha^{10}_{3,7}\alpha^{14}_{4,10} &&=0\\
(e_3,e_5,e_6):&\quad \alpha^{14}_{3,11}\alpha^{11}_{5,6}+\alpha^{8}_{3,5}\alpha^{14}_{6,8}-\alpha^{9}_{3,6}\alpha^{14}_{5,9} &&=0
\end{aligned}
$$

Solution 1:

$$\begin{aligned}
\alpha^{11}_{3,8} &= 0\\
\alpha^{11}_{4,7} &= 0\\
\alpha^{13}_{6,7} &= 0\\
\alpha^{12}_{5,7} &= 0\\
\alpha^{8}_{2,6} &= 1\\
\alpha^{12}_{3,9} &= 0\\
\alpha^{12}_{4,8} &= 0\\
\alpha^{7}_{2,5} &= 1\\
\alpha^{9}_{2,7} &= 1\\
\alpha^{10}_{3,7} &= 0\\
\alpha^{10}_{2,8} &= 1\\
\alpha^{9}_{4,5} &= 0\\
\alpha^{13}_{3,10} &= 0\\
\alpha^{14}_{6,8} &= 0\\
\alpha^{13}_{5,8} &= 0\\
\alpha^{8}_{3,5} &= 0\\
\alpha^{11}_{5,6} &= 0\\
\alpha^{10}_{4,6} &= 0\\
\alpha^{14}_{3,11} &= 0\\
\alpha^{11}_{2,9} &= 1\\
\alpha^{13}_{2,11} &= 1\\
\alpha^{13}_{4,9} &= 0\\
\alpha^{14}_{5,9} &= 0\\
\alpha^{14}_{4,10} &= 0\\
\alpha^{9}_{3,6} &= 0\\
\alpha^{7}_{3,4} &= 0\\
\alpha^{14}_{2,12} &= 1\\
\alpha^{12}_{2,10} &= 1
\end{aligned}$$

Solution 2:

$$\alpha^{11}_{3,8} = 5/84$$
$$\alpha^{11}_{4,7} = 1/84$$
$$\alpha^{13}_{6,7} = 1/2310$$
$$\alpha^{12}_{5,7} = 1/420$$
$$\alpha^{8}_{2,6} = 4/5$$
$$\alpha^{12}_{3,9} = 1/20$$
$$\alpha^{12}_{4,8} = 1/105$$
$$\alpha^{7}_{2,5} = 9/10$$
$$\alpha^{9}_{2,7} = 5/7$$
$$\alpha^{10}_{3,7} = 1/14$$
$$\alpha^{10}_{2,8} = 9/14$$
$$\alpha^{9}_{4,5} = 1/70$$
$$\alpha^{13}_{3,10} = 7/165$$
$$\alpha^{14}_{6,8} = 1/2310$$
$$\alpha^{13}_{5,8} = 3/1540$$
$$\alpha^{8}_{3,5} = 1/10$$
$$\alpha^{11}_{5,6} = 1/420$$
$$\alpha^{10}_{4,6} = 1/70$$
$$\alpha^{14}_{3,11} = 2/55$$
$$\alpha^{11}_{2,9} = 7/12$$
$$\alpha^{13}_{2,11} = 27/55$$
$$\alpha^{13}_{4,9} = 1/132$$
$$\alpha^{14}_{5,9} = 1/660$$
$$\alpha^{14}_{4,10} = 1/165$$
$$\alpha^{9}_{3,6} = 3/35$$
$$\alpha^{7}_{3,4} = 1/10$$
$$\alpha^{14}_{2,12} = 5/11$$
$$\alpha^{12}_{2,10} = 8/15$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{3,8} \to x_1$$

$$\alpha^{11}_{4,7} \to x_2$$

$$\alpha^{13}_{6,7} \to x_3$$

$$\alpha^{12}_{5,7} \to x_4$$

$$\alpha^{8}_{2,6} \to x_5$$

$$\alpha^{12}_{3,9} \to x_6$$

$$\alpha^{12}_{4,8} \to x_7$$

$$\alpha^{7}_{2,5} \to x_8$$

$$\alpha^{9}_{2,7} \to x_9$$

$$\alpha^{10}_{3,7} \to x_{10}$$

$$\alpha^{10}_{2,8} \to x_{11}$$

$$\alpha^{9}_{4,5} \to x_{12}$$

$$\alpha^{13}_{3,10} \to x_{13}$$

$$\alpha^{14}_{6,8} \to x_{14}$$

$$\alpha^{13}_{5,8} \to x_{15}$$

$$\alpha^{8}_{3,5} \to x_{16}$$

$$\alpha^{11}_{5,6} \to x_{17}$$

$$\alpha^{10}_{4,6} \to x_{18}$$

$$\alpha^{14}_{3,11} \to x_{19}$$

$$\alpha^{11}_{2,9} \to x_{20}$$

$$\alpha^{13}_{2,11} \to x_{21}$$

$$\alpha^{13}_{4,9} \to x_{22}$$

$$\alpha^{14}_{5,9} \to x_{23}$$

$$\alpha^{14}_{4,10} \to x_{24}$$

$$\alpha^{9}_{3,6} \to x_{25}$$

$$\alpha^{7}_{3,4} \to x_{26}$$

$$\alpha^{14}_{2,12} \to x_{27}$$

$$\alpha^{12}_{2,10} \to x_{28}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{26}-x_8+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{16}-x_5+x_8 &&=0\\
(e_1,e_3,e_4):&\quad -x_{16}+x_{26} &&=0\\
(e_1,e_2,e_6):&\quad -x_{25}+x_5-x_9 &&=0\\
(e_1,e_3,e_5):&\quad -x_{12}+x_{16}-x_{25} &&=0\\
(e_2,e_3,e_4):&\quad x_{12}-x_{25}+x_{26}x_9 &&=0\\
(e_1,e_2,e_7):&\quad -x_{10}-x_{11}+x_9 &&=0\\
(e_1,e_3,e_6):&\quad -x_{10}-x_{18}+x_{25} &&=0\\
(e_1,e_4,e_5):&\quad x_{12}-x_{18} &&=0\\
(e_2,e_3,e_5):&\quad -x_{10}x_8+x_{11}x_{16} &&=0\\
(e_1,e_2,e_8):&\quad -x_1+x_{11}-x_{20} &&=0\\
(e_1,e_3,e_7):&\quad -x_1+x_{10}-x_2 &&=0\\
(e_1,e_4,e_6):&\quad -x_{17}+x_{18}-x_2 &&=0\\
(e_2,e_3,e_6):&\quad -x_1x_5-x_{17}+x_{20}x_{25} &&=0\\
(e_2,e_4,e_5):&\quad x_{12}x_{20}+x_{17}-x_2x_8 &&=0\\
(e_1,e_2,e_9):&\quad x_{20}-x_{28}-x_6 &&=0\\
(e_1,e_3,e_8):&\quad x_1-x_6-x_7 &&=0\\
(e_1,e_4,e_7):&\quad x_2-x_4-x_7 &&=0\\
(e_1,e_5,e_6):&\quad x_{17}-x_4 &&=0\\
(e_2,e_3,e_7):&\quad x_{10}x_{28}-x_4-x_6x_9 &&=0\\
(e_2,e_4,e_6):&\quad x_{18}x_{28}-x_5x_7 &&=0\\
(e_3,e_4,e_5):&\quad x_{12}x_6-x_{16}x_7+x_{26}x_4 &&=0\\
(e_1,e_2,e_{10}):&\quad -x_{13}-x_{21}+x_{28} &&=0\\
(e_1,e_3,e_9):&\quad -x_{13}-x_{22}+x_6 &&=0\\
(e_1,e_4,e_8):&\quad -x_{15}-x_{22}+x_7 &&=0\\
(e_1,e_5,e_7):&\quad -x_{15}-x_3+x_4 &&=0\\
(e_2,e_3,e_8):&\quad x_1x_{21}-x_{11}x_{13}-x_{15} &&=0\\
(e_2,e_4,e_7):&\quad x_2x_{21}-x_{22}x_9-x_3 &&=0\\
(e_2,e_5,e_6):&\quad -x_{15}x_5+x_{17}x_{21}+x_3x_8 &&=0\\
(e_3,e_4,e_6):&\quad x_{13}x_{18}-x_{22}x_{25}+x_{26}x_3 &&=0\\
(e_1,e_2,e_{11}):&\quad -x_{19}+x_{21}-x_{27} &&=0\\
(e_1,e_3,e_{10}):&\quad x_{13}-x_{19}-x_{24} &&=0\\
(e_1,e_4,e_9):&\quad x_{22}-x_{23}-x_{24} &&=0\\
(e_1,e_5,e_8):&\quad -x_{14}+x_{15}-x_{23} &&=0\\
(e_1,e_6,e_7):&\quad -x_{14}+x_3 &&=0\\
(e_2,e_3,e_9):&\quad -x_{19}x_{20}-x_{23}+x_{27}x_6 &&=0\\
(e_2,e_4,e_8):&\quad -x_{11}x_{24}-x_{14}+x_{27}x_7 &&=0\\
(e_2,e_5,e_7):&\quad -x_{23}x_9+x_{27}x_4 &&=0\\
(e_3,e_4,e_7):&\quad -x_{10}x_{24}+x_{19}x_2 &&=0\\
(e_3,e_5,e_6):&\quad x_{14}x_{16}+x_{17}x_{19}-x_{23}x_{25} &&=0
\end{aligned}
$$

Groebner basis (28 variables, 0 linear, 28 nonlinear)

$$x_1-\frac{9442385x_{28}^5}{203297472}+\frac{39791369x_{28}^4}{152473104}-\frac{187989481x_{28}^3}{304946208}+\frac{4537949x_{28}^2}{5647152}-\frac{245637491x_{28}}{609892416}+\frac{84955}{76236552}=0$$

$$x_2+\frac{2915995x_{28}^5}{135531648}-\frac{3049927x_{28}^4}{25412184}+\frac{57387209x_{28}^3}{203297472}-\frac{360999x_{28}^2}{941192}+\frac{119035285x_{28}}{406594944}-\frac{4725911}{50824368}=0$$

$$\frac{2668075x_{28}^5}{372712032}-\frac{11622955x_{28}^4}{279534024}+\frac{52962515x_{28}^3}{559068048}-\frac{1100615x_{28}^2}{10353112}+\frac{65903905x_{28}}{1118136096}+x_3-\frac{1809365}{139767012}=0$$

$$\frac{401155x_{28}^5}{22588608}-\frac{1710217x_{28}^4}{16941456}+\frac{2711669x_{28}^3}{11294304}-\frac{1649957x_{28}^2}{5647152}+\frac{12135073x_{28}}{67765824}+x_4-\frac{41225}{941192}=0$$

$$-\frac{3114395x_{28}^5}{29042496}+\frac{6555889x_{28}^4}{10890936}-\frac{61909453x_{28}^3}{43563744}+\frac{751421x_{28}^2}{403368}-\frac{145272173x_{28}}{87127488}+x_5-\frac{2930909}{10890936}=0$$

$$-\frac{2915995x_{28}^5}{58084992}+\frac{3049927x_{28}^4}{10890936}-\frac{57387209x_{28}^3}{87127488}+\frac{120333x_{28}^2}{134456}-\frac{89992789x_{28}}{174254976}+x_6+\frac{1095599}{21781872}=0$$

$$\frac{509065x_{28}^5}{135531648}-\frac{969203x_{28}^4}{50824368}+\frac{8577167x_{28}^3}{203297472}-\frac{516037x_{28}^2}{5647152}+\frac{46224847x_{28}}{406594944}+x_7-\frac{2499761}{50824368}=0$$

$$-\frac{3114395x_{28}^5}{58084992}+\frac{6555889x_{28}^4}{21781872}-\frac{61909453x_{28}^3}{87127488}+\frac{751421x_{28}^2}{806736}-\frac{145272173x_{28}}{174254976}+x_8-\frac{13821845}{21781872}=0$$

$$-\frac{1029865x_{28}^5}{8470728}+\frac{17330359x_{28}^4}{25412184}-\frac{40896115x_{28}^3}{25412184}+\frac{5981945x_{28}^2}{2823576}-\frac{25783109x_{28}}{12706092}+x_9-\frac{128515}{3176523}=0$$

$$x_{10}-\frac{10136785x_{28}^5}{406594944}+\frac{21491807x_{28}^4}{152473104}-\frac{203817335x_{28}^3}{609892416}+\frac{2371955x_{28}^2}{5647152}-\frac{134169127x_{28}}{1219784832}-\frac{14007823}{152473104}=0$$

$$x_{11}-\frac{39296735x_{28}^5}{406594944}+\frac{82490347x_{28}^4}{152473104}-\frac{777689425x_{28}^3}{609892416}+\frac{9591935x_{28}^2}{5647152}-\frac{2341009337x_{28}}{1219784832}+\frac{7839103}{152473104}=0$$

$$x_{12}+\frac{5322925x_{28}^5}{135531648}-\frac{11230505x_{28}^4}{50824368}+\frac{106197251x_{28}^3}{203297472}-\frac{3815951x_{28}^2}{5647152}+\frac{191845723x_{28}}{406594944}-\frac{6952061}{50824368}=0$$

$$x_{13}-\frac{1978765x_{28}^5}{45638208}+\frac{512881x_{28}^4}{2139291}-\frac{38027351x_{28}^3}{68457312}+\frac{63424x_{28}^2}{79233}-\frac{69826195x_{28}}{136914624}+\frac{1174517}{17114328}=0$$

$$x_{14}+\frac{2668075x_{28}^5}{372712032}-\frac{11622955x_{28}^4}{279534024}+\frac{52962515x_{28}^3}{559068048}-\frac{1100615x_{28}^2}{10353112}+\frac{65903905x_{28}}{1118136096}-\frac{1809365}{139767012}=0$$

$$x_{15}+\frac{7901965x_{28}^5}{745424064}-\frac{33191251x_{28}^4}{559068048}+\frac{162530201x_{28}^3}{1118136096}-\frac{11545837x_{28}^2}{62118672}+\frac{268649599x_{28}}{2236272192}-\frac{8625095}{279534024}=0$$

$$x_{16}+\frac{3114395x_{28}^5}{58084992}-\frac{6555889x_{28}^4}{21781872}+\frac{61909453x_{28}^3}{87127488}-\frac{751421x_{28}^2}{806736}+\frac{145272173x_{28}}{174254976}-\frac{7960027}{21781872}=0$$

$$x_{17}+\frac{401155x_{28}^5}{22588608}-\frac{1710217x_{28}^4}{16941456}+\frac{2711669x_{28}^3}{11294304}-\frac{1649957x_{28}^2}{5647152}+\frac{12135073x_{28}}{67765824}-\frac{41225}{941192}=0$$

$$x_{18}+\frac{5322925x_{28}^5}{135531648}-\frac{11230505x_{28}^4}{50824368}+\frac{106197251x_{28}^3}{203297472}-\frac{3815951x_{28}^2}{5647152}+\frac{191845723x_{28}}{406594944}-\frac{6952061}{50824368}=0$$

$$x_{19}-\frac{21130205x_{28}^5}{638934912}+\frac{43524661x_{28}^4}{239600592}-\frac{384987811x_{28}^3}{958402368}+\frac{5558977x_{28}^2}{8874096}-\frac{847919819x_{28}}{1916804736}+\frac{16543717}{239600592}=0$$

$$x_{20}-\frac{2915995x_{28}^5}{58084992}+\frac{3049927x_{28}^4}{10890936}-\frac{57387209x_{28}^3}{87127488}+\frac{120333x_{28}^2}{134456}-\frac{264247765x_{28}}{174254976}+\frac{1095599}{21781872}=0$$

$$x_{21}+\frac{1978765x_{28}^5}{45638208}-\frac{512881x_{28}^4}{2139291}+\frac{38027351x_{28}^3}{68457312}-\frac{63424x_{28}^2}{79233}-\frac{67088429x_{28}}{136914624}-\frac{1174517}{17114328}=0$$

$$x_{22}-\frac{485915x_{28}^5}{70992768}+\frac{536429x_{28}^4}{13311144}-\frac{10986265x_{28}^3}{106489152}+\frac{419245x_{28}^2}{4437048}-\frac{1372661x_{28}}{212978304}-\frac{487961}{26622288}=0$$

$$x_{23}+\frac{366545x_{28}^5}{106489152}-\frac{1420763x_{28}^4}{79866864}+\frac{8086453x_{28}^3}{159733728}-\frac{706021x_{28}^2}{8874096}+\frac{19548827x_{28}}{319467456}-\frac{715195}{39933432}=0$$

$$x_{24}-\frac{2190835x_{28}^5}{212978304}+\frac{4639337x_{28}^4}{79866864}-\frac{49131701x_{28}^3}{319467456}+\frac{514837x_{28}^2}{2958032}-\frac{43215637x_{28}}{638934912}-\frac{33493}{79866864}=0$$

$$x_{25}+\frac{2915995x_{28}^5}{203297472}-\frac{3049927x_{28}^4}{38118276}+\frac{57387209x_{28}^3}{304946208}-\frac{120333x_{28}^2}{470596}+\frac{220684021x_{28}}{609892416}-\frac{17432003}{76236552}=0$$

$$x_{26}+\frac{3114395x_{28}^5}{58084992}-\frac{6555889x_{28}^4}{21781872}+\frac{61909453x_{28}^3}{87127488}-\frac{751421x_{28}^2}{806736}+\frac{145272173x_{28}}{174254976}-\frac{7960027}{21781872}=0$$

$$x_{27}+\frac{48832915x_{28}^5}{638934912}-\frac{100967333x_{28}^4}{239600592}+\frac{917370725x_{28}^3}{958402368}-\frac{12662465x_{28}^2}{8874096}-\frac{91318187x_{28}}{1916804736}-\frac{32986955}{239600592}=0$$

$$x_{28}^6-\frac{83x_{28}^5}{15}+\frac{38x_{28}^4}{3}-\frac{46x_{28}^3}{3}+\frac{31x_{28}^2}{3}-\frac{11x_{28}}{3}+\frac{8}{15}=0$$

Solution 1:

$$x_1=0$$

$$x_2=0$$

$$x_3=0$$

$$x_4=0$$

$$x_5=1$$

$$x_6=0$$

$$x_7=0$$

$$x_8=1$$

$$x_9=1$$

$$x_10=0$$

$$x_11=1$$

$$x_12=0$$

$$x_13=0$$

$$x_14=0$$

$$x_15=0$$

$$x_16=0$$

$$x_17 = 0$$
$$x_18 = 0$$
$$x_19 = 0$$
$$x_20 = 1$$
$$x_21 = 1$$
$$x_22 = 0$$
$$x_23 = 0$$
$$x_24 = 0$$
$$x_25 = 0$$
$$x_26 = 0$$
$$x_27 = 1$$
$$x_28 = 1$$

Solution 2:

$$x_1 = 5/84$$
$$x_2 = 1/84$$
$$x_3 = 1/2310$$
$$x_4 = 1/420$$
$$x_5 = 4/5$$
$$x_6 = 1/20$$
$$x_7 = 1/105$$
$$x_8 = 9/10$$
$$x_9 = 5/7$$
$$x_10 = 1/14$$
$$x_11 = 9/14$$
$$x_12 = 1/70$$
$$x_13 = 7/165$$
$$x_14 = 1/2310$$
$$x_15 = 3/1540$$
$$x_16 = 1/10$$
$$x_17 = 1/420$$
$$x_18 = 1/70$$
$$x_19 = 2/55$$

$$x_2 0 = 7/12$$
$$x_2 1 = 27/55$$
$$x_2 2 = 1/132$$
$$x_2 3 = 1/660$$
$$x_2 4 = 1/165$$
$$x_2 5 = 3/35$$
$$x_2 6 = 1/10$$
$$x_2 7 = 5/11$$
$$x_2 8 = 8/15$$

$\mathfrak{m}_{1A}(3, 14)$

m1A314 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_2, e_{11}] = e_{14} && [e_3, e_{10}] = -e_{14} \\
&[e_4, e_9] = e_{14} && [e_5, e_8] = -e_{14} \\
&[e_6, e_7] = e_{14} &&
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(3, 14)$

m3A314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_9] &= e_{12} & [e_2, e_{10}] &= 4e_{13} \\
[e_2, e_{11}] &= \alpha_{2,11}^{14} e_{14} & [e_3, e_8] &= -e_{12} \\
[e_3, e_9] &= -3e_{13} & [e_3, e_{10}] &= \alpha_{3,10}^{14} e_{14} \\
[e_4, e_7] &= e_{12} & [e_4, e_8] &= 2e_{13} \\
[e_4, e_9] &= \alpha_{4,9}^{14} e_{14} & [e_5, e_6] &= -e_{12} \\
[e_5, e_7] &= -e_{13} & [e_5, e_8] &= \alpha_{5,8}^{14} e_{14} \\
[e_6, e_7] &= \alpha_{6,7}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}) &: \quad -\alpha_{2,11}^{14} - \alpha_{3,10}^{14} + 4 &&= 0 \\
(e_1, e_3, e_9) &: \quad -\alpha_{3,10}^{14} - \alpha_{4,9}^{14} - 3 &&= 0 \\
(e_1, e_4, e_8) &: \quad -\alpha_{4,9}^{14} - \alpha_{5,8}^{14} + 2 &&= 0 \\
(e_1, e_5, e_7) &: \quad -\alpha_{5,8}^{14} - \alpha_{6,7}^{14} - 1 &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,10}^{14} \to x_1$$

$$\alpha_{2,11}^{14} \to x_2$$

$$\alpha_{5,8}^{14} \to x_3$$

$$\alpha_{4,9}^{14} \to x_4$$

$$\alpha_{6,7}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): \quad & -x_1 - x_2 + 4 && = 0\\
(e_1, e_3, e_9): \quad & -x_1 - x_4 - 3 && = 0\\
(e_1, e_4, e_8): \quad & -x_3 - x_4 + 2 && = 0\\
(e_1, e_5, e_7): \quad & -x_3 - x_5 - 1 && = 0
\end{aligned}$$

Groebner basis (5 variables, 4 linear, 0 nonlinear)

$$x_1 + x_5 + 6 = 0$$
$$x_2 - x_5 - 10 = 0$$
$$x_3 + x_5 + 1 = 0$$
$$x_4 - x_5 - 3 = 0$$

$\mathfrak{m}_{5A}(3, 14)$

m5A314 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_2, e_7] &= e_{10} & [e_2, e_8] &= 3e_{11}\\
[e_2, e_9] &= \frac{7e_{12}}{2} & [e_2, e_{10}] &= 0\\
[e_2, e_{11}] &= 0 & [e_3, e_6] &= -e_{10}\\
[e_3, e_7] &= -2e_{11} & [e_3, e_8] &= -\frac{e_{12}}{2}\\
[e_3, e_9] &= \frac{7e_{13}}{2} & [e_3, e_{10}] &= 0\\
[e_4, e_5] &= e_{10} & [e_4, e_6] &= e_{11}\\
[e_4, e_7] &= -\frac{3e_{12}}{2} & [e_4, e_8] &= -4e_{13}\\
[e_4, e_9] &= \frac{7e_{14}}{2} & [e_5, e_6] &= \frac{5e_{12}}{2}\\
[e_5, e_7] &= \frac{5e_{13}}{2} & [e_5, e_8] &= -\frac{15e_{14}}{2}\\
[e_6, e_7] &= 10e_{14}
\end{aligned}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_7] = e_{10} & [e_2, e_8] = 3e_{11} \\
[e_2, e_9] = \alpha_{2,9}^{12} e_{12} & [e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} \\
[e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} & [e_3, e_6] = -e_{10} \\
[e_3, e_7] = -2e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14} \\
[e_4, e_5] = e_{10} & [e_4, e_6] = e_{11} \\
[e_4, e_7] = \alpha_{4,7}^{12} e_{12} & [e_4, e_8] = \alpha_{4,8}^{13} e_{13} \\
[e_4, e_9] = \alpha_{4,9}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} & [e_5, e_8] = \alpha_{5,8}^{14} e_{14} \\
[e_6, e_7] = \alpha_{6,7}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha^{12}_{2,9} - \alpha^{12}_{3,8} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha^{12}_{3,8} - \alpha^{12}_{4,7} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha^{12}_{4,7} - \alpha^{12}_{5,6} + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha^{13}_{2,10} + \alpha^{12}_{2,9} - \alpha^{13}_{3,9} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha^{12}_{3,8} - \alpha^{13}_{3,9} - \alpha^{13}_{4,8} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha^{12}_{4,7} - \alpha^{13}_{4,8} - \alpha^{13}_{5,7} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha^{12}_{5,6} - \alpha^{13}_{5,7} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha^{13}_{2,10} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha^{13}_{2,10} &&= 0\\
(e_1, e_2, e_{10}): &\quad \alpha^{13}_{2,10} - \alpha^{14}_{2,11} - \alpha^{14}_{3,10} &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha^{14}_{3,10} + \alpha^{13}_{3,9} - \alpha^{14}_{4,9} &&= 0\\
(e_1, e_4, e_8): &\quad \alpha^{13}_{4,8} - \alpha^{14}_{4,9} - \alpha^{14}_{5,8} &&= 0\\
(e_1, e_5, e_7): &\quad \alpha^{13}_{5,7} - \alpha^{14}_{5,8} - \alpha^{14}_{6,7} &&= 0\\
(e_2, e_3, e_7): &\quad -2\alpha^{14}_{2,11} - \alpha^{14}_{3,10} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha^{14}_{2,11} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha^{14}_{3,10} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha_{2,9}^{12} = 7/2$$
$$\alpha_{5,7}^{13} = 5/2$$
$$\alpha_{5,8}^{14} = -15/2$$
$$\alpha_{3,8}^{12} = -1/2$$
$$\alpha_{4,8}^{13} = -4$$
$$\alpha_{3,10}^{14} = 0$$
$$\alpha_{2,10}^{13} = 0$$
$$\alpha_{4,7}^{12} = -3/2$$
$$\alpha_{5,6}^{12} = 5/2$$
$$\alpha_{2,11}^{14} = 0$$
$$\alpha_{3,9}^{13} = 7/2$$
$$\alpha_{4,9}^{14} = 7/2$$
$$\alpha_{6,7}^{14} = 10$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{12} \to x_1$$
$$\alpha_{5,7}^{13} \to x_2$$
$$\alpha_{5,8}^{14} \to x_3$$
$$\alpha_{3,8}^{12} \to x_4$$
$$\alpha_{4,8}^{13} \to x_5$$
$$\alpha_{3,10}^{14} \to x_6$$
$$\alpha_{2,10}^{13} \to x_7$$
$$\alpha_{4,7}^{12} \to x_8$$
$$\alpha_{5,6}^{12} \to x_9$$
$$\alpha_{2,11}^{14} \to x_{10}$$
$$\alpha_{3,9}^{13} \to x_{11}$$
$$\alpha_{4,9}^{14} \to x_{12}$$
$$\alpha_{6,7}^{14} \to x_{13}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_4 + 3 &&= 0 \\
(e_1, e_3, e_7): &\quad -x_4 - x_8 - 2 &&= 0 \\
(e_1, e_4, e_6): &\quad -x_8 - x_9 + 1 &&= 0 \\
(e_1, e_2, e_9): &\quad x_1 - x_{11} - x_7 &&= 0 \\
(e_1, e_3, e_8): &\quad -x_{11} + x_4 - x_5 &&= 0 \\
(e_1, e_4, e_7): &\quad -x_2 - x_5 + x_8 &&= 0 \\
(e_1, e_5, e_6): &\quad -x_2 + x_9 &&= 0 \\
(e_2, e_3, e_6): &\quad -x_7 &&= 0 \\
(e_2, e_4, e_5): &\quad x_7 &&= 0 \\
(e_1, e_2, e_{10}): &\quad -x_{10} - x_6 + x_7 &&= 0 \\
(e_1, e_3, e_9): &\quad x_{11} - x_{12} - x_6 &&= 0 \\
(e_1, e_4, e_8): &\quad -x_{12} - x_3 + x_5 &&= 0 \\
(e_1, e_5, e_7): &\quad -x_{13} + x_2 - x_3 &&= 0 \\
(e_2, e_3, e_7): &\quad -2x_{10} - x_6 &&= 0 \\
(e_2, e_4, e_6): &\quad x_{10} &&= 0 \\
(e_3, e_4, e_5): &\quad x_6 &&= 0
\end{aligned}
$$

Groebner basis (13 variables, 13 linear, 0 nonlinear)

$$x_1 - \frac{7}{2} = 0$$

$$x_2 - \frac{5}{2} = 0$$

$$x_3 + \frac{15}{2} = 0$$

$$x_4 + \frac{1}{2} = 0$$

$$x_5 + 4 = 0$$

$$x_6 = 0$$

$$x_7 = 0$$

$$x_8 + \frac{3}{2} = 0$$

$$x_9 - \frac{5}{2} = 0$$

$$x_{10} = 0$$

$$x_{11} - \frac{7}{2} = 0$$

$$x_{12} - \frac{7}{2} = 0$$

$$x_{13} - 10 = 0$$

Solution 1:

$$x_1 = 7/2$$

$$x_2 = 5/2$$

$$x_3 = -15/2$$

$$x_4 = -1/2$$

$$x_5 = -4$$

$$x_6 = 0$$

$$x_7 = 0$$

$$x_8 = -3/2$$

$$x_9 = 5/2$$

$$x_10 = 0$$

$$x_11 = 7/2$$

$$x_12 = 7/2$$

$$x_13 = 10$$

$\mathfrak{m}_{7A}(3, 14)$

m7A314 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_5] &= e_8 & [e_2, e_6] &= 2e_9 \\
[e_2, e_7] &= \frac{5e_{10}}{3} & [e_2, e_8] &= 0 \\
[e_2, e_9] &= -\frac{49e_{12}}{33} & [e_2, e_{10}] &= -\frac{14e_{13}}{11} \\
[e_2, e_{11}] &= -\frac{7e_{14}}{11} & [e_3, e_4] &= -e_8 \\
[e_3, e_5] &= -e_9 & [e_3, e_6] &= \frac{e_{10}}{3} \\
[e_3, e_7] &= \frac{5e_{11}}{3} & [e_3, e_8] &= \frac{49e_{12}}{33} \\
[e_3, e_9] &= -\frac{7e_{13}}{33} & [e_3, e_{10}] &= -\frac{7e_{14}}{11} \\
[e_4, e_5] &= -\frac{4e_{10}}{3} & [e_4, e_6] &= -\frac{4e_{11}}{3} \\
[e_4, e_7] &= \frac{2e_{12}}{11} & [e_4, e_8] &= \frac{56e_{13}}{33} \\
[e_4, e_9] &= \frac{14e_{14}}{33} & [e_5, e_6] &= -\frac{50e_{12}}{33} \\
[e_5, e_7] &= -\frac{50e_{13}}{33} & [e_5, e_8] &= \frac{14e_{14}}{11} \\
[e_6, e_7] &= -\frac{92e_{14}}{33}
\end{aligned}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_5] = e_8 & [e_2, e_6] = 2e_9 \\
[e_2, e_7] = \alpha_{2,7}^{10} e_{10} & [e_2, e_8] = \alpha_{2,8}^{11} e_{11} \\
[e_2, e_9] = \alpha_{2,9}^{12} e_{12} & [e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} \\
[e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} & [e_3, e_4] = -e_8 \\
[e_3, e_5] = -e_9 & [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] = \alpha_{3,7}^{11} e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14} \\
[e_4, e_5] = \alpha_{4,5}^{10} e_{10} & [e_4, e_6] = \alpha_{4,6}^{11} e_{11} \\
[e_4, e_7] = \alpha_{4,7}^{12} e_{12} & [e_4, e_8] = \alpha_{4,8}^{13} e_{13} \\
[e_4, e_9] = \alpha_{4,9}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} & [e_5, e_8] = \alpha_{5,8}^{14} e_{14} \\
[e_6, e_7] = \alpha_{6,7}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6): &\quad -\alpha_{2,7}^{10}-\alpha_{3,6}^{10}+2 &&=0\\
(e_1,e_3,e_5): &\quad -\alpha_{3,6}^{10}-\alpha_{4,5}^{10}-1 &&=0\\
(e_1,e_2,e_7): &\quad \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} &&=0\\
(e_1,e_3,e_6): &\quad \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} &&=0\\
(e_1,e_4,e_5): &\quad \alpha_{4,5}^{10}-\alpha_{4,6}^{11} &&=0\\
(e_2,e_3,e_4): &\quad -\alpha_{2,8}^{11} &&=0\\
(e_1,e_2,e_8): &\quad \alpha_{2,8}^{11}-\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_3,e_7): &\quad \alpha_{3,7}^{11}-\alpha_{3,8}^{12}-\alpha_{4,7}^{12} &&=0\\
(e_1,e_4,e_6): &\quad \alpha_{4,6}^{11}-\alpha_{4,7}^{12}-\alpha_{5,6}^{12} &&=0\\
(e_2,e_3,e_5): &\quad -\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_2,e_9): &\quad -\alpha_{2,10}^{13}+\alpha_{2,9}^{12}-\alpha_{3,9}^{13} &&=0\\
(e_1,e_3,e_8): &\quad \alpha_{3,8}^{12}-\alpha_{3,9}^{13}-\alpha_{4,8}^{13} &&=0\\
(e_1,e_4,e_7): &\quad \alpha_{4,7}^{12}-\alpha_{4,8}^{13}-\alpha_{5,7}^{13} &&=0\\
(e_1,e_5,e_6): &\quad \alpha_{5,6}^{12}-\alpha_{5,7}^{13} &&=0\\
(e_2,e_3,e_6): &\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10}-2\alpha_{3,9}^{13} &&=0\\
(e_2,e_4,e_5): &\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10}-\alpha_{4,8}^{13} &&=0\\
(e_1,e_2,e_{10}): &\quad \alpha_{2,10}^{13}-\alpha_{2,11}^{14}-\alpha_{3,10}^{14} &&=0\\
(e_1,e_3,e_9): &\quad -\alpha_{3,10}^{14}+\alpha_{3,9}^{13}-\alpha_{4,9}^{14} &&=0\\
(e_1,e_4,e_8): &\quad \alpha_{4,8}^{13}-\alpha_{4,9}^{14}-\alpha_{5,8}^{14} &&=0\\
(e_1,e_5,e_7): &\quad \alpha_{5,7}^{13}-\alpha_{5,8}^{14}-\alpha_{6,7}^{14} &&=0\\
(e_2,e_3,e_7): &\quad \alpha_{2,11}^{14}\alpha_{3,7}^{11}-\alpha_{2,7}^{10}\alpha_{3,10}^{14} &&=0\\
(e_2,e_4,e_6): &\quad \alpha_{2,11}^{14}\alpha_{4,6}^{11}-2\alpha_{4,9}^{14} &&=0\\
(e_3,e_4,e_5): &\quad \alpha_{3,10}^{14}\alpha_{4,5}^{10}+\alpha_{4,9}^{14}-\alpha_{5,8}^{14} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha^{12}_{2,9} = -49/33$$
$$\alpha^{13}_{5,7} = -50/33$$
$$\alpha^{14}_{5,8} = 14/11$$
$$\alpha^{10}_{2,7} = 5/3$$
$$\alpha^{12}_{3,8} = 49/33$$
$$\alpha^{13}_{4,8} = 56/33$$
$$\alpha^{14}_{3,10} = -7/11$$
$$\alpha^{11}_{3,7} = 5/3$$
$$\alpha^{13}_{2,10} = -14/11$$
$$\alpha^{10}_{3,6} = 1/3$$
$$\alpha^{12}_{4,7} = 2/11$$
$$\alpha^{11}_{2,8} = 0$$
$$\alpha^{12}_{5,6} = -50/33$$
$$\alpha^{14}_{2,11} = -7/11$$
$$\alpha^{10}_{4,5} = -4/3$$
$$\alpha^{13}_{3,9} = -7/33$$
$$\alpha^{11}_{4,6} = -4/3$$
$$\alpha^{14}_{4,9} = 14/33$$
$$\alpha^{14}_{6,7} = -92/33$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{2,9} \to x_1$$
$$\alpha^{13}_{5,7} \to x_2$$
$$\alpha^{14}_{5,8} \to x_3$$
$$\alpha^{10}_{2,7} \to x_4$$
$$\alpha^{12}_{3,8} \to x_5$$
$$\alpha^{13}_{4,8} \to x_6$$
$$\alpha^{14}_{3,10} \to x_7$$
$$\alpha^{11}_{3,7} \to x_8$$

$$\alpha^{13}_{2,10} \to x_9$$

$$\alpha^{10}_{3,6} \to x_{10}$$

$$\alpha^{12}_{4,7} \to x_{11}$$

$$\alpha^{11}_{2,8} \to x_{12}$$

$$\alpha^{12}_{5,6} \to x_{13}$$

$$\alpha^{14}_{2,11} \to x_{14}$$

$$\alpha^{10}_{4,5} \to x_{15}$$

$$\alpha^{13}_{3,9} \to x_{16}$$

$$\alpha^{11}_{4,6} \to x_{17}$$

$$\alpha^{14}_{4,9} \to x_{18}$$

$$\alpha^{14}_{6,7} \to x_{19}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{10} - x_4 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{10} - x_{15} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{12} + x_4 - x_8 &&= 0\\
(e_1, e_3, e_6): &\quad x_{10} - x_{17} - x_8 &&= 0\\
(e_1, e_4, e_5): &\quad x_{15} - x_{17} &&= 0\\
(e_2, e_3, e_4): &\quad -x_{12} &&= 0\\
(e_1, e_2, e_8): &\quad -x_1 + x_{12} - x_5 &&= 0\\
(e_1, e_3, e_7): &\quad -x_{11} - x_5 + x_8 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{11} - x_{13} + x_{17} &&= 0\\
(e_2, e_3, e_5): &\quad -x_1 - x_5 &&= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_{16} - x_9 &&= 0\\
(e_1, e_3, e_8): &\quad -x_{16} + x_5 - x_6 &&= 0\\
(e_1, e_4, e_7): &\quad x_{11} - x_2 - x_6 &&= 0\\
(e_1, e_5, e_6): &\quad x_{13} - x_2 &&= 0\\
(e_2, e_3, e_6): &\quad x_{10}x_9 - 2x_{16} &&= 0\\
(e_2, e_4, e_5): &\quad x_{15}x_9 - x_6 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_{14} - x_7 + x_9 &&= 0\\
(e_1, e_3, e_9): &\quad x_{16} - x_{18} - x_7 &&= 0\\
(e_1, e_4, e_8): &\quad -x_{18} - x_3 + x_6 &&= 0\\
(e_1, e_5, e_7): &\quad -x_{19} + x_2 - x_3 &&= 0\\
(e_2, e_3, e_7): &\quad x_{14}x_8 - x_4x_7 &&= 0\\
(e_2, e_4, e_6): &\quad x_{14}x_{17} - 2x_{18} &&= 0\\
(e_3, e_4, e_5): &\quad x_{15}x_7 + x_{18} - x_3 &&= 0
\end{aligned}
$$

Groebner basis (19 variables, 19 linear, 0 nonlinear)

$$x_1 + \frac{49}{33} = 0$$

$$x_2 + \frac{50}{33} = 0$$

$$x_3 - \frac{14}{11} = 0$$

$$x_4 - \frac{5}{3} = 0$$

$$x_5 - \frac{49}{33} = 0$$

$$x_6 - \frac{56}{33} = 0$$
$$x_7 + \frac{7}{11} = 0$$
$$x_8 - \frac{5}{3} = 0$$
$$x_9 + \frac{14}{11} = 0$$
$$x_{10} - \frac{1}{3} = 0$$
$$x_{11} - \frac{2}{11} = 0$$
$$x_{12} = 0$$
$$x_{13} + \frac{50}{33} = 0$$
$$x_{14} + \frac{7}{11} = 0$$
$$x_{15} + \frac{4}{3} = 0$$
$$x_{16} + \frac{7}{33} = 0$$
$$x_{17} + \frac{4}{3} = 0$$
$$x_{18} - \frac{14}{33} = 0$$
$$x_{19} + \frac{92}{33} = 0$$

Solution 1:

$$x_1 = -49/33$$
$$x_2 = -50/33$$
$$x_3 = 14/11$$
$$x_4 = 5/3$$
$$x_5 = 49/33$$
$$x_6 = 56/33$$
$$x_7 = -7/11$$
$$x_8 = 5/3$$
$$x_9 = -14/11$$
$$x_10 = 1/3$$

$$x_11 = 2/11$$
$$x_12 = 0$$
$$x_13 = -50/33$$
$$x_14 = -7/11$$
$$x_15 = -4/3$$
$$x_16 = -7/33$$
$$x_17 = -4/3$$
$$x_18 = 14/33$$
$$x_19 = -92/33$$

$\mathfrak{m}_{9A}(3, 14)$

m9A314 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_3] = e_6 \qquad [e_2, e_4] = e_7$$
$$[e_2, e_5] = \alpha_{2,5}^{8} e_8 \qquad [e_2, e_6] = \alpha_{2,6}^{9} e_9$$
$$[e_2, e_7] = \alpha_{2,7}^{10} e_{10} \qquad [e_2, e_8] = \alpha_{2,8}^{11} e_{11}$$
$$[e_2, e_9] = \alpha_{2,9}^{12} e_{12} \qquad [e_2, e_{10}] = \alpha_{2,10}^{13} e_{13}$$
$$[e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} \qquad [e_3, e_4] = \alpha_{3,4}^{8} e_8$$
$$[e_3, e_5] = \alpha_{3,5}^{9} e_9 \qquad [e_3, e_6] = \alpha_{3,6}^{10} e_{10}$$
$$[e_3, e_7] = \alpha_{3,7}^{11} e_{11} \qquad [e_3, e_8] = \alpha_{3,8}^{12} e_{12}$$
$$[e_3, e_9] = \alpha_{3,9}^{13} e_{13} \qquad [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14}$$
$$[e_4, e_5] = \alpha_{4,5}^{10} e_{10} \qquad [e_4, e_6] = \alpha_{4,6}^{11} e_{11}$$
$$[e_4, e_7] = \alpha_{4,7}^{12} e_{12} \qquad [e_4, e_8] = \alpha_{4,8}^{13} e_{13}$$
$$[e_4, e_9] = \alpha_{4,9}^{14} e_{14} \qquad [e_5, e_6] = \alpha_{5,6}^{12} e_{12}$$
$$[e_5, e_7] = \alpha_{5,7}^{13} e_{13} \qquad [e_5, e_8] = \alpha_{5,8}^{14} e_{14}$$
$$[e_6, e_7] = \alpha_{6,7}^{14} e_{14}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{8}_{2,5}-\alpha^{8}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{8}_{2,5}-\alpha^{9}_{2,6}-\alpha^{9}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{8}_{3,4}-\alpha^{9}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{9}_{2,6}-\alpha^{10}_{2,7}-\alpha^{10}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{9}_{3,5}-\alpha^{10}_{3,6}-\alpha^{10}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{10}_{2,7}-\alpha^{11}_{2,8}-\alpha^{11}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{10}_{3,6}-\alpha^{11}_{3,7}-\alpha^{11}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{10}_{4,5}-\alpha^{11}_{4,6} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{11}_{2,8}\alpha^{8}_{3,4}-\alpha^{11}_{3,7}+\alpha^{11}_{4,6} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{11}_{2,8}-\alpha^{12}_{2,9}-\alpha^{12}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{11}_{3,7}-\alpha^{12}_{3,8}-\alpha^{12}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{11}_{4,6}-\alpha^{12}_{4,7}-\alpha^{12}_{5,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{8}_{2,5}\alpha^{12}_{3,8}+\alpha^{12}_{2,9}\alpha^{9}_{3,5}+\alpha^{12}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{13}_{2,10}+\alpha^{12}_{2,9}-\alpha^{13}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{12}_{3,8}-\alpha^{13}_{3,9}-\alpha^{13}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{12}_{4,7}-\alpha^{13}_{4,8}-\alpha^{13}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{12}_{5,6}-\alpha^{13}_{5,7} &&=0\\
(e_2,e_3,e_6):&\quad \alpha^{13}_{2,10}\alpha^{10}_{3,6}-\alpha^{9}_{2,6}\alpha^{13}_{3,9} &&=0\\
(e_2,e_4,e_5):&\quad \alpha^{13}_{2,10}\alpha^{10}_{4,5}-\alpha^{8}_{2,5}\alpha^{13}_{4,8}+\alpha^{13}_{5,7} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha^{13}_{2,10}-\alpha^{14}_{2,11}-\alpha^{14}_{3,10} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{14}_{3,10}+\alpha^{13}_{3,9}-\alpha^{14}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad \alpha^{13}_{4,8}-\alpha^{14}_{4,9}-\alpha^{14}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad \alpha^{13}_{5,7}-\alpha^{14}_{5,8}-\alpha^{14}_{6,7} &&=0\\
(e_2,e_3,e_7):&\quad \alpha^{14}_{2,11}\alpha^{11}_{3,7}-\alpha^{10}_{2,7}\alpha^{14}_{3,10}-\alpha^{14}_{6,7} &&=0\\
(e_2,e_4,e_6):&\quad \alpha^{14}_{2,11}\alpha^{11}_{4,6}-\alpha^{9}_{2,6}\alpha^{14}_{4,9}+\alpha^{14}_{6,7} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^{14}_{3,10}\alpha^{10}_{4,5}+\alpha^{8}_{3,4}\alpha^{14}_{5,8}-\alpha^{9}_{3,5}\alpha^{14}_{4,9} &&=0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{3,8}\to x_1$$

$$\alpha^{13}_{2,10}\to x_2$$

$$\alpha^{10}_{3,6} \to x_3$$

$$\alpha^{13}_{3,9} \to x_4$$

$$\alpha^{12}_{2,9} \to x_5$$

$$\alpha^{8}_{3,4} \to x_6$$

$$\alpha^{13}_{4,8} \to x_7$$

$$\alpha^{14}_{3,10} \to x_8$$

$$\alpha^{11}_{3,7} \to x_9$$

$$\alpha^{12}_{4,7} \to x_{10}$$

$$\alpha^{11}_{2,8} \to x_{11}$$

$$\alpha^{12}_{5,6} \to x_{12}$$

$$\alpha^{14}_{2,11} \to x_{13}$$

$$\alpha^{10}_{4,5} \to x_{14}$$

$$\alpha^{9}_{3,5} \to x_{15}$$

$$\alpha^{14}_{5,8} \to x_{16}$$

$$\alpha^{14}_{6,7} \to x_{17}$$

$$\alpha^{10}_{2,7} \to x_{18}$$

$$\alpha^{8}_{2,5} \to x_{19}$$

$$\alpha^{13}_{5,7} \to x_{20}$$

$$\alpha^{9}_{2,6} \to x_{21}$$

$$\alpha^{11}_{4,6} \to x_{22}$$

$$\alpha^{14}_{4,9} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -x_{19} - x_6 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_{15} + x_{19} - x_{21} &&= 0\\
(e_1, e_3, e_4):&\quad -x_{15} + x_6 &&= 0\\
(e_1, e_2, e_6):&\quad -x_{18} + x_{21} - x_3 &&= 0\\
(e_1, e_3, e_5):&\quad -x_{14} + x_{15} - x_3 &&= 0\\
(e_1, e_2, e_7):&\quad -x_{11} + x_{18} - x_9 &&= 0\\
(e_1, e_3, e_6):&\quad -x_{22} + x_3 - x_9 &&= 0\\
(e_1, e_4, e_5):&\quad x_{14} - x_{22} &&= 0\\
(e_2, e_3, e_4):&\quad x_{11}x_6 + x_{22} - x_9 &&= 0\\
(e_1, e_2, e_8):&\quad -x_1 + x_{11} - x_5 &&= 0\\
(e_1, e_3, e_7):&\quad -x_1 - x_{10} + x_9 &&= 0\\
(e_1, e_4, e_6):&\quad -x_{10} - x_{12} + x_{22} &&= 0\\
(e_2, e_3, e_5):&\quad -x_1x_{19} + x_{12} + x_{15}x_5 &&= 0\\
(e_1, e_2, e_9):&\quad -x_2 - x_4 + x_5 &&= 0\\
(e_1, e_3, e_8):&\quad x_1 - x_4 - x_7 &&= 0\\
(e_1, e_4, e_7):&\quad x_{10} - x_{20} - x_7 &&= 0\\
(e_1, e_5, e_6):&\quad x_{12} - x_{20} &&= 0\\
(e_2, e_3, e_6):&\quad x_2x_3 - x_{21}x_4 &&= 0\\
(e_2, e_4, e_5):&\quad x_{14}x_2 - x_{19}x_7 + x_{20} &&= 0\\
(e_1, e_2, e_{10}):&\quad -x_{13} + x_2 - x_8 &&= 0\\
(e_1, e_3, e_9):&\quad -x_{23} + x_4 - x_8 &&= 0\\
(e_1, e_4, e_8):&\quad -x_{16} - x_{23} + x_7 &&= 0\\
(e_1, e_5, e_7):&\quad -x_{16} - x_{17} + x_{20} &&= 0\\
(e_2, e_3, e_7):&\quad x_{13}x_9 - x_{17} - x_{18}x_8 &&= 0\\
(e_2, e_4, e_6):&\quad x_{13}x_{22} + x_{17} - x_{21}x_{23} &&= 0\\
(e_3, e_4, e_5):&\quad x_{14}x_8 - x_{15}x_{23} + x_{16}x_6 &&= 0
\end{aligned}
$$

Groebner basis (23 variables, 8 linear, 16 nonlinear)

$$x_1+\frac{128x_{21}x_{23}^3}{2403}-\frac{772x_{21}x_{23}^2}{801}+\frac{5x_{21}x_{23}}{6}+\frac{x_{21}}{2}-\frac{12740x_{22}^5x_{23}}{21627}+\frac{353045x_{22}^5}{57672}-\frac{36400x_{22}^4x_{23}^2}{21627}+\frac{69041x_{22}^4x_{23}}{4806}+\frac{1259195x}{38448}$$

$$x_2-\frac{512x_{21}x_{23}^3}{2403}+\frac{3088x_{21}x_{23}^2}{801}-\frac{10x_{21}x_{23}}{3}-3x_{21}+\frac{50960x_{22}^5x_{23}}{21627}-\frac{353045x_{22}^5}{14418}+\frac{145600x_{22}^4x_{23}^2}{21627}-\frac{138082x_{22}^4x_{23}}{2403}-\frac{1259}{9}$$

$$\frac{x_{21}}{2}+x_{22}+x_3-\frac{1}{2}=0$$

$$\frac{128x_{21}x_{23}^3}{801}-\frac{772x_{21}x_{23}^2}{267}+\frac{5x_{21}x_{23}}{2}+\frac{x_{21}}{2}-\frac{12740x_{22}^5x_{23}}{7209}+\frac{353045x_{22}^5}{19224}-\frac{36400x_{22}^4x_{23}^2}{7209}+\frac{69041x_{22}^4x_{23}}{1602}+\frac{1259195x_{22}^4}{12816}+$$

$$-\frac{128x_{21}x_{23}^3}{2403}+\frac{772x_{21}x_{23}^2}{801}-\frac{5x_{21}x_{23}}{6}-\frac{5x_{21}}{2}+\frac{12740x_{22}^5x_{23}}{21627}-\frac{353045x_{22}^5}{57672}+\frac{36400x_{22}^4x_{23}^2}{21627}-\frac{69041x_{22}^4x_{23}}{4806}-\frac{1259195x_{22}^4}{38448}$$

$$\frac{x_{21}}{2}+x_6-\frac{1}{2}=0$$

$$-\frac{256x_{21}x_{23}^3}{2403}+\frac{1544x_{21}x_{23}^2}{801}-\frac{5x_{21}x_{23}}{3}+\frac{25480x_{22}^5x_{23}}{21627}-\frac{353045x_{22}^5}{28836}+\frac{72800x_{22}^4x_{23}^2}{21627}-\frac{69041x_{22}^4x_{23}}{2403}-\frac{1259195x_{22}^4}{19224}-$$

$$\frac{128x_{21}x_{23}^3}{801}-\frac{772x_{21}x_{23}^2}{267}+\frac{5x_{21}x_{23}}{2}+\frac{x_{21}}{2}-\frac{12740x_{22}^5x_{23}}{7209}+\frac{353045x_{22}^5}{19224}-\frac{36400x_{22}^4x_{23}^2}{7209}+\frac{69041x_{22}^4x_{23}}{1602}+\frac{1259195x_{22}^4}{12816}+$$

$$\frac{x_{21}}{2}+2x_{22}+x_9-\frac{1}{2}=0$$

$$x_{10}-\frac{128x_{21}x_{23}^3}{2403}+\frac{772x_{21}x_{23}^2}{801}-\frac{5x_{21}x_{23}}{6}+\frac{12740x_{22}^5x_{23}}{21627}-\frac{353045x_{22}^5}{57672}+\frac{36400x_{22}^4x_{23}^2}{21627}-\frac{69041x_{22}^4x_{23}}{4806}-\frac{1259195x_{22}^4}{38448}-$$

$$x_{11}-2x_{21}-3x_{22}+1=0$$

$$x_{12}+\frac{128x_{21}x_{23}^3}{2403}-\frac{772x_{21}x_{23}^2}{801}+\frac{5x_{21}x_{23}}{6}-\frac{12740x_{22}^5x_{23}}{21627}+\frac{353045x_{22}^5}{57672}-\frac{36400x_{22}^4x_{23}^2}{21627}+\frac{69041x_{22}^4x_{23}}{4806}+\frac{1259195x_{22}^4}{38448}+$$

$$x_{13}-\frac{896x_{21}x_{23}^3}{2403}+\frac{5404x_{21}x_{23}^2}{801}-\frac{35x_{21}x_{23}}{6}-\frac{7x_{21}}{2}+\frac{89180x_{22}^5x_{23}}{21627}-\frac{2471315x_{22}^5}{57672}+\frac{254800x_{22}^4x_{23}^2}{21627}-\frac{483287x_{22}^4x_{23}}{4806}-$$

$$x_{14}-x_{22}=0$$

$$x_{15}+\frac{x_{21}}{2}-\frac{1}{2}=0$$

$$x_{16}-\frac{256x_{21}x_{23}^3}{2403}+\frac{1544x_{21}x_{23}^2}{801}-\frac{5x_{21}x_{23}}{3}+\frac{25480x_{22}^5x_{23}}{21627}-\frac{353045x_{22}^5}{28836}+\frac{72800x_{22}^4x_{23}^2}{21627}-\frac{69041x_{22}^4x_{23}}{2403}-\frac{1259195x_{22}^4}{19224}-$$

$$x_{17}+\frac{128x_{21}x_{23}^3}{801}-\frac{772x_{21}x_{23}^2}{267}+\frac{5x_{21}x_{23}}{2}-\frac{12740x_{22}^5x_{23}}{7209}+\frac{353045x_{22}^5}{19224}-\frac{36400x_{22}^4x_{23}^2}{7209}+\frac{69041x_{22}^4x_{23}}{1602}+\frac{1259195x_{22}^4}{12816}+$$

$$x_{18}-\frac{3x_{21}}{2}-x_{22}+\frac{1}{2}=0$$

$$x_{19}-\frac{x_{21}}{2}-\frac{1}{2}=0$$

$$x_{20}+\frac{128x_{21}x_{23}^3}{2403}-\frac{772x_{21}x_{23}^2}{801}+\frac{5x_{21}x_{23}}{6}-\frac{12740x_{22}^5x_{23}}{21627}+\frac{353045x_{22}^5}{57672}-\frac{36400x_{22}^4x_{23}^2}{21627}+\frac{69041x_{22}^4x_{23}}{4806}+\frac{1259195x_{22}^4}{38448}+$$

$$x_{21}^2+\frac{640x_{21}x_{23}^3}{1869}-\frac{1620x_{21}x_{23}^2}{623}+\frac{3x_{21}x_{23}}{2}-2x_{21}-\frac{9100x_{22}^5x_{23}}{2403}-\frac{875x_{22}^5}{2136}-\frac{26000x_{22}^4x_{23}^2}{2403}-\frac{34055x_{22}^4x_{23}}{1602}-\frac{1615x_{22}^4}{4272}+$$

$$x_{21}x_{22}-\frac{1280x_{21}x_{23}^3}{5607}+\frac{1080x_{21}x_{23}^2}{623}-x_{21}x_{23}+\frac{18200x_{22}^5x_{23}}{7209}+\frac{875x_{22}^5}{3204}+\frac{52000x_{22}^4x_{23}^2}{7209}+\frac{34055x_{22}^4x_{23}}{2403}+\frac{1615x_{22}^4}{6408}-$$

$$x_{21}x_{23}^4-\frac{267x_{21}x_{23}^3}{32}+\frac{2403x_{21}x_{23}^2}{256}-\frac{3185x_{22}^5x_{23}^2}{288}+\frac{21805x_{22}^5x_{23}}{3072}+\frac{4361x_{22}^5}{512}-\frac{2275x_{22}^4x_{23}^3}{72}-\frac{29477x_{22}^4x_{23}^2}{768}+\frac{137683x_{22}^4}{2048}$$

$$x_{22}^6+\frac{20x_{22}^5x_{23}}{7}+\frac{339x_{22}^5}{70}-\frac{44x_{22}^4x_{23}^2}{49}+\frac{6369x_{22}^4x_{23}}{490}-\frac{738x_{22}^4}{245}+\frac{579x_{22}^3x_{23}^2}{245}-\frac{117x_{22}^3x_{23}}{35}+\frac{108x_{22}^3}{245}-\frac{468x_{22}^2x_{23}^2}{245}-$$

$\mathfrak{m}_{2A}(4, 14)$

m2A414 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_9] &= e_{13} & [e_2, e_{10}] &= 4e_{14} \\
[e_3, e_8] &= -e_{13} & [e_3, e_9] &= -3e_{14} \\
[e_4, e_7] &= e_{13} & [e_4, e_8] &= 2e_{14} \\
[e_5, e_6] &= -e_{13} & [e_5, e_7] &= -e_{14}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(4, 14)$

m4A414 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_7] &= e_{11} & [e_2, e_8] &= 3e_{12} \\
[e_2, e_9] &= \alpha_{2,9}^{13} e_{13} & [e_2, e_{10}] &= \alpha_{2,10}^{14} e_{14} \\
[e_3, e_6] &= -e_{11} & [e_3, e_7] &= -2e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_3, e_9] &= \alpha_{3,9}^{14} e_{14} \\
[e_4, e_5] &= e_{11} & [e_4, e_6] &= e_{12} \\
[e_4, e_7] &= \alpha_{4,7}^{13} e_{13} & [e_4, e_8] &= \alpha_{4,8}^{14} e_{14} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13} & [e_5, e_7] &= \alpha_{5,7}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{13} - \alpha_{3,8}^{13} + 3 &= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{13} - \alpha_{4,7}^{13} - 2 &= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{13} - \alpha_{5,6}^{13} + 1 &= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,9}^{14} \to x_1$$

$$\alpha_{2,10}^{14} \to x_2$$

$$\alpha_{5,6}^{13} \to x_3$$

$$\alpha_{4,8}^{14} \to x_4$$

$$\alpha_{2,9}^{13} \to x_5$$

$$\alpha_{4,7}^{13} \to x_6$$

$$\alpha_{5,7}^{14} \to x_7$$

$$\alpha_{3,8}^{13} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_5 - x_8 + 3 &= 0\\
(e_1, e_3, e_7): &\quad -x_6 - x_8 - 2 &= 0\\
(e_1, e_4, e_6): &\quad -x_3 - x_6 + 1 &= 0\\
(e_1, e_2, e_9): &\quad -x_1 - x_2 + x_5 &= 0\\
(e_1, e_3, e_8): &\quad -x_1 - x_4 + x_8 &= 0\\
(e_1, e_4, e_7): &\quad -x_4 + x_6 - x_7 &= 0\\
(e_1, e_5, e_6): &\quad x_3 - x_7 &= 0
\end{aligned}$$

Groebner basis (8 variables, 7 linear, 0 nonlinear)

$$x_1 - 3x_8 - 5 = 0$$

$$x_2 + 4x_8 + 2 = 0$$
$$x_3 - x_8 - 3 = 0$$
$$x_4 + 2x_8 + 5 = 0$$
$$x_5 + x_8 - 3 = 0$$
$$x_6 + x_8 + 2 = 0$$
$$x_7 - x_8 - 3 = 0$$

$\mathfrak{m}_{6A}(4, 14)$

m6A414 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_5] &= e_9 & [e_2, e_6] &= 2e_{10} \\
[e_2, e_7] &= \alpha_{2,7}^{11} e_{11} & [e_2, e_8] &= \alpha_{2,8}^{12} e_{12} \\
[e_2, e_9] &= \alpha_{2,9}^{13} e_{13} & [e_2, e_{10}] &= \alpha_{2,10}^{14} e_{14} \\
[e_3, e_4] &= -e_9 & [e_3, e_5] &= -e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_3, e_9] &= \alpha_{3,9}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{11} e_{11} & [e_4, e_6] &= \alpha_{4,6}^{12} e_{12} \\
[e_4, e_7] &= \alpha_{4,7}^{13} e_{13} & [e_4, e_8] &= \alpha_{4,8}^{14} e_{14} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13} & [e_5, e_7] &= \alpha_{5,7}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,9}^{13} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,10}^{14} - \alpha_{3,9}^{14} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,9}^{14} \to x_1$$

$$\alpha_{2,10}^{14} \to x_2$$

$$\alpha_{4,5}^{11} \to x_3$$

$$\alpha_{2,7}^{11} \to x_4$$

$$\alpha_{2,8}^{12} \to x_5$$

$$\alpha_{5,6}^{13} \to x_6$$

$$\alpha_{3,7}^{12} \to x_7$$

$$\alpha_{4,8}^{14} \to x_8$$

$$\alpha_{3,8}^{13} \to x_9$$

$$\alpha_{2,9}^{13} \to x_{10}$$

$$\alpha_{3,6}^{11} \to x_{11}$$

$$\alpha_{4,7}^{13} \to x_{12}$$

$$\alpha_{5,7}^{14} \to x_{13}$$

$$\alpha_{4,6}^{12} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_6): &\quad -x_{11}-x_4+2 &&= 0\\
(e_1,e_3,e_5): &\quad -x_{11}-x_3-1 &&= 0\\
(e_1,e_2,e_7): &\quad x_4-x_5-x_7 &&= 0\\
(e_1,e_3,e_6): &\quad x_{11}-x_{14}-x_7 &&= 0\\
(e_1,e_4,e_5): &\quad -x_{14}+x_3 &&= 0\\
(e_1,e_2,e_8): &\quad -x_{10}+x_5-x_9 &&= 0\\
(e_1,e_3,e_7): &\quad -x_{12}+x_7-x_9 &&= 0\\
(e_1,e_4,e_6): &\quad -x_{12}+x_{14}-x_6 &&= 0\\
(e_2,e_3,e_4): &\quad -x_{10} &&= 0\\
(e_1,e_2,e_9): &\quad -x_1+x_{10}-x_2 &&= 0\\
(e_1,e_3,e_8): &\quad -x_1-x_8+x_9 &&= 0\\
(e_1,e_4,e_7): &\quad x_{12}-x_{13}-x_8 &&= 0\\
(e_1,e_5,e_6): &\quad -x_{13}+x_6 &&= 0\\
(e_2,e_3,e_5): &\quad -x_1-x_2 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 13 linear, 0 nonlinear)

$$x_1-14x_{14}-14=0$$

$$14x_{14}+x_2+14=0$$

$$-x_{14}+x_3=0$$

$$-x_{14}+x_4-3=0$$

$$-3x_{14}+x_5-4=0$$

$$-6x_{14}+x_6-5=0$$

$$2x_{14}+x_7+1=0$$

$$11x_{14}+x_8+10=0$$

$$-3x_{14}+x_9-4=0$$

$$x_{10}=0$$

$$x_{11}+x_{14}+1=0$$

$$x_{12}+5x_{14}+5=0$$

$$x_{13}-6x_{14}-5=0$$

$\mathfrak{m}_{8A}(4, 14)$

m8A414 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_3] &= e_7 & [e_2, e_4] &= e_8 \\
[e_2, e_5] &= \alpha_{2,5}^{9} e_9 & [e_2, e_6] &= \alpha_{2,6}^{10} e_{10} \\
[e_2, e_7] &= \alpha_{2,7}^{11} e_{11} & [e_2, e_8] &= \alpha_{2,8}^{12} e_{12} \\
[e_2, e_9] &= \alpha_{2,9}^{13} e_{13} & [e_2, e_{10}] &= \alpha_{2,10}^{14} e_{14} \\
[e_3, e_4] &= \alpha_{3,4}^{9} e_9 & [e_3, e_5] &= \alpha_{3,5}^{10} e_{10} \\
[e_3, e_6] &= \alpha_{3,6}^{11} e_{11} & [e_3, e_7] &= \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_3, e_9] &= \alpha_{3,9}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{11} e_{11} & [e_4, e_6] &= \alpha_{4,6}^{12} e_{12} \\
[e_4, e_7] &= \alpha_{4,7}^{13} e_{13} & [e_4, e_8] &= \alpha_{4,8}^{14} e_{14} \\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13} & [e_5, e_7] &= \alpha_{5,7}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): & \quad -\alpha^9_{2,5} - \alpha^9_{3,4} + 1 &= 0\\
(e_1, e_2, e_5): & \quad \alpha^9_{2,5} - \alpha^{10}_{2,6} - \alpha^{10}_{3,5} &= 0\\
(e_1, e_3, e_4): & \quad \alpha^9_{3,4} - \alpha^{10}_{3,5} &= 0\\
(e_1, e_2, e_6): & \quad \alpha^{10}_{2,6} - \alpha^{11}_{2,7} - \alpha^{11}_{3,6} &= 0\\
(e_1, e_3, e_5): & \quad \alpha^{10}_{3,5} - \alpha^{11}_{3,6} - \alpha^{11}_{4,5} &= 0\\
(e_1, e_2, e_7): & \quad \alpha^{11}_{2,7} - \alpha^{12}_{2,8} - \alpha^{12}_{3,7} &= 0\\
(e_1, e_3, e_6): & \quad \alpha^{11}_{3,6} - \alpha^{12}_{3,7} - \alpha^{12}_{4,6} &= 0\\
(e_1, e_4, e_5): & \quad \alpha^{11}_{4,5} - \alpha^{12}_{4,6} &= 0\\
(e_1, e_2, e_8): & \quad \alpha^{12}_{2,8} - \alpha^{13}_{2,9} - \alpha^{13}_{3,8} &= 0\\
(e_1, e_3, e_7): & \quad \alpha^{12}_{3,7} - \alpha^{13}_{3,8} - \alpha^{13}_{4,7} &= 0\\
(e_1, e_4, e_6): & \quad \alpha^{12}_{4,6} - \alpha^{13}_{4,7} - \alpha^{13}_{5,6} &= 0\\
(e_2, e_3, e_4): & \quad \alpha^{13}_{2,9}\alpha^9_{3,4} - \alpha^{13}_{3,8} + \alpha^{13}_{4,7} &= 0\\
(e_1, e_2, e_9): & \quad -\alpha^{14}_{2,10} + \alpha^{13}_{2,9} - \alpha^{14}_{3,9} &= 0\\
(e_1, e_3, e_8): & \quad \alpha^{13}_{3,8} - \alpha^{14}_{3,9} - \alpha^{14}_{4,8} &= 0\\
(e_1, e_4, e_7): & \quad \alpha^{13}_{4,7} - \alpha^{14}_{4,8} - \alpha^{14}_{5,7} &= 0\\
(e_1, e_5, e_6): & \quad \alpha^{13}_{5,6} - \alpha^{14}_{5,7} &= 0\\
(e_2, e_3, e_5): & \quad \alpha^{14}_{2,10}\alpha^{10}_{3,5} - \alpha^9_{2,5}\alpha^{14}_{3,9} + \alpha^{14}_{5,7} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{10}_{2,6} \to x_1$$
$$\alpha^{14}_{3,9} \to x_2$$
$$\alpha^{14}_{2,10} \to x_3$$
$$\alpha^9_{2,5} \to x_4$$
$$\alpha^9_{3,4} \to x_5$$
$$\alpha^{11}_{4,5} \to x_6$$
$$\alpha^{11}_{2,7} \to x_7$$
$$\alpha^{12}_{2,8} \to x_8$$
$$\alpha^{13}_{5,6} \to x_9$$
$$\alpha^{12}_{3,7} \to x_{10}$$

$$\alpha^{10}_{3,5} \to x_{11}$$
$$\alpha^{14}_{4,8} \to x_{12}$$
$$\alpha^{13}_{3,8} \to x_{13}$$
$$\alpha^{13}_{2,9} \to x_{14}$$
$$\alpha^{11}_{3,6} \to x_{15}$$
$$\alpha^{13}_{4,7} \to x_{16}$$
$$\alpha^{14}_{5,7} \to x_{17}$$
$$\alpha^{12}_{4,6} \to x_{18}$$

Jacobi Tests

$$\begin{array}{lll}
(e_1,e_2,e_4): & -x_4 - x_5 + 1 & = 0 \\
(e_1,e_2,e_5): & -x_1 - x_{11} + x_4 & = 0 \\
(e_1,e_3,e_4): & -x_{11} + x_5 & = 0 \\
(e_1,e_2,e_6): & x_1 - x_{15} - x_7 & = 0 \\
(e_1,e_3,e_5): & x_{11} - x_{15} - x_6 & = 0 \\
(e_1,e_2,e_7): & -x_{10} + x_7 - x_8 & = 0 \\
(e_1,e_3,e_6): & -x_{10} + x_{15} - x_{18} & = 0 \\
(e_1,e_4,e_5): & -x_{18} + x_6 & = 0 \\
(e_1,e_2,e_8): & -x_{13} - x_{14} + x_8 & = 0 \\
(e_1,e_3,e_7): & x_{10} - x_{13} - x_{16} & = 0 \\
(e_1,e_4,e_6): & -x_{16} + x_{18} - x_9 & = 0 \\
(e_2,e_3,e_4): & -x_{13} + x_{14}x_5 + x_{16} & = 0 \\
(e_1,e_2,e_9): & x_{14} - x_2 - x_3 & = 0 \\
(e_1,e_3,e_8): & -x_{12} + x_{13} - x_2 & = 0 \\
(e_1,e_4,e_7): & -x_{12} + x_{16} - x_{17} & = 0 \\
(e_1,e_5,e_6): & -x_{17} + x_9 & = 0 \\
(e_2,e_3,e_5): & x_{11}x_3 + x_{17} - x_2x_4 & = 0
\end{array}$$

Groebner basis (18 variables, 15 linear, 1 nonlinear)

$$x_1 + 2x_{15} + 2x_{18} - 1 = 0$$
$$-x_{15} - 3x_{17} + 3x_{18} + x_2 = 0$$
$$6x_{15} + 4x_{17} - 4x_{18} + x_3 - 1 = 0$$
$$x_{15} + x_{18} + x_4 - 1 = 0$$

$$-x_{15} - x_{18} + x_5 = 0$$
$$-x_{18} + x_6 = 0$$
$$3x_{15} + 2x_{18} + x_7 - 1 = 0$$
$$4x_{15} + x_{18} + x_8 - 1 = 0$$
$$-x_{17} + x_9 = 0$$
$$x_{10} - x_{15} + x_{18} = 0$$
$$x_{11} - x_{15} - x_{18} = 0$$
$$x_{12} + 2x_{17} - x_{18} = 0$$
$$x_{13} - x_{15} - x_{17} + 2x_{18} = 0$$
$$x_{14} + 5x_{15} + x_{17} - x_{18} - 1 = 0$$
$$x_{15}^2 + \frac{x_{15}x_{17}}{5} + \frac{4x_{15}x_{18}}{5} + \frac{x_{17}x_{18}}{5} + \frac{2x_{17}}{5} - \frac{x_{18}^2}{5} - \frac{4x_{18}}{5} = 0$$
$$x_{16} + x_{17} - x_{18} = 0$$

$\mathfrak{m}_{1A}(5, 14)$

m1A514 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_9] &= e_{14} & [e_3, e_8] &= -e_{14} \\ [e_4, e_7] &= e_{14} & [e_5, e_6] &= -e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(5, 14)$

m3A514 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_7] = e_{12} & [e_2, e_8] = 3e_{13} \\
[e_2, e_9] = \alpha_{2,9}^{14} e_{14} & [e_3, e_6] = -e_{12} \\
[e_3, e_7] = -2e_{13} & [e_3, e_8] = \alpha_{3,8}^{14} e_{14} \\
[e_4, e_5] = e_{12} & [e_4, e_6] = e_{13} \\
[e_4, e_7] = \alpha_{4,7}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{14} e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_8): & -\alpha_{2,9}^{14} - \alpha_{3,8}^{14} + 3 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{14} - \alpha_{4,7}^{14} - 2 & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{14} - \alpha_{5,6}^{14} + 1 & = 0
\end{array}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{array}{l}
\alpha_{3,8}^{14} \to x_1 \\
\alpha_{2,9}^{14} \to x_2 \\
\alpha_{4,7}^{14} \to x_3 \\
\alpha_{5,6}^{14} \to x_4
\end{array}
$$

Jacobi Tests

$$
\begin{array}{lll}
(e_1, e_2, e_8): & -x_1 - x_2 + 3 & = 0 \\
(e_1, e_3, e_7): & -x_1 - x_3 - 2 & = 0 \\
(e_1, e_4, e_6): & -x_3 - x_4 + 1 & = 0
\end{array}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 + 3 = 0$$
$$x_2 + x_4 - 6 = 0$$
$$x_3 + x_4 - 1 = 0$$

$\mathfrak{m}_{5A}(5, 14)$

m5A514 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_2, e_5] = e_{10} && [e_2, e_6] = 2e_{11} \\
&[e_2, e_7] = \alpha_{2,7}^{12} e_{12} && [e_2, e_8] = \alpha_{2,8}^{13} e_{13} \\
&[e_2, e_9] = \alpha_{2,9}^{14} e_{14} && [e_3, e_4] = -e_{10} \\
&[e_3, e_5] = -e_{11} && [e_3, e_6] = \alpha_{3,6}^{12} e_{12} \\
&[e_3, e_7] = \alpha_{3,7}^{13} e_{13} && [e_3, e_8] = \alpha_{3,8}^{14} e_{14} \\
&[e_4, e_5] = \alpha_{4,5}^{12} e_{12} && [e_4, e_6] = \alpha_{4,6}^{13} e_{13} \\
&[e_4, e_7] = \alpha_{4,7}^{14} e_{14} && [e_5, e_6] = \alpha_{5,6}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
&(e_1, e_2, e_6): && -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 && = 0 \\
&(e_1, e_3, e_5): && -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 && = 0 \\
&(e_1, e_2, e_7): && \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} && = 0 \\
&(e_1, e_3, e_6): && \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} && = 0 \\
&(e_1, e_4, e_5): && \alpha_{4,5}^{12} - \alpha_{4,6}^{13} && = 0 \\
&(e_1, e_2, e_8): && \alpha_{2,8}^{13} - \alpha_{2,9}^{14} - \alpha_{3,8}^{14} && = 0 \\
&(e_1, e_3, e_7): && \alpha_{3,7}^{13} - \alpha_{3,8}^{14} - \alpha_{4,7}^{14} && = 0 \\
&(e_1, e_4, e_6): && \alpha_{4,6}^{13} - \alpha_{4,7}^{14} - \alpha_{5,6}^{14} && = 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,6}^{13} \to x_1$$
$$\alpha_{4,5}^{12} \to x_2$$
$$\alpha_{4,7}^{14} \to x_3$$
$$\alpha_{3,8}^{14} \to x_4$$
$$\alpha_{2,8}^{13} \to x_5$$
$$\alpha_{3,7}^{13} \to x_6$$
$$\alpha_{2,7}^{12} \to x_7$$
$$\alpha_{5,6}^{14} \to x_8$$
$$\alpha_{2,9}^{14} \to x_9$$
$$\alpha_{3,6}^{12} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):& \quad -x_{10} - x_7 + 2 &= 0\\
(e_1, e_3, e_5):& \quad -x_{10} - x_2 - 1 &= 0\\
(e_1, e_2, e_7):& \quad -x_5 - x_6 + x_7 &= 0\\
(e_1, e_3, e_6):& \quad -x_1 + x_{10} - x_6 &= 0\\
(e_1, e_4, e_5):& \quad -x_1 + x_2 &= 0\\
(e_1, e_2, e_8):& \quad -x_4 + x_5 - x_9 &= 0\\
(e_1, e_3, e_7):& \quad -x_3 - x_4 + x_6 &= 0\\
(e_1, e_4, e_6):& \quad x_1 - x_3 - x_8 &= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + x_{10} + 1 = 0$$
$$x_{10} + x_2 + 1 = 0$$
$$-5x_{10} + x_3 - x_9 = 0$$
$$3x_{10} + x_4 + x_9 - 1 = 0$$
$$3x_{10} + x_5 - 1 = 0$$
$$-2x_{10} + x_6 - 1 = 0$$
$$x_{10} + x_7 - 2 = 0$$
$$6x_{10} + x_8 + x_9 + 1 = 0$$

$\mathfrak{m}_{7A}(5, 14)$

m7A514 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_3] &= e_8 & [e_2, e_4] &= e_9 \\
[e_2, e_5] &= \alpha_{2,5}^{10} e_{10} & [e_2, e_6] &= \alpha_{2,6}^{11} e_{11} \\
[e_2, e_7] &= \alpha_{2,7}^{12} e_{12} & [e_2, e_8] &= \alpha_{2,8}^{13} e_{13} \\
[e_2, e_9] &= \alpha_{2,9}^{14} e_{14} & [e_3, e_4] &= \alpha_{3,4}^{10} e_{10} \\
[e_3, e_5] &= \alpha_{3,5}^{11} e_{11} & [e_3, e_6] &= \alpha_{3,6}^{12} e_{12} \\
[e_3, e_7] &= \alpha_{3,7}^{13} e_{13} & [e_3, e_8] &= \alpha_{3,8}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{12} e_{12} & [e_4, e_6] &= \alpha_{4,6}^{13} e_{13} \\
[e_4, e_7] &= \alpha_{4,7}^{14} e_{14} & [e_5, e_6] &= \alpha_{5,6}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):& \quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &&= 0 \\
(e_1, e_2, e_5):& \quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &&= 0 \\
(e_1, e_3, e_4):& \quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &&= 0 \\
(e_1, e_2, e_6):& \quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &&= 0 \\
(e_1, e_3, e_5):& \quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &&= 0 \\
(e_1, e_2, e_7):& \quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &&= 0 \\
(e_1, e_3, e_6):& \quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &&= 0 \\
(e_1, e_4, e_5):& \quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &&= 0 \\
(e_1, e_2, e_8):& \quad \alpha_{2,8}^{13} - \alpha_{2,9}^{14} - \alpha_{3,8}^{14} &&= 0 \\
(e_1, e_3, e_7):& \quad \alpha_{3,7}^{13} - \alpha_{3,8}^{14} - \alpha_{4,7}^{14} &&= 0 \\
(e_1, e_4, e_6):& \quad \alpha_{4,6}^{13} - \alpha_{4,7}^{14} - \alpha_{5,6}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{4,6}^{13} \to x_1$$
$$\alpha_{4,5}^{12} \to x_2$$
$$\alpha_{4,7}^{14} \to x_3$$
$$\alpha_{3,8}^{14} \to x_4$$
$$\alpha_{3,5}^{11} \to x_5$$
$$\alpha_{2,8}^{13} \to x_6$$
$$\alpha_{2,5}^{10} \to x_7$$
$$\alpha_{3,7}^{13} \to x_8$$
$$\alpha_{3,6}^{12} \to x_9$$
$$\alpha_{2,7}^{12} \to x_{10}$$
$$\alpha_{5,6}^{14} \to x_{11}$$
$$\alpha_{3,4}^{10} \to x_{12}$$
$$\alpha_{2,9}^{14} \to x_{13}$$
$$\alpha_{2,6}^{11} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{12} - x_7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{14} - x_5 + x_7 &&= 0\\
(e_1, e_3, e_4): &\quad x_{12} - x_5 &&= 0\\
(e_1, e_2, e_6): &\quad -x_{10} + x_{14} - x_9 &&= 0\\
(e_1, e_3, e_5): &\quad -x_2 + x_5 - x_9 &&= 0\\
(e_1, e_2, e_7): &\quad x_{10} - x_6 - x_8 &&= 0\\
(e_1, e_3, e_6): &\quad -x_1 - x_8 + x_9 &&= 0\\
(e_1, e_4, e_5): &\quad -x_1 + x_2 &&= 0\\
(e_1, e_2, e_8): &\quad -x_{13} - x_4 + x_6 &&= 0\\
(e_1, e_3, e_7): &\quad -x_3 - x_4 + x_8 &&= 0\\
(e_1, e_4, e_6): &\quad x_1 - x_{11} - x_3 &&= 0
\end{aligned}$$

Groebner basis (14 variables, 11 linear, 0 nonlinear)

$$x_1 - \frac{x_{11}}{6} - \frac{x_{13}}{6} + \frac{5x_{14}}{12} - \frac{1}{4} = 0$$

$$-\frac{x_{11}}{6} - \frac{x_{13}}{6} + \frac{5x_{14}}{12} + x_2 - \frac{1}{4} = 0$$
$$\frac{5x_{11}}{6} - \frac{x_{13}}{6} + \frac{5x_{14}}{12} + x_3 - \frac{1}{4} = 0$$
$$-\frac{x_{11}}{2} + \frac{x_{13}}{2} - \frac{3x_{14}}{4} + x_4 + \frac{1}{4} = 0$$
$$\frac{x_{14}}{2} + x_5 - \frac{1}{2} = 0$$
$$-\frac{x_{11}}{2} - \frac{x_{13}}{2} - \frac{3x_{14}}{4} + x_6 + \frac{1}{4} = 0$$
$$-\frac{x_{14}}{2} + x_7 - \frac{1}{2} = 0$$
$$\frac{x_{11}}{3} + \frac{x_{13}}{3} - \frac{x_{14}}{3} + x_8 = 0$$
$$\frac{x_{11}}{6} + \frac{x_{13}}{6} + \frac{x_{14}}{12} + x_9 - \frac{1}{4} = 0$$
$$x_{10} - \frac{x_{11}}{6} - \frac{x_{13}}{6} - \frac{13x_{14}}{12} + \frac{1}{4} = 0$$
$$x_{12} + \frac{x_{14}}{2} - \frac{1}{2} = 0$$

$\mathfrak{m}_{2A}(6, 14)$

m2A614 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_7] &= e_{13} & [e_2, e_8] &= 3e_{14} \\ [e_3, e_6] &= -e_{13} & [e_3, e_7] &= -2e_{14} \\ [e_4, e_5] &= e_{13} & [e_4, e_6] &= e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(6,14)$

m4A614 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14} \\
[e_2,e_5] &= e_{11} & [e_2,e_6] &= 2e_{12} \\
[e_2,e_7] &= \alpha_{2,7}^{13}e_{13} & [e_2,e_8] &= \alpha_{2,8}^{14}e_{14} \\
[e_3,e_4] &= -e_{11} & [e_3,e_5] &= -e_{12} \\
[e_3,e_6] &= \alpha_{3,6}^{13}e_{13} & [e_3,e_7] &= \alpha_{3,7}^{14}e_{14} \\
[e_4,e_5] &= \alpha_{4,5}^{13}e_{13} & [e_4,e_6] &= \alpha_{4,6}^{14}e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6):&\quad -\alpha_{2,7}^{13}-\alpha_{3,6}^{13}+2 &&= 0 \\
(e_1,e_3,e_5):&\quad -\alpha_{3,6}^{13}-\alpha_{4,5}^{13}-1 &&= 0 \\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{13}-\alpha_{2,8}^{14}-\alpha_{3,7}^{14} &&= 0 \\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{13}-\alpha_{3,7}^{14}-\alpha_{4,6}^{14} &&= 0 \\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{13}-\alpha_{4,6}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{3,6}^{13} &\to x_1 \\
\alpha_{2,7}^{13} &\to x_2 \\
\alpha_{4,6}^{14} &\to x_3 \\
\alpha_{2,8}^{14} &\to x_4 \\
\alpha_{4,5}^{13} &\to x_5 \\
\alpha_{3,7}^{14} &\to x_6
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_1 - x_2 + 2 &= 0\\
(e_1, e_3, e_5):&\quad -x_1 - x_5 - 1 &= 0\\
(e_1, e_2, e_7):&\quad x_2 - x_4 - x_6 &= 0\\
(e_1, e_3, e_6):&\quad x_1 - x_3 - x_6 &= 0\\
(e_1, e_4, e_5):&\quad -x_3 + x_5 &= 0
\end{aligned}$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$x_1 - \frac{x_6}{2} + \frac{1}{2} = 0$$

$$x_2 + \frac{x_6}{2} - \frac{5}{2} = 0$$

$$x_3 + \frac{x_6}{2} + \frac{1}{2} = 0$$

$$x_4 + \frac{3x_6}{2} - \frac{5}{2} = 0$$

$$x_5 + \frac{x_6}{2} + \frac{1}{2} = 0$$

$\mathfrak{m}_{6A}(6, 14)$

m6A614 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_2, e_3] &= e_9 & [e_2, e_4] &= e_{10}\\
[e_2, e_5] &= \alpha_{2,5}^{11} e_{11} & [e_2, e_6] &= \alpha_{2,6}^{12} e_{12}\\
[e_2, e_7] &= \alpha_{2,7}^{13} e_{13} & [e_2, e_8] &= \alpha_{2,8}^{14} e_{14}\\
[e_3, e_4] &= \alpha_{3,4}^{11} e_{11} & [e_3, e_5] &= \alpha_{3,5}^{12} e_{12}\\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_3, e_7] &= \alpha_{3,7}^{14} e_{14}\\
[e_4, e_5] &= \alpha_{4,5}^{13} e_{13} & [e_4, e_6] &= \alpha_{4,6}^{14} e_{14}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{12} - \alpha_{2,7}^{13} - \alpha_{3,6}^{13} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{12} - \alpha_{3,6}^{13} - \alpha_{4,5}^{13} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{13} - \alpha_{2,8}^{14} - \alpha_{3,7}^{14} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{13} - \alpha_{3,7}^{14} - \alpha_{4,6}^{14} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{13} - \alpha_{4,6}^{14} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\begin{aligned}
\alpha_{2,6}^{12} &\to x_1\\
\alpha_{4,5}^{13} &\to x_2\\
\alpha_{3,6}^{13} &\to x_3\\
\alpha_{2,7}^{13} &\to x_4\\
\alpha_{3,4}^{11} &\to x_5\\
\alpha_{4,6}^{14} &\to x_6\\
\alpha_{2,5}^{11} &\to x_7\\
\alpha_{2,8}^{14} &\to x_8\\
\alpha_{3,5}^{12} &\to x_9\\
\alpha_{3,7}^{14} &\to x_{10}
\end{aligned}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_5 - x_7 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_1 + x_7 - x_9 &= 0\\
(e_1, e_3, e_4): &\quad x_5 - x_9 &= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_3 - x_4 &= 0\\
(e_1, e_3, e_5): &\quad -x_2 - x_3 + x_9 &= 0\\
(e_1, e_2, e_7): &\quad -x_{10} + x_4 - x_8 &= 0\\
(e_1, e_3, e_6): &\quad -x_{10} + x_3 - x_6 &= 0\\
(e_1, e_4, e_5): &\quad x_2 - x_6 &= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + 2x_9 - 1 = 0$$
$$\frac{x_{10}}{2} + x_2 - \frac{x_9}{2} = 0$$
$$-\frac{x_{10}}{2} + x_3 - \frac{x_9}{2} = 0$$
$$\frac{x_{10}}{2} + x_4 + \frac{5x_9}{2} - 1 = 0$$
$$x_5 - x_9 = 0$$
$$\frac{x_{10}}{2} + x_6 - \frac{x_9}{2} = 0$$
$$x_7 + x_9 - 1 = 0$$
$$\frac{3x_{10}}{2} + x_8 + \frac{5x_9}{2} - 1 = 0$$

$\mathfrak{m}_{1A}(7, 14)$

m1A714 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_2, e_7] = e_{14} \qquad [e_3, e_6] = -e_{14}$$
$$[e_4, e_5] = e_{14}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(7, 14)$

m3A714 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_5] &= e_{12} & [e_2, e_6] &= 2e_{13} \\
[e_2, e_7] &= \alpha_{2,7}^{14} e_{14} & [e_3, e_4] &= -e_{12} \\
[e_3, e_5] &= -e_{13} & [e_3, e_6] &= \alpha_{3,6}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{14} - \alpha_{3,6}^{14} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{14} - \alpha_{4,5}^{14} - 1 &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{14} \to x_1$$

$$\alpha_{3,6}^{14} \to x_2$$

$$\alpha_{4,5}^{14} \to x_3$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_2 + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -x_2 - x_3 - 1 &= 0
\end{aligned}
$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 - x_3 - 3 = 0$$

$$x_2 + x_3 + 1 = 0$$

$\mathfrak{m}_{5A}(7, 14)$

m5A714 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_2, e_3] &= e_{10} & [e_2, e_4] &= e_{11} \\
[e_2, e_5] &= \alpha_{2,5}^{12} e_{12} & [e_2, e_6] &= \alpha_{2,6}^{13} e_{13} \\
[e_2, e_7] &= \alpha_{2,7}^{14} e_{14} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12} \\
[e_3, e_5] &= \alpha_{3,5}^{13} e_{13} & [e_3, e_6] &= \alpha_{3,6}^{14} e_{14} \\
[e_4, e_5] &= \alpha_{4,5}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{12} - \alpha_{3,5}^{13} &= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{13} - \alpha_{2,7}^{14} - \alpha_{3,6}^{14} &= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{13} - \alpha_{3,6}^{14} - \alpha_{4,5}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,6}^{13} &\to x_1 \\
\alpha_{2,7}^{14} &\to x_2 \\
\alpha_{3,5}^{13} &\to x_3 \\
\alpha_{3,6}^{14} &\to x_4 \\
\alpha_{3,4}^{12} &\to x_5 \\
\alpha_{2,5}^{12} &\to x_6 \\
\alpha_{4,5}^{14} &\to x_7
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_5 - x_6 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_3 + x_6 &= 0\\
(e_1, e_3, e_4): &\quad -x_3 + x_5 &= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_2 - x_4 &= 0\\
(e_1, e_3, e_5): &\quad x_3 - x_4 - x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - 2x_6 + 1 = 0$$
$$x_2 - 3x_6 - x_7 + 2 = 0$$
$$x_3 + x_6 - 1 = 0$$
$$x_4 + x_6 + x_7 - 1 = 0$$
$$x_5 + x_6 - 1 = 0$$

$\mathfrak{m}_{2A}(8, 14)$

m2A814 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_2, e_5] &= e_{13} & [e_2, e_6] &= 2e_{14}\\
[e_3, e_4] &= -e_{13} & [e_3, e_5] &= -e_{14}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(8, 14)$

m4A814 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_2, e_3] = e_{11} & [e_2, e_4] = e_{12} \\
[e_2, e_5] = \alpha_{2,5}^{13} e_{13} & [e_2, e_6] = \alpha_{2,6}^{14} e_{14} \\
[e_3, e_4] = \alpha_{3,4}^{13} e_{13} & [e_3, e_5] = \alpha_{3,5}^{14} e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_4): & -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 & = 0 \\
(e_1, e_2, e_5): & \alpha_{2,5}^{13} - \alpha_{2,6}^{14} - \alpha_{3,5}^{14} & = 0 \\
(e_1, e_3, e_4): & \alpha_{3,4}^{13} - \alpha_{3,5}^{14} & = 0
\end{array}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,5}^{14} \to x_1$$

$$\alpha_{2,6}^{14} \to x_2$$

$$\alpha_{2,5}^{13} \to x_3$$

$$\alpha_{3,4}^{13} \to x_4$$

Jacobi Tests

$$
\begin{array}{lll}
(e_1, e_2, e_4): & -x_3 - x_4 + 1 & = 0 \\
(e_1, e_2, e_5): & -x_1 - x_2 + x_3 & = 0 \\
(e_1, e_3, e_4): & -x_1 + x_4 & = 0
\end{array}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 = 0$$

$$x_2 + 2x_4 - 1 = 0$$

$$x_3 + x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(9, 14)$

m1A914 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_5] &= e_{14} & [e_3, e_4] &= -e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(9, 14)$

m3A914 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_3] &= e_{12} & [e_2, e_4] &= e_{13} \\ [e_2, e_5] &= \alpha_{2,5}^{14} e_{14} & [e_3, e_4] &= \alpha_{3,4}^{14} e_{14} \end{aligned}$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{14} - \alpha_{3,4}^{14} + 1 = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,5}^{14} \to x_1$$
$$\alpha_{3,4}^{14} \to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 \qquad = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(10, 14)$

m2A1014 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_3] &= e_{13} & [e_2, e_4] &= e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(11, 14)$

m1A1114 (this line included for string searching purposes)
Original brackets:

$$\begin{aligned} [e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\ [e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\ [e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\ [e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\ [e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\ [e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\ [e_2, e_3] &= e_{14} \end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(2,15)$

m1A215 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_1,e_{13}]=e_{14} \\
[e_1,e_{14}]=e_{15} & [e_2,e_{13}]=e_{15} \\
[e_3,e_{12}]=-e_{15} & [e_4,e_{11}]=e_{15} \\
[e_5,e_{10}]=-e_{15} & [e_6,e_9]=e_{15} \\
[e_7,e_8]=-e_{15} &
\end{array}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(2,15)$

m3A215 (this line included for string searching purposes)

Solution 1

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_1,e_{13}]=e_{14} \\
[e_1,e_{14}]=e_{15} & [e_2,e_{11}]=e_{13} \\
[e_2,e_{12}]=5e_{14} & [e_2,e_{13}]=0 \\
[e_3,e_{10}]=-e_{13} & [e_3,e_{11}]=-4e_{14} \\
[e_3,e_{12}]=5e_{15} & [e_4,e_9]=e_{13} \\
[e_4,e_{10}]=3e_{14} & [e_4,e_{11}]=-9e_{15} \\
[e_5,e_8]=-e_{13} & [e_5,e_9]=-2e_{14} \\
[e_5,e_{10}]=12e_{15} & [e_6,e_7]=e_{13} \\
[e_6,e_8]=e_{14} & [e_6,e_9]=-14e_{15} \\
[e_7,e_8]=15e_{15} &
\end{array}
$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_{11}] = e_{13} \\
[e_2, e_{12}] = 5e_{14} & [e_2, e_{13}] = \alpha_{2,13}^{15} e_{15} \\
[e_3, e_{10}] = -e_{13} & [e_3, e_{11}] = -4e_{14} \\
[e_3, e_{12}] = \alpha_{3,12}^{15} e_{15} & [e_4, e_9] = e_{13} \\
[e_4, e_{10}] = 3e_{14} & [e_4, e_{11}] = \alpha_{4,11}^{15} e_{15} \\
[e_5, e_8] = -e_{13} & [e_5, e_9] = -2e_{14} \\
[e_5, e_{10}] = \alpha_{5,10}^{15} e_{15} & [e_6, e_7] = e_{13} \\
[e_6, e_8] = e_{14} & [e_6, e_9] = \alpha_{6,9}^{15} e_{15} \\
[e_7, e_8] = \alpha_{7,8}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rll}
(e_1, e_2, e_{12}): & -\alpha_{2,13}^{15} - \alpha_{3,12}^{15} + 5 & = 0 \\
(e_1, e_3, e_{11}): & -\alpha_{3,12}^{15} - \alpha_{4,11}^{15} - 4 & = 0 \\
(e_1, e_4, e_{10}): & -\alpha_{4,11}^{15} - \alpha_{5,10}^{15} + 3 & = 0 \\
(e_1, e_5, e_9): & -\alpha_{5,10}^{15} - \alpha_{6,9}^{15} - 2 & = 0 \\
(e_1, e_6, e_8): & -\alpha_{6,9}^{15} - \alpha_{7,8}^{15} + 1 & = 0 \\
(e_2, e_3, e_{10}): & -\alpha_{2,13}^{15} & = 0 \\
(e_2, e_4, e_9): & \alpha_{2,13}^{15} & = 0 \\
(e_2, e_5, e_8): & -\alpha_{2,13}^{15} & = 0 \\
(e_2, e_6, e_7): & \alpha_{2,13}^{15} & = 0
\end{array}
$$

Solution 1:

$$\alpha^{15}_{6,9} = -14$$
$$\alpha^{15}_{7,8} = 15$$
$$\alpha^{15}_{4,11} = -9$$
$$\alpha^{15}_{2,13} = 0$$
$$\alpha^{15}_{5,10} = 12$$
$$\alpha^{15}_{3,12} = 5$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{15}_{6,9} \to x_1$$
$$\alpha^{15}_{7,8} \to x_2$$
$$\alpha^{15}_{4,11} \to x_3$$
$$\alpha^{15}_{2,13} \to x_4$$
$$\alpha^{15}_{5,10} \to x_5$$
$$\alpha^{15}_{3,12} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}): &\quad -x_4 - x_6 + 5 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_3 - x_6 - 4 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_3 - x_5 + 3 &&= 0\\
(e_1, e_5, e_9): &\quad -x_1 - x_5 - 2 &&= 0\\
(e_1, e_6, e_8): &\quad -x_1 - x_2 + 1 &&= 0\\
(e_2, e_3, e_{10}): &\quad -x_4 &&= 0\\
(e_2, e_4, e_9): &\quad x_4 &&= 0\\
(e_2, e_5, e_8): &\quad -x_4 &&= 0\\
(e_2, e_6, e_7): &\quad x_4 &&= 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 + 14 = 0$$
$$x_2 - 15 = 0$$
$$x_3 + 9 = 0$$

$$x_4 = 0$$
$$x_5 - 12 = 0$$
$$x_6 - 5 = 0$$

Solution 1:

$$x_1 = -14$$
$$x_2 = 15$$
$$x_3 = -9$$
$$x_4 = 0$$
$$x_5 = 12$$
$$x_6 = 5$$

$\mathfrak{m}_{11A}(2, 15)$

m11A215 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_3] = e_5$$
$$[e_2, e_4] = e_6 \qquad [e_2, e_5] = e_7$$
$$[e_2, e_6] = e_8 \qquad [e_2, e_7] = e_9$$
$$[e_2, e_8] = e_{10} \qquad [e_2, e_9] = e_{11}$$
$$[e_2, e_{10}] = e_{12} \qquad [e_2, e_{11}] = e_{13}$$
$$[e_2, e_{12}] = e_{14} \qquad [e_2, e_{13}] = e_{15}$$
$$[e_3, e_4] = 0 \qquad [e_3, e_5] = 0$$
$$[e_3, e_6] = 0 \qquad [e_3, e_7] = 0$$
$$[e_3, e_8] = 0 \qquad [e_3, e_9] = 0$$
$$[e_3, e_{10}] = 0 \qquad [e_3, e_{11}] = 0$$
$$[e_3, e_{12}] = 0 \qquad [e_4, e_5] = 0$$
$$[e_4, e_6] = 0 \qquad [e_4, e_7] = 0$$
$$[e_4, e_8] = 0 \qquad [e_4, e_9] = 0$$
$$[e_4, e_{10}] = 0 \qquad [e_4, e_{11}] = 0$$
$$[e_5, e_6] = 0 \qquad [e_5, e_7] = 0$$
$$[e_5, e_8] = 0 \qquad [e_5, e_9] = 0$$
$$[e_5, e_{10}] = 0 \qquad [e_6, e_7] = 0$$
$$[e_6, e_8] = 0 \qquad [e_6, e_9] = 0$$
$$[e_7, e_8] = 0$$

Solution 2

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_1,e_{13}]=e_{14}$$
$$[e_1,e_{14}]=e_{15} \qquad [e_2,e_3]=e_5$$
$$[e_2,e_4]=e_6 \qquad [e_2,e_5]=\frac{9e_7}{10}$$
$$[e_2,e_6]=\frac{4e_8}{5} \qquad [e_2,e_7]=\frac{5e_9}{7}$$
$$[e_2,e_8]=\frac{9e_{10}}{14} \qquad [e_2,e_9]=\frac{7e_{11}}{12}$$
$$[e_2,e_{10}]=\frac{8e_{12}}{15} \qquad [e_2,e_{11}]=\frac{27e_{13}}{55}$$
$$[e_2,e_{12}]=\frac{5e_{14}}{11} \qquad [e_2,e_{13}]=\frac{11e_{15}}{26}$$
$$[e_3,e_4]=\frac{e_7}{10} \qquad [e_3,e_5]=\frac{e_8}{10}$$
$$[e_3,e_6]=\frac{3e_9}{35} \qquad [e_3,e_7]=\frac{e_{10}}{14}$$
$$[e_3,e_8]=\frac{5e_{11}}{84} \qquad [e_3,e_9]=\frac{e_{12}}{20}$$
$$[e_3,e_{10}]=\frac{7e_{13}}{165} \qquad [e_3,e_{11}]=\frac{2e_{14}}{55}$$
$$[e_3,e_{12}]=\frac{9e_{15}}{286} \qquad [e_4,e_5]=\frac{e_9}{70}$$
$$[e_4,e_6]=\frac{e_{10}}{70} \qquad [e_4,e_7]=\frac{e_{11}}{84}$$
$$[e_4,e_8]=\frac{e_{12}}{105} \qquad [e_4,e_9]=\frac{e_{13}}{132}$$
$$[e_4,e_{10}]=\frac{e_{14}}{165} \qquad [e_4,e_{11}]=\frac{7e_{15}}{1430}$$
$$[e_5,e_6]=\frac{e_{11}}{420} \qquad [e_5,e_7]=\frac{e_{12}}{420}$$
$$[e_5,e_8]=\frac{3e_{13}}{1540} \qquad [e_5,e_9]=\frac{e_{14}}{660}$$
$$[e_5,e_{10}]=\frac{e_{15}}{858} \qquad [e_6,e_7]=\frac{e_{13}}{2310}$$
$$[e_6,e_8]=\frac{e_{14}}{2310} \qquad [e_6,e_9]=\frac{e_{15}}{2860}$$
$$[e_7,e_8]=\frac{e_{15}}{12012}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_2, e_{12}] = \alpha_{2,12}^{14} e_{14} & [e_2, e_{13}] = \alpha_{2,13}^{15} e_{15} \\
[e_3, e_4] = \alpha_{3,4}^{7} e_7 & [e_3, e_5] = \alpha_{3,5}^{8} e_8 \\
[e_3, e_6] = \alpha_{3,6}^{9} e_9 & [e_3, e_7] = \alpha_{3,7}^{10} e_{10} \\
[e_3, e_8] = \alpha_{3,8}^{11} e_{11} & [e_3, e_9] = \alpha_{3,9}^{12} e_{12} \\
[e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} & [e_3, e_{11}] = \alpha_{3,11}^{14} e_{14} \\
[e_3, e_{12}] = \alpha_{3,12}^{15} e_{15} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_8] = \alpha_{4,8}^{12} e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_4, e_{10}] = \alpha_{4,10}^{14} e_{14} & [e_4, e_{11}] = \alpha_{4,11}^{15} e_{15} \\
[e_5, e_6] = \alpha_{5,6}^{11} e_{11} & [e_5, e_7] = \alpha_{5,7}^{12} e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{13} e_{13} & [e_5, e_9] = \alpha_{5,9}^{14} e_{14} \\
[e_5, e_{10}] = \alpha_{5,10}^{15} e_{15} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13} \\
[e_6, e_8] = \alpha_{6,8}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{15} e_{15} \\
[e_7, e_8] = \alpha_{7,8}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{7}_{2,5}-\alpha^{7}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{7}_{2,5}-\alpha^{8}_{2,6}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{7}_{3,4}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{8}_{2,6}-\alpha^{9}_{2,7}-\alpha^{9}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{8}_{3,5}-\alpha^{9}_{3,6}-\alpha^{9}_{4,5} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{9}_{2,7}\alpha^{7}_{3,4}-\alpha^{9}_{3,6}+\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{9}_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{9}_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{9}_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha^{8}_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^{9}_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^{9}_{4,5}+\alpha^{11}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{12}_{2,10}+\alpha^{11}_{2,9}-\alpha^{12}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{11}_{3,8}-\alpha^{12}_{3,9}-\alpha^{12}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{11}_{4,7}-\alpha^{12}_{4,8}-\alpha^{12}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{11}_{5,6}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_3,e_7):&\quad \alpha^{12}_{2,10}\alpha^{10}_{3,7}-\alpha^{9}_{2,7}\alpha^{12}_{3,9}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_4,e_6):&\quad \alpha^{12}_{2,10}\alpha^{10}_{4,6}-\alpha^{8}_{2,6}\alpha^{12}_{4,8} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^{7}_{3,4}\alpha^{12}_{5,7}-\alpha^{8}_{3,5}\alpha^{12}_{4,8}+\alpha^{12}_{3,9}\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha^{12}_{2,10}-\alpha^{13}_{2,11}-\alpha^{13}_{3,10} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{13}_{3,10}+\alpha^{12}_{3,9}-\alpha^{13}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad \alpha^{12}_{4,8}-\alpha^{13}_{4,9}-\alpha^{13}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad \alpha^{12}_{5,7}-\alpha^{13}_{5,8}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_3,e_8):&\quad \alpha^{13}_{2,11}\alpha^{11}_{3,8}-\alpha^{10}_{2,8}\alpha^{13}_{3,10}-\alpha^{13}_{5,8} &&=0\\
(e_2,e_4,e_7):&\quad \alpha^{13}_{2,11}\alpha^{11}_{4,7}-\alpha^{9}_{2,7}\alpha^{13}_{4,9}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_5,e_6):&\quad \alpha^{13}_{2,11}\alpha^{11}_{5,6}+\alpha^{7}_{2,5}\alpha^{13}_{6,7}-\alpha^{8}_{2,6}\alpha^{13}_{5,8} &&=0\\
(e_3,e_4,e_6):&\quad \alpha^{13}_{3,10}\alpha^{10}_{4,6}+\alpha^{7}_{3,4}\alpha^{13}_{6,7}-\alpha^{9}_{3,6}\alpha^{13}_{4,9} &&=0\\
(e_1,e_2,e_{11}):&\quad \alpha^{13}_{2,11}-\alpha^{14}_{2,12}-\alpha^{14}_{3,11} &&=0\\
(e_1,e_3,e_{10}):&\quad \alpha^{13}_{3,10}-\alpha^{14}_{3,11}-\alpha^{14}_{4,10} &&=0\\
(e_1,e_4,e_9):&\quad -\alpha^{14}_{4,10}+\alpha^{13}_{4,9}-\alpha^{14}_{5,9} &&=0\\
(e_1,e_5,e_8):&\quad \alpha^{13}_{5,8}-\alpha^{14}_{5,9}-\alpha^{14}_{6,8} &&=0\\
(e_1,e_6,e_7):&\quad \alpha^{13}_{6,7}-\alpha^{14}_{6,8} &&=0\\
(e_2,e_3,e_9):&\quad \alpha^{14}_{2,12}\alpha^{12}_{3,9}-\alpha^{11}_{2,9}\alpha^{14}_{3,11}-\alpha^{14}_{5,9} &&=0\\
(e_2,e_4,e_8):&\quad \alpha^{14}_{2,12}\alpha^{12}_{4,8}-\alpha^{10}_{2,8}\alpha^{14}_{4,10}-\alpha^{14}_{6,8} &&=0\\
(e_2,e_5,e_7):&\quad \alpha^{14}_{2,12}\alpha^{12}_{5,7}-\alpha^{9}_{2,7}\alpha^{14}_{5,9} &&=0\\
(e_3,e_4,e_7):&\quad \alpha^{14}_{3,11}\alpha^{11}_{4,7}-\alpha^{10}_{3,7}\alpha^{14}_{4,10} &&=0\\
(e_3,e_5,e_6):&\quad \alpha^{14}_{3,11}\alpha^{11}_{5,6}+\alpha^{8}_{3,5}\alpha^{14}_{6,8}-\alpha^{9}_{3,6}\alpha^{14}_{5,9} &&=0
\end{aligned}
$$

Solution 1:

$$\alpha^{15}_{4,11} = 0$$
$$\alpha^{11}_{3,8} = 0$$
$$\alpha^{11}_{4,7} = 0$$
$$\alpha^{15}_{3,12} = 0$$
$$\alpha^{13}_{6,7} = 0$$
$$\alpha^{12}_{5,7} = 0$$
$$\alpha^{8}_{2,6} = 1$$
$$\alpha^{12}_{3,9} = 0$$
$$\alpha^{12}_{4,8} = 0$$
$$\alpha^{7}_{2,5} = 1$$
$$\alpha^{9}_{2,7} = 1$$
$$\alpha^{10}_{3,7} = 0$$
$$\alpha^{10}_{2,8} = 1$$
$$\alpha^{9}_{4,5} = 0$$
$$\alpha^{13}_{3,10} = 0$$
$$\alpha^{14}_{6,8} = 0$$
$$\alpha^{15}_{7,8} = 0$$
$$\alpha^{13}_{5,8} = 0$$
$$\alpha^{8}_{3,5} = 0$$
$$\alpha^{11}_{5,6} = 0$$
$$\alpha^{15}_{2,13} = 1$$
$$\alpha^{15}_{5,10} = 0$$
$$\alpha^{10}_{4,6} = 0$$
$$\alpha^{14}_{3,11} = 0$$
$$\alpha^{11}_{2,9} = 1$$
$$\alpha^{13}_{2,11} = 1$$
$$\alpha^{13}_{4,9} = 0$$
$$\alpha^{14}_{5,9} = 0$$
$$\alpha^{15}_{6,9} = 0$$
$$\alpha^{14}_{4,10} = 0$$
$$\alpha^{9}_{3,6} = 0$$
$$\alpha^{7}_{3,4} = 0$$
$$\alpha^{14}_{2,12} = 1$$
$$\alpha^{12}_{2,10} = 1$$

Solution 2:

$$\alpha^{15}_{4,11} = 7/1430$$
$$\alpha^{11}_{3,8} = 5/84$$
$$\alpha^{11}_{4,7} = 1/84$$
$$\alpha^{15}_{3,12} = 9/286$$
$$\alpha^{13}_{6,7} = 1/2310$$
$$\alpha^{12}_{5,7} = 1/420$$
$$\alpha^{8}_{2,6} = 4/5$$
$$\alpha^{12}_{3,9} = 1/20$$
$$\alpha^{12}_{4,8} = 1/105$$
$$\alpha^{7}_{2,5} = 9/10$$
$$\alpha^{9}_{2,7} = 5/7$$
$$\alpha^{10}_{3,7} = 1/14$$
$$\alpha^{10}_{2,8} = 9/14$$
$$\alpha^{9}_{4,5} = 1/70$$
$$\alpha^{13}_{3,10} = 7/165$$
$$\alpha^{14}_{6,8} = 1/2310$$
$$\alpha^{15}_{7,8} = 1/12012$$
$$\alpha^{13}_{5,8} = 3/1540$$
$$\alpha^{8}_{3,5} = 1/10$$
$$\alpha^{11}_{5,6} = 1/420$$
$$\alpha^{15}_{2,13} = 11/26$$
$$\alpha^{15}_{5,10} = 1/858$$
$$\alpha^{10}_{4,6} = 1/70$$
$$\alpha^{14}_{3,11} = 2/55$$
$$\alpha^{11}_{2,9} = 7/12$$
$$\alpha^{13}_{2,11} = 27/55$$
$$\alpha^{13}_{4,9} = 1/132$$
$$\alpha^{14}_{5,9} = 1/660$$
$$\alpha^{15}_{6,9} = 1/2860$$
$$\alpha^{14}_{4,10} = 1/165$$
$$\alpha^{9}_{3,6} = 3/35$$
$$\alpha^{7}_{3,4} = 1/10$$
$$\alpha^{14}_{2,12} = 5/11$$
$$\alpha^{12}_{2,10} = 8/15$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,11}^{15} \to x_1$$

$$\alpha_{3,8}^{11} \to x_2$$

$$\alpha_{4,7}^{11} \to x_3$$

$$\alpha_{3,12}^{15} \to x_4$$

$$\alpha_{6,7}^{13} \to x_5$$

$$\alpha_{5,7}^{12} \to x_6$$

$$\alpha_{2,6}^{8} \to x_7$$

$$\alpha_{3,9}^{12} \to x_8$$

$$\alpha_{4,8}^{12} \to x_9$$

$$\alpha_{2,5}^{7} \to x_{10}$$

$$\alpha_{2,7}^{9} \to x_{11}$$

$$\alpha_{3,7}^{10} \to x_{12}$$

$$\alpha_{2,8}^{10} \to x_{13}$$

$$\alpha_{4,5}^{9} \to x_{14}$$

$$\alpha_{3,10}^{13} \to x_{15}$$

$$\alpha_{6,8}^{14} \to x_{16}$$

$$\alpha_{7,8}^{15} \to x_{17}$$

$$\alpha_{5,8}^{13} \to x_{18}$$

$$\alpha_{3,5}^{8} \to x_{19}$$

$$\alpha_{5,6}^{11} \to x_{20}$$

$$\alpha_{2,13}^{15} \to x_{21}$$

$$\alpha_{5,10}^{15} \to x_{22}$$

$$\alpha_{4,6}^{10} \to x_{23}$$

$$\alpha_{3,11}^{14} \to x_{24}$$

$$\alpha_{2,9}^{11} \to x_{25}$$

$$\alpha_{2,11}^{13} \to x_{26}$$

$$\alpha_{4,9}^{13} \to x_{27}$$

$$\alpha^{14}_{5,9} \to x_{28}$$

$$\alpha^{15}_{6,9} \to x_{29}$$

$$\alpha^{14}_{4,10} \to x_{30}$$

$$\alpha^{9}_{3,6} \to x_{31}$$

$$\alpha^{7}_{3,4} \to x_{32}$$

$$\alpha^{14}_{2,12} \to x_{33}$$

$$\alpha^{12}_{2,10} \to x_{34}$$

Jacobi Tests

$$
\begin{array}{rll}
(e_1, e_2, e_4): & -x_{10} - x_{32} + 1 & = 0 \\
(e_1, e_2, e_5): & x_{10} - x_{19} - x_7 & = 0 \\
(e_1, e_3, e_4): & -x_{19} + x_{32} & = 0 \\
(e_1, e_2, e_6): & -x_{11} - x_{31} + x_7 & = 0 \\
(e_1, e_3, e_5): & -x_{14} + x_{19} - x_{31} & = 0 \\
(e_2, e_3, e_4): & x_{11}x_{32} + x_{14} - x_{31} & = 0 \\
(e_1, e_2, e_7): & x_{11} - x_{12} - x_{13} & = 0 \\
(e_1, e_3, e_6): & -x_{12} - x_{23} + x_{31} & = 0 \\
(e_1, e_4, e_5): & x_{14} - x_{23} & = 0 \\
(e_2, e_3, e_5): & -x_{10}x_{12} + x_{13}x_{19} & = 0 \\
(e_1, e_2, e_8): & x_{13} - x_2 - x_{25} & = 0 \\
(e_1, e_3, e_7): & x_{12} - x_2 - x_3 & = 0 \\
(e_1, e_4, e_6): & -x_{20} + x_{23} - x_3 & = 0 \\
(e_2, e_3, e_6): & -x_2x_7 - x_{20} + x_{25}x_{31} & = 0 \\
(e_2, e_4, e_5): & -x_{10}x_3 + x_{14}x_{25} + x_{20} & = 0 \\
(e_1, e_2, e_9): & x_{25} - x_{34} - x_8 & = 0 \\
(e_1, e_3, e_8): & x_2 - x_8 - x_9 & = 0 \\
(e_1, e_4, e_7): & x_3 - x_6 - x_9 & = 0 \\
(e_1, e_5, e_6): & x_{20} - x_6 & = 0 \\
(e_2, e_3, e_7): & -x_{11}x_8 + x_{12}x_{34} - x_6 & = 0 \\
(e_2, e_4, e_6): & x_{23}x_{34} - x_7x_9 & = 0 \\
(e_3, e_4, e_5): & x_{14}x_8 - x_{19}x_9 + x_{32}x_6 & = 0 \\
(e_1, e_2, e_{10}): & -x_{15} - x_{26} + x_{34} & = 0 \\
(e_1, e_3, e_9): & -x_{15} - x_{27} + x_8 & = 0 \\
(e_1, e_4, e_8): & -x_{18} - x_{27} + x_9 & = 0 \\
(e_1, e_5, e_7): & -x_{18} - x_5 + x_6 & = 0 \\
(e_2, e_3, e_8): & -x_{13}x_{15} - x_{18} + x_2x_{26} & = 0 \\
(e_2, e_4, e_7): & -x_{11}x_{27} + x_{26}x_3 - x_5 & = 0 \\
(e_2, e_5, e_6): & x_{10}x_5 - x_{18}x_7 + x_{20}x_{26} & = 0 \\
(e_3, e_4, e_6): & x_{15}x_{23} - x_{27}x_{31} + x_{32}x_5 & = 0 \\
(e_1, e_2, e_{11}): & -x_{24} + x_{26} - x_{33} & = 0 \\
(e_1, e_3, e_{10}): & x_{15} - x_{24} - x_{30} & = 0 \\
(e_1, e_4, e_9): & x_{27} - x_{28} - x_{30} & = 0 \\
(e_1, e_5, e_8): & -x_{16} + x_{18} - x_{28} & = 0 \\
(e_1, e_6, e_7): & -x_{16} + x_5 & = 0 \\
(e_2, e_3, e_9): & -x_{24}x_{25} - x_{28} + x_{33}x_8 & = 0 \\
(e_2, e_4, e_8): & -x_{13}x_{30} - x_{16} + x_{33}x_9 & = 0 \\
(e_2, e_5, e_7): & -x_{11}x_{28} + x_{33}x_6 & = 0 \\
(e_3, e_4, e_7): & -x_{12}x_{30} + x_{24}x_3 & = 0 \\
(e_3, e_5, e_6): & x_{16}x_{19} + x_{20}x_{24} - x_{28}x_{31} & = 0 \\
(e_1, e_2, e_{12}): & -x_{21} + x_{33} - x_4 & = 0 \\
(e_1, e_3, e_{11}): & -x_1 + x_{24} - x_4 & = 0 \\
(e_1, e_4, e_{10}): & -x_1 - x_{22} + x_{30} & = 0 \\
(e_1, e_5, e_9): & [illegible] & = 0
\end{array}
$$

Groebner basis (34 variables, 0 linear, 34 nonlinear)

$$x_1 - \frac{4086475x_{34}^5}{395531136} + \frac{3938365x_{34}^4}{74162088} - \frac{87391193x_{34}^3}{593296704} + \frac{1523729x_{34}^2}{8240232} - \frac{105245029x_{34}}{1186593408} + \frac{1232003}{148324176} = 0$$

$$x_2 - \frac{9442385x_{34}^5}{203297472} + \frac{39791369x_{34}^4}{152473104} - \frac{187989481x_{34}^3}{304946208} + \frac{4537949x_{34}^2}{5647152} - \frac{245637491x_{34}}{609892416} + \frac{84955}{76236552} = 0$$

$$x_3 + \frac{2915995x_{34}^5}{135531648} - \frac{3049927x_{34}^4}{25412184} + \frac{57387209x_{34}^3}{203297472} - \frac{360999x_{34}^2}{941192} + \frac{119035285x_{34}}{406594944} - \frac{4725911}{50824368} = 0$$

$$-\frac{94438345x_{34}^5}{4153076928} + \frac{400409263x_{34}^4}{3114807696} - \frac{1584813245x_{34}^3}{6229615392} + \frac{16978165x_{34}^2}{38454416} - \frac{4406406019x_{34}}{12459230784} + x_4 + \frac{94598129}{1557403848} = 0$$

$$\frac{2668075x_{34}^5}{372712032} - \frac{11622955x_{34}^4}{279534024} + \frac{52962515x_{34}^3}{559068048} - \frac{1100615x_{34}^2}{10353112} + \frac{65903905x_{34}}{1118136096} + x_5 - \frac{1809365}{139767012} = 0$$

$$\frac{401155x_{34}^5}{22588608} - \frac{1710217x_{34}^4}{16941456} + \frac{2711669x_{34}^3}{11294304} - \frac{1649957x_{34}^2}{5647152} + \frac{12135073x_{34}}{67765824} + x_6 - \frac{41225}{941192} = 0$$

$$-\frac{3114395x_{34}^5}{29042496} + \frac{6555889x_{34}^4}{10890936} - \frac{61909453x_{34}^3}{43563744} + \frac{751421x_{34}^2}{403368} - \frac{145272173x_{34}}{87127488} + x_7 - \frac{2930909}{10890936} = 0$$

$$-\frac{2915995x_{34}^5}{58084992} + \frac{3049927x_{34}^4}{10890936} - \frac{57387209x_{34}^3}{87127488} + \frac{120333x_{34}^2}{134456} - \frac{89992789x_{34}}{174254976} + x_8 + \frac{1095599}{21781872} = 0$$

$$\frac{509065x_{34}^5}{135531648} - \frac{969203x_{34}^4}{50824368} + \frac{8577167x_{34}^3}{203297472} - \frac{516037x_{34}^2}{5647152} + \frac{46224847x_{34}}{406594944} + x_9 - \frac{2499761}{50824368} = 0$$

$$x_{10} - \frac{3114395x_{34}^5}{58084992} + \frac{6555889x_{34}^4}{21781872} - \frac{61909453x_{34}^3}{87127488} + \frac{751421x_{34}^2}{806736} - \frac{145272173x_{34}}{174254976} - \frac{13821845}{21781872} = 0$$

$$x_{11} - \frac{1029865x_{34}^5}{8470728} + \frac{17330359x_{34}^4}{25412184} - \frac{40896115x_{34}^3}{25412184} + \frac{5981945x_{34}^2}{2823576} - \frac{25783109x_{34}}{12706092} - \frac{128515}{3176523} = 0$$

$$x_{12} - \frac{10136785x_{34}^5}{406594944} + \frac{21491807x_{34}^4}{152473104} - \frac{203817335x_{34}^3}{609892416} + \frac{2371955x_{34}^2}{5647152} - \frac{134169127x_{34}}{1219784832} - \frac{14007823}{152473104} = 0$$

$$x_{13} - \frac{39296735x_{34}^5}{406594944} + \frac{82490347x_{34}^4}{152473104} - \frac{777689425x_{34}^3}{609892416} + \frac{9591935x_{34}^2}{5647152} - \frac{2341009337x_{34}}{1219784832} + \frac{7839103}{152473104} = 0$$

$$x_{14} + \frac{5322925x_{34}^5}{135531648} - \frac{11230505x_{34}^4}{50824368} + \frac{106197251x_{34}^3}{203297472} - \frac{3815951x_{34}^2}{5647152} + \frac{191845723x_{34}}{406594944} - \frac{6952061}{50824368} = 0$$

$$x_{15} - \frac{1978765x_{34}^5}{45638208} + \frac{512881x_{34}^4}{2139291} - \frac{38027351x_{34}^3}{68457312} + \frac{63424x_{34}^2}{79233} - \frac{69826195x_{34}}{136914624} + \frac{1174517}{17114328} = 0$$

$$x_{16} + \frac{2668075x_{34}^5}{372712032} - \frac{11622955x_{34}^4}{279534024} + \frac{52962515x_{34}^3}{559068048} - \frac{1100615x_{34}^2}{10353112} + \frac{65903905x_{34}}{1118136096} - \frac{1809365}{139767012} = 0$$

$$x_{17} + \frac{253125x_{34}^5}{67295228} - \frac{1265625x_{34}^4}{67295228} + \frac{1265625x_{34}^3}{33647614} - \frac{1265625x_{34}^2}{33647614} + \frac{1265625x_{34}}{67295228} - \frac{253125}{67295228} = 0$$

$$x_{18} + \frac{7901965x_{34}^5}{745424064} - \frac{33191251x_{34}^4}{559068048} + \frac{162530201x_{34}^3}{1118136096} - \frac{11545837x_{34}^2}{62118672} + \frac{268649599x_{34}}{2236272192} - \frac{8625095}{279534024} = 0$$

$$x_{19} + \frac{3114395x_{34}^5}{58084992} - \frac{6555889x_{34}^4}{21781872} + \frac{61909453x_{34}^3}{87127488} - \frac{751421x_{34}^2}{806736} + \frac{145272173x_{34}}{174254976} - \frac{7960027}{21781872} = 0$$

$$x_{20}+\frac{401155x_{34}^5}{22588608}-\frac{1710217x_{34}^4}{16941456}+\frac{2711669x_{34}^3}{11294304}-\frac{1649957x_{34}^2}{5647152}+\frac{12135073x_{34}}{67765824}-\frac{41225}{941192}=0$$

$$x_{21}+\frac{24960745x_{34}^5}{251701632}-\frac{3244289x_{34}^4}{5899257}+\frac{457437755x_{34}^3}{377552448}-\frac{4898785x_{34}^2}{2621892}+\frac{231081079x_{34}}{755104896}-\frac{18728081}{94388112}=0$$

$$x_{22}+\frac{6915x_{34}^5}{153817664}+\frac{574919x_{34}^4}{115363248}-\frac{4495627x_{34}^3}{692179488}-\frac{1253563x_{34}^2}{115363248}+\frac{9717329x_{34}}{461452992}-\frac{1509905}{173044872}=0$$

$$x_{23}+\frac{5322925x_{34}^5}{135531648}-\frac{11230505x_{34}^4}{50824368}+\frac{106197251x_{34}^3}{203297472}-\frac{3815951x_{34}^2}{5647152}+\frac{191845723x_{34}}{406594944}-\frac{6952061}{50824368}=0$$

$$x_{24}-\frac{21130205x_{34}^5}{638934912}+\frac{43524661x_{34}^4}{239600592}-\frac{384987811x_{34}^3}{958402368}+\frac{5558977x_{34}^2}{8874096}-\frac{847919819x_{34}}{1916804736}+\frac{16543717}{239600592}=0$$

$$x_{25}-\frac{2915995x_{34}^5}{58084992}+\frac{3049927x_{34}^4}{10890936}-\frac{57387209x_{34}^3}{87127488}+\frac{120333x_{34}^2}{134456}-\frac{264247765x_{34}}{174254976}+\frac{1095599}{21781872}=0$$

$$x_{26}+\frac{1978765x_{34}^5}{45638208}-\frac{512881x_{34}^4}{2139291}+\frac{38027351x_{34}^3}{68457312}-\frac{63424x_{34}^2}{79233}-\frac{67088429x_{34}}{136914624}-\frac{1174517}{17114328}=0$$

$$x_{27}-\frac{485915x_{34}^5}{70992768}+\frac{536429x_{34}^4}{13311144}-\frac{10986265x_{34}^3}{106489152}+\frac{419245x_{34}^2}{4437048}-\frac{1372661x_{34}}{212978304}-\frac{487961}{26622288}=0$$

$$x_{28}+\frac{366545x_{34}^5}{106489152}-\frac{1420763x_{34}^4}{79866864}+\frac{8086453x_{34}^3}{159733728}-\frac{706021x_{34}^2}{8874096}+\frac{19548827x_{34}}{319467456}-\frac{715195}{39933432}=0$$

$$x_{29}+\frac{2351425x_{34}^5}{692179488}-\frac{11822095x_{34}^4}{519134616}+\frac{59305385x_{34}^3}{1038269232}-\frac{1320785x_{34}^2}{19227208}+\frac{83339395x_{34}}{2076538464}-\frac{1191955}{129783654}=0$$

$$x_{30}-\frac{2190835x_{34}^5}{212978304}+\frac{4639337x_{34}^4}{79866864}-\frac{49131701x_{34}^3}{319467456}+\frac{514837x_{34}^2}{2958032}-\frac{43215637x_{34}}{638934912}-\frac{33493}{79866864}=0$$

$$x_{31}+\frac{2915995x_{34}^5}{203297472}-\frac{3049927x_{34}^4}{38118276}+\frac{57387209x_{34}^3}{304946208}-\frac{120333x_{34}^2}{470596}+\frac{220684021x_{34}}{609892416}-\frac{17432003}{76236552}=0$$

$$x_{32}+\frac{3114395x_{34}^5}{58084992}-\frac{6555889x_{34}^4}{21781872}+\frac{61909453x_{34}^3}{87127488}-\frac{751421x_{34}^2}{806736}+\frac{145272173x_{34}}{174254976}-\frac{7960027}{21781872}=0$$

$$x_{33}+\frac{48832915x_{34}^5}{638934912}-\frac{100967333x_{34}^4}{239600592}+\frac{917370725x_{34}^3}{958402368}-\frac{12662465x_{34}^2}{8874096}-\frac{91318187x_{34}}{1916804736}-\frac{32986955}{239600592}=0$$

$$x_{34}^6-\frac{83x_{34}^5}{15}+\frac{38x_{34}^4}{3}-\frac{46x_{34}^3}{3}+\frac{31x_{34}^2}{3}-\frac{11x_{34}}{3}+\frac{8}{15}=0$$

Solution 1:

$$x_1=0$$

$$x_2=0$$

$$x_3=0$$

$$x_4=0$$

$$x_5=0$$

$$x_6=0$$

$$x_7=1$$

$$x_8 = 0$$
$$x_9 = 0$$
$$x_10 = 1$$
$$x_11 = 1$$
$$x_12 = 0$$
$$x_13 = 1$$
$$x_14 = 0$$
$$x_15 = 0$$
$$x_16 = 0$$
$$x_17 = 0$$
$$x_18 = 0$$
$$x_19 = 0$$
$$x_20 = 0$$
$$x_21 = 1$$
$$x_22 = 0$$
$$x_23 = 0$$
$$x_24 = 0$$
$$x_25 = 1$$
$$x_26 = 1$$
$$x_27 = 0$$
$$x_28 = 0$$
$$x_29 = 0$$
$$x_30 = 0$$
$$x_31 = 0$$
$$x_32 = 0$$
$$x_33 = 1$$
$$x_34 = 1$$

Solution 2:

$$x_1 = 7/1430$$
$$x_2 = 5/84$$
$$x_3 = 1/84$$
$$x_4 = 9/286$$

$$x_5 = 1/2310$$
$$x_6 = 1/420$$
$$x_7 = 4/5$$
$$x_8 = 1/20$$
$$x_9 = 1/105$$
$$x_10 = 9/10$$
$$x_11 = 5/7$$
$$x_12 = 1/14$$
$$x_13 = 9/14$$
$$x_14 = 1/70$$
$$x_15 = 7/165$$
$$x_16 = 1/2310$$
$$x_17 = 1/12012$$
$$x_18 = 3/1540$$
$$x_19 = 1/10$$
$$x_20 = 1/420$$
$$x_21 = 11/26$$
$$x_22 = 1/858$$
$$x_23 = 1/70$$
$$x_24 = 2/55$$
$$x_25 = 7/12$$
$$x_26 = 27/55$$
$$x_27 = 1/132$$
$$x_28 = 1/660$$
$$x_29 = 1/2860$$
$$x_30 = 1/165$$
$$x_31 = 3/35$$
$$x_32 = 1/10$$
$$x_33 = 5/11$$
$$x_34 = 8/15$$

$\mathfrak{m}_{2A}(3, 15)$

m2A315 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_{11}] &= e_{14} \\
[e_2, e_{12}] &= 5e_{15} & [e_3, e_{10}] &= -e_{14} \\
[e_3, e_{11}] &= -4e_{15} & [e_4, e_9] &= e_{14} \\
[e_4, e_{10}] &= 3e_{15} & [e_5, e_8] &= -e_{14} \\
[e_5, e_9] &= -2e_{15} & [e_6, e_7] &= e_{14} \\
[e_6, e_8] &= e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(3, 15)$

m4A315 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_1, e_{14}] = e_{15} && [e_2, e_9] = e_{12} \\
&[e_2, e_{10}] = 4e_{13} && [e_2, e_{11}] = 6e_{14} \\
&[e_2, e_{12}] = 0 && [e_3, e_8] = -e_{12} \\
&[e_3, e_9] = -3e_{13} && [e_3, e_{10}] = -2e_{14} \\
&[e_3, e_{11}] = 6e_{15} && [e_4, e_7] = e_{12} \\
&[e_4, e_8] = 2e_{13} && [e_4, e_9] = -e_{14} \\
&[e_4, e_{10}] = -8e_{15} && [e_5, e_6] = -e_{12} \\
&[e_5, e_7] = -e_{13} && [e_5, e_8] = 3e_{14} \\
&[e_5, e_9] = 7e_{15} && [e_6, e_7] = -4e_{14} \\
&[e_6, e_8] = -4e_{15}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_1, e_{13}] = e_{14} \\
&[e_1, e_{14}] = e_{15} && [e_2, e_9] = e_{12} \\
&[e_2, e_{10}] = 4e_{13} && [e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} \\
&[e_2, e_{12}] = \alpha_{2,12}^{15} e_{15} && [e_3, e_8] = -e_{12} \\
&[e_3, e_9] = -3e_{13} && [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14} \\
&[e_3, e_{11}] = \alpha_{3,11}^{15} e_{15} && [e_4, e_7] = e_{12} \\
&[e_4, e_8] = 2e_{13} && [e_4, e_9] = \alpha_{4,9}^{14} e_{14} \\
&[e_4, e_{10}] = \alpha_{4,10}^{15} e_{15} && [e_5, e_6] = -e_{12} \\
&[e_5, e_7] = -e_{13} && [e_5, e_8] = \alpha_{5,8}^{14} e_{14} \\
&[e_5, e_9] = \alpha_{5,9}^{15} e_{15} && [e_6, e_7] = \alpha_{6,7}^{14} e_{14} \\
&[e_6, e_8] = \alpha_{6,8}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}): \quad & -\alpha_{2,11}^{14} - \alpha_{3,10}^{14} + 4 && = 0 \\
(e_1, e_3, e_9): \quad & -\alpha_{3,10}^{14} - \alpha_{4,9}^{14} - 3 && = 0 \\
(e_1, e_4, e_8): \quad & -\alpha_{4,9}^{14} - \alpha_{5,8}^{14} + 2 && = 0 \\
(e_1, e_5, e_7): \quad & -\alpha_{5,8}^{14} - \alpha_{6,7}^{14} - 1 && = 0 \\
(e_1, e_2, e_{11}): \quad & \alpha_{2,11}^{14} - \alpha_{2,12}^{15} - \alpha_{3,11}^{15} && = 0 \\
(e_1, e_3, e_{10}): \quad & \alpha_{3,10}^{14} - \alpha_{3,11}^{15} - \alpha_{4,10}^{15} && = 0 \\
(e_1, e_4, e_9): \quad & -\alpha_{4,10}^{15} + \alpha_{4,9}^{14} - \alpha_{5,9}^{15} && = 0 \\
(e_1, e_5, e_8): \quad & \alpha_{5,8}^{14} - \alpha_{5,9}^{15} - \alpha_{6,8}^{15} && = 0 \\
(e_1, e_6, e_7): \quad & \alpha_{6,7}^{14} - \alpha_{6,8}^{15} && = 0 \\
(e_2, e_3, e_8): \quad & -\alpha_{2,12}^{15} && = 0 \\
(e_2, e_4, e_7): \quad & \alpha_{2,12}^{15} && = 0 \\
(e_2, e_5, e_6): \quad & -\alpha_{2,12}^{15} && = 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{6,8}^{15} &= -4 \\
\alpha_{2,12}^{15} &= 0 \\
\alpha_{3,10}^{14} &= -2 \\
\alpha_{5,9}^{15} &= 7 \\
\alpha_{2,11}^{14} &= 6 \\
\alpha_{5,8}^{14} &= 3 \\
\alpha_{4,10}^{15} &= -8 \\
\alpha_{3,11}^{15} &= 6 \\
\alpha_{4,9}^{14} &= -1 \\
\alpha_{6,7}^{14} &= -4
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{6,8}^{15} \to x_1$$

$$\alpha_{2,12}^{15} \to x_2$$

$$\alpha_{3,10}^{14} \to x_3$$

$$\alpha_{5,9}^{15} \to x_4$$

$$\alpha_{2,11}^{14} \to x_5$$

$$\alpha_{5,8}^{14} \to x_6$$

$$\alpha_{4,10}^{15} \to x_7$$

$$\alpha_{3,11}^{15} \to x_8$$

$$\alpha_{4,9}^{14} \to x_9$$

$$\alpha_{6,7}^{14} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}):&\quad -x_3 - x_5 + 4 &&= 0\\
(e_1, e_3, e_9):&\quad -x_3 - x_9 - 3 &&= 0\\
(e_1, e_4, e_8):&\quad -x_6 - x_9 + 2 &&= 0\\
(e_1, e_5, e_7):&\quad -x_{10} - x_6 - 1 &&= 0\\
(e_1, e_2, e_{11}):&\quad -x_2 + x_5 - x_8 &&= 0\\
(e_1, e_3, e_{10}):&\quad x_3 - x_7 - x_8 &&= 0\\
(e_1, e_4, e_9):&\quad -x_4 - x_7 + x_9 &&= 0\\
(e_1, e_5, e_8):&\quad -x_1 - x_4 + x_6 &&= 0\\
(e_1, e_6, e_7):&\quad -x_1 + x_{10} &&= 0\\
(e_2, e_3, e_8):&\quad -x_2 &&= 0\\
(e_2, e_4, e_7):&\quad x_2 &&= 0\\
(e_2, e_5, e_6):&\quad -x_2 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 10 linear, 0 nonlinear)

$$x_1 + 4 = 0$$

$$x_2 = 0$$

$$x_3 + 2 = 0$$

$$x_4 - 7 = 0$$

$$x_5 - 6 = 0$$

$$x_6 - 3 = 0$$

$$x_7 + 8 = 0$$

$$x_8 - 6 = 0$$

$$x_9 + 1 = 0$$

$$x_{10} + 4 = 0$$

Solution 1:

$$x_1 = -4$$
$$x_2 = 0$$
$$x_3 = -2$$
$$x_4 = 7$$
$$x_5 = 6$$
$$x_6 = 3$$
$$x_7 = -8$$
$$x_8 = 6$$
$$x_9 = -1$$
$$x_10 = -4$$

$\mathfrak{m}_{6A}(3, 15)$

m6A315 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_7] = e_{10}$$
$$[e_2, e_8] = 3e_{11} \qquad [e_2, e_9] = \alpha_{2,9}^{12} e_{12}$$
$$[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} \qquad [e_2, e_{11}] = \alpha_{2,11}^{14} e_{14}$$
$$[e_2, e_{12}] = \alpha_{2,12}^{15} e_{15} \qquad [e_3, e_6] = -e_{10}$$
$$[e_3, e_7] = -2e_{11} \qquad [e_3, e_8] = \alpha_{3,8}^{12} e_{12}$$
$$[e_3, e_9] = \alpha_{3,9}^{13} e_{13} \qquad [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14}$$
$$[e_3, e_{11}] = \alpha_{3,11}^{15} e_{15} \qquad [e_4, e_5] = e_{10}$$
$$[e_4, e_6] = e_{11} \qquad [e_4, e_7] = \alpha_{4,7}^{12} e_{12}$$
$$[e_4, e_8] = \alpha_{4,8}^{13} e_{13} \qquad [e_4, e_9] = \alpha_{4,9}^{14} e_{14}$$
$$[e_4, e_{10}] = \alpha_{4,10}^{15} e_{15} \qquad [e_5, e_6] = \alpha_{5,6}^{12} e_{12}$$
$$[e_5, e_7] = \alpha_{5,7}^{13} e_{13} \qquad [e_5, e_8] = \alpha_{5,8}^{14} e_{14}$$
$$[e_5, e_9] = \alpha_{5,9}^{15} e_{15} \qquad [e_6, e_7] = \alpha_{6,7}^{14} e_{14}$$
$$[e_6, e_8] = \alpha_{6,8}^{15} e_{15}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): \quad & -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &= 0\\
(e_1, e_3, e_7): \quad & -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &= 0\\
(e_1, e_4, e_6): \quad & -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &= 0\\
(e_1, e_2, e_9): \quad & -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &= 0\\
(e_1, e_3, e_8): \quad & \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &= 0\\
(e_1, e_4, e_7): \quad & \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &= 0\\
(e_1, e_5, e_6): \quad & \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &= 0\\
(e_2, e_3, e_6): \quad & -\alpha_{2,10}^{13} &= 0\\
(e_2, e_4, e_5): \quad & \alpha_{2,10}^{13} &= 0\\
(e_1, e_2, e_{10}): \quad & \alpha_{2,10}^{13} - \alpha_{2,11}^{14} - \alpha_{3,10}^{14} &= 0\\
(e_1, e_3, e_9): \quad & -\alpha_{3,10}^{14} + \alpha_{3,9}^{13} - \alpha_{4,9}^{14} &= 0\\
(e_1, e_4, e_8): \quad & \alpha_{4,8}^{13} - \alpha_{4,9}^{14} - \alpha_{5,8}^{14} &= 0\\
(e_1, e_5, e_7): \quad & \alpha_{5,7}^{13} - \alpha_{5,8}^{14} - \alpha_{6,7}^{14} &= 0\\
(e_2, e_3, e_7): \quad & -2\alpha_{2,11}^{14} - \alpha_{3,10}^{14} &= 0\\
(e_2, e_4, e_6): \quad & \alpha_{2,11}^{14} &= 0\\
(e_3, e_4, e_5): \quad & \alpha_{3,10}^{14} &= 0\\
(e_1, e_2, e_{11}): \quad & \alpha_{2,11}^{14} - \alpha_{2,12}^{15} - \alpha_{3,11}^{15} &= 0\\
(e_1, e_3, e_{10}): \quad & \alpha_{3,10}^{14} - \alpha_{3,11}^{15} - \alpha_{4,10}^{15} &= 0\\
(e_1, e_4, e_9): \quad & -\alpha_{4,10}^{15} + \alpha_{4,9}^{14} - \alpha_{5,9}^{15} &= 0\\
(e_1, e_5, e_8): \quad & \alpha_{5,8}^{14} - \alpha_{5,9}^{15} - \alpha_{6,8}^{15} &= 0\\
(e_1, e_6, e_7): \quad & \alpha_{6,7}^{14} - \alpha_{6,8}^{15} &= 0\\
(e_2, e_3, e_8): \quad & \alpha_{2,12}^{15}\alpha_{3,8}^{12} - 3\alpha_{3,11}^{15} &= 0\\
(e_2, e_4, e_7): \quad & \alpha_{2,12}^{15}\alpha_{4,7}^{12} - \alpha_{4,10}^{15} &= 0\\
(e_2, e_5, e_6): \quad & \alpha_{2,12}^{15}\alpha_{5,6}^{12} &= 0\\
(e_3, e_4, e_6): \quad & \alpha_{3,11}^{15} + \alpha_{4,10}^{15} &= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{12} \to x_1$$

$$\alpha_{5,7}^{13} \to x_2$$

$$\alpha_{5,8}^{14} \to x_3$$

$$\alpha^{12}_{3,8} \to x_4$$

$$\alpha^{15}_{2,12} \to x_5$$

$$\alpha^{15}_{6,8} \to x_6$$

$$\alpha^{13}_{4,8} \to x_7$$

$$\alpha^{14}_{3,10} \to x_8$$

$$\alpha^{15}_{5,9} \to x_9$$

$$\alpha^{13}_{2,10} \to x_{10}$$

$$\alpha^{12}_{4,7} \to x_{11}$$

$$\alpha^{12}_{5,6} \to x_{12}$$

$$\alpha^{14}_{2,11} \to x_{13}$$

$$\alpha^{13}_{3,9} \to x_{14}$$

$$\alpha^{15}_{4,10} \to x_{15}$$

$$\alpha^{15}_{3,11} \to x_{16}$$

$$\alpha^{14}_{4,9} \to x_{17}$$

$$\alpha^{14}_{6,7} \to x_{18}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_4 + 3 &= 0\\
(e_1, e_3, e_7): &\quad -x_{11} - x_4 - 2 &= 0\\
(e_1, e_4, e_6): &\quad -x_{11} - x_{12} + 1 &= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_{10} - x_{14} &= 0\\
(e_1, e_3, e_8): &\quad -x_{14} + x_4 - x_7 &= 0\\
(e_1, e_4, e_7): &\quad x_{11} - x_2 - x_7 &= 0\\
(e_1, e_5, e_6): &\quad x_{12} - x_2 &= 0\\
(e_2, e_3, e_6): &\quad -x_{10} &= 0\\
(e_2, e_4, e_5): &\quad x_{10} &= 0\\
(e_1, e_2, e_{10}): &\quad x_{10} - x_{13} - x_8 &= 0\\
(e_1, e_3, e_9): &\quad x_{14} - x_{17} - x_8 &= 0\\
(e_1, e_4, e_8): &\quad -x_{17} - x_3 + x_7 &= 0\\
(e_1, e_5, e_7): &\quad -x_{18} + x_2 - x_3 &= 0\\
(e_2, e_3, e_7): &\quad -2x_{13} - x_8 &= 0\\
(e_2, e_4, e_6): &\quad x_{13} &= 0\\
(e_3, e_4, e_5): &\quad x_8 &= 0\\
(e_1, e_2, e_{11}): &\quad x_{13} - x_{16} - x_5 &= 0\\
(e_1, e_3, e_{10}): &\quad -x_{15} - x_{16} + x_8 &= 0\\
(e_1, e_4, e_9): &\quad -x_{15} + x_{17} - x_9 &= 0\\
(e_1, e_5, e_8): &\quad x_3 - x_6 - x_9 &= 0\\
(e_1, e_6, e_7): &\quad x_{18} - x_6 &= 0\\
(e_2, e_3, e_8): &\quad -3x_{16} + x_4x_5 &= 0\\
(e_2, e_4, e_7): &\quad x_{11}x_5 - x_{15} &= 0\\
(e_2, e_5, e_6): &\quad x_{12}x_5 &= 0\\
(e_3, e_4, e_6): &\quad x_{15} + x_{16} &= 0
\end{aligned}
$$

Groebner basis (18 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{8A}(3, 15)$

m8A315 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= 2e_9 & [e_2, e_7] &= \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] &= \alpha_{2,8}^{11} e_{11} & [e_2, e_9] &= \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] &= \alpha_{2,10}^{13} e_{13} & [e_2, e_{11}] &= \alpha_{2,11}^{14} e_{14} \\
[e_2, e_{12}] &= \alpha_{2,12}^{15} e_{15} & [e_3, e_4] &= -e_8 \\
[e_3, e_5] &= -e_9 & [e_3, e_6] &= \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] &= \alpha_{3,7}^{11} e_{11} & [e_3, e_8] &= \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] &= \alpha_{3,9}^{13} e_{13} & [e_3, e_{10}] &= \alpha_{3,10}^{14} e_{14} \\
[e_3, e_{11}] &= \alpha_{3,11}^{15} e_{15} & [e_4, e_5] &= \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] &= \alpha_{4,6}^{11} e_{11} & [e_4, e_7] &= \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] &= \alpha_{4,8}^{13} e_{13} & [e_4, e_9] &= \alpha_{4,9}^{14} e_{14} \\
[e_4, e_{10}] &= \alpha_{4,10}^{15} e_{15} & [e_5, e_6] &= \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] &= \alpha_{5,7}^{13} e_{13} & [e_5, e_8] &= \alpha_{5,8}^{14} e_{14} \\
[e_5, e_9] &= \alpha_{5,9}^{15} e_{15} & [e_6, e_7] &= \alpha_{6,7}^{14} e_{14} \\
[e_6, e_8] &= \alpha_{6,8}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_6):&\quad -\alpha_{2,7}^{10}-\alpha_{3,6}^{10}+2 &&=0\\
(e_1,e_3,e_5):&\quad -\alpha_{3,6}^{10}-\alpha_{4,5}^{10}-1 &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{10}-\alpha_{4,6}^{11} &&=0\\
(e_2,e_3,e_4):&\quad -\alpha_{2,8}^{11} &&=0\\
(e_1,e_2,e_8):&\quad \alpha_{2,8}^{11}-\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_3,e_7):&\quad \alpha_{3,7}^{11}-\alpha_{3,8}^{12}-\alpha_{4,7}^{12} &&=0\\
(e_1,e_4,e_6):&\quad \alpha_{4,6}^{11}-\alpha_{4,7}^{12}-\alpha_{5,6}^{12} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha_{2,10}^{13}+\alpha_{2,9}^{12}-\alpha_{3,9}^{13} &&=0\\
(e_1,e_3,e_8):&\quad \alpha_{3,8}^{12}-\alpha_{3,9}^{13}-\alpha_{4,8}^{13} &&=0\\
(e_1,e_4,e_7):&\quad \alpha_{4,7}^{12}-\alpha_{4,8}^{13}-\alpha_{5,7}^{13} &&=0\\
(e_1,e_5,e_6):&\quad \alpha_{5,6}^{12}-\alpha_{5,7}^{13} &&=0\\
(e_2,e_3,e_6):&\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10}-2\alpha_{3,9}^{13} &&=0\\
(e_2,e_4,e_5):&\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10}-\alpha_{4,8}^{13} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha_{2,10}^{13}-\alpha_{2,11}^{14}-\alpha_{3,10}^{14} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{14}+\alpha_{3,9}^{13}-\alpha_{4,9}^{14} &&=0\\
(e_1,e_4,e_8):&\quad \alpha_{4,8}^{13}-\alpha_{4,9}^{14}-\alpha_{5,8}^{14} &&=0\\
(e_1,e_5,e_7):&\quad \alpha_{5,7}^{13}-\alpha_{5,8}^{14}-\alpha_{6,7}^{14} &&=0\\
(e_2,e_3,e_7):&\quad \alpha_{2,11}^{14}\alpha_{3,7}^{11}-\alpha_{2,7}^{10}\alpha_{3,10}^{14} &&=0\\
(e_2,e_4,e_6):&\quad \alpha_{2,11}^{14}\alpha_{4,6}^{11}-2\alpha_{4,9}^{14} &&=0\\
(e_3,e_4,e_5):&\quad \alpha_{3,10}^{14}\alpha_{4,5}^{10}+\alpha_{4,9}^{14}-\alpha_{5,8}^{14} &&=0\\
(e_1,e_2,e_{11}):&\quad \alpha_{2,11}^{14}-\alpha_{2,12}^{15}-\alpha_{3,11}^{15} &&=0\\
(e_1,e_3,e_{10}):&\quad \alpha_{3,10}^{14}-\alpha_{3,11}^{15}-\alpha_{4,10}^{15} &&=0\\
(e_1,e_4,e_9):&\quad -\alpha_{4,10}^{15}+\alpha_{4,9}^{14}-\alpha_{5,9}^{15} &&=0\\
(e_1,e_5,e_8):&\quad \alpha_{5,8}^{14}-\alpha_{5,9}^{15}-\alpha_{6,8}^{15} &&=0\\
(e_1,e_6,e_7):&\quad \alpha_{6,7}^{14}-\alpha_{6,8}^{15} &&=0\\
(e_2,e_3,e_8):&\quad \alpha_{2,12}^{15}\alpha_{3,8}^{12}-\alpha_{2,8}^{11}\alpha_{3,11}^{15} &&=0\\
(e_2,e_4,e_7):&\quad \alpha_{2,12}^{15}\alpha_{4,7}^{12}-\alpha_{2,7}^{10}\alpha_{4,10}^{15} &&=0\\
(e_2,e_5,e_6):&\quad \alpha_{2,12}^{15}\alpha_{5,6}^{12}-2\alpha_{5,9}^{15}+\alpha_{6,8}^{15} &&=0\\
(e_3,e_4,e_6):&\quad \alpha_{3,11}^{15}\alpha_{4,6}^{11}-\alpha_{3,6}^{10}\alpha_{4,10}^{15}-\alpha_{6,8}^{15} &&=0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,8}^{12} \to x_1$$

$$\alpha_{2,10}^{13} \to x_2$$

$$\alpha_{3,6}^{10} \to x_3$$

$$\alpha_{3,9}^{13} \to x_4$$

$$\alpha_{2,9}^{12} \to x_5$$

$$\alpha_{4,8}^{13} \to x_6$$

$$\alpha_{3,10}^{14} \to x_7$$

$$\alpha_{3,7}^{11} \to x_8$$

$$\alpha_{4,7}^{12} \to x_9$$

$$\alpha_{2,8}^{11} \to x_{10}$$

$$\alpha_{5,6}^{12} \to x_{11}$$

$$\alpha_{2,11}^{14} \to x_{12}$$

$$\alpha_{5,8}^{14} \to x_{13}$$

$$\alpha_{4,5}^{10} \to x_{14}$$

$$\alpha_{3,11}^{15} \to x_{15}$$

$$\alpha_{6,8}^{15} \to x_{16}$$

$$\alpha_{6,7}^{14} \to x_{17}$$

$$\alpha_{2,7}^{10} \to x_{18}$$

$$\alpha_{5,9}^{15} \to x_{19}$$

$$\alpha_{2,12}^{15} \to x_{20}$$

$$\alpha_{5,7}^{13} \to x_{21}$$

$$\alpha_{4,10}^{15} \to x_{22}$$

$$\alpha_{4,6}^{11} \to x_{23}$$

$$\alpha_{4,9}^{14} \to x_{24}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{18} - x_3 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_{14} - x_3 - 1 &= 0\\
(e_1, e_2, e_7): &\quad -x_{10} + x_{18} - x_8 &= 0\\
(e_1, e_3, e_6): &\quad -x_{23} + x_3 - x_8 &= 0\\
(e_1, e_4, e_5): &\quad x_{14} - x_{23} &= 0\\
(e_2, e_3, e_4): &\quad -x_{10} &= 0\\
(e_1, e_2, e_8): &\quad -x_1 + x_{10} - x_5 &= 0\\
(e_1, e_3, e_7): &\quad -x_1 + x_8 - x_9 &= 0\\
(e_1, e_4, e_6): &\quad -x_{11} + x_{23} - x_9 &= 0\\
(e_2, e_3, e_5): &\quad -x_1 - x_5 &= 0\\
(e_1, e_2, e_9): &\quad -x_2 - x_4 + x_5 &= 0\\
(e_1, e_3, e_8): &\quad x_1 - x_4 - x_6 &= 0\\
(e_1, e_4, e_7): &\quad -x_{21} - x_6 + x_9 &= 0\\
(e_1, e_5, e_6): &\quad x_{11} - x_{21} &= 0\\
(e_2, e_3, e_6): &\quad x_2x_3 - 2x_4 &= 0\\
(e_2, e_4, e_5): &\quad x_{14}x_2 - x_6 &= 0\\
(e_1, e_2, e_{10}): &\quad -x_{12} + x_2 - x_7 &= 0\\
(e_1, e_3, e_9): &\quad -x_{24} + x_4 - x_7 &= 0\\
(e_1, e_4, e_8): &\quad -x_{13} - x_{24} + x_6 &= 0\\
(e_1, e_5, e_7): &\quad -x_{13} - x_{17} + x_{21} &= 0\\
(e_2, e_3, e_7): &\quad x_{12}x_8 - x_{18}x_7 &= 0\\
(e_2, e_4, e_6): &\quad x_{12}x_{23} - 2x_{24} &= 0\\
(e_3, e_4, e_5): &\quad -x_{13} + x_{14}x_7 + x_{24} &= 0\\
(e_1, e_2, e_{11}): &\quad x_{12} - x_{15} - x_{20} &= 0\\
(e_1, e_3, e_{10}): &\quad -x_{15} - x_{22} + x_7 &= 0\\
(e_1, e_4, e_9): &\quad -x_{19} - x_{22} + x_{24} &= 0\\
(e_1, e_5, e_8): &\quad x_{13} - x_{16} - x_{19} &= 0\\
(e_1, e_6, e_7): &\quad -x_{16} + x_{17} &= 0\\
(e_2, e_3, e_8): &\quad x_1x_{20} - x_{10}x_{15} &= 0\\
(e_2, e_4, e_7): &\quad -x_{18}x_{22} + x_{20}x_9 &= 0\\
(e_2, e_5, e_6): &\quad x_{11}x_{20} + x_{16} - 2x_{19} &= 0\\
(e_3, e_4, e_6): &\quad x_{15}x_{23} - x_{16} - x_{22}x_3 &= 0
\end{aligned}
$$

Groebner basis (24 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{10A}(3, 15)$

m10A315 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{2e_8}{5}$$
$$[e_2, e_6] = -\frac{e_9}{5} \qquad [e_2, e_7] = -\frac{e_{10}}{2}$$
$$[e_2, e_8] = -\frac{e_{11}}{2} \qquad [e_2, e_9] = -\frac{e_{12}}{5}$$
$$[e_2, e_{10}] = \frac{2e_{13}}{5} \qquad [e_2, e_{11}] = e_{14}$$
$$[e_2, e_{12}] = e_{15} \qquad [e_3, e_4] = \frac{3e_8}{5}$$
$$[e_3, e_5] = \frac{3e_9}{5} \qquad [e_3, e_6] = \frac{3e_{10}}{10}$$
$$[e_3, e_7] = 0 \qquad [e_3, e_8] = -\frac{3e_{12}}{10}$$
$$[e_3, e_9] = -\frac{3e_{13}}{5} \qquad [e_3, e_{10}] = -\frac{3e_{14}}{5}$$
$$[e_3, e_{11}] = 0 \qquad [e_4, e_5] = \frac{3e_{10}}{10}$$
$$[e_4, e_6] = \frac{3e_{11}}{10} \qquad [e_4, e_7] = \frac{3e_{12}}{10}$$
$$[e_4, e_8] = \frac{3e_{13}}{10} \qquad [e_4, e_9] = 0$$
$$[e_4, e_{10}] = -\frac{3e_{15}}{5} \qquad [e_5, e_6] = 0$$
$$[e_5, e_7] = 0 \qquad [e_5, e_8] = \frac{3e_{14}}{10}$$
$$[e_5, e_9] = \frac{3e_{15}}{5} \qquad [e_6, e_7] = -\frac{3e_{14}}{10}$$
$$[e_6, e_8] = -\frac{3e_{15}}{10}$$

Solution 2

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = e_8$$
$$[e_2, e_6] = e_9 \qquad [e_2, e_7] = e_{10}$$
$$[e_2, e_8] = e_{11} \qquad [e_2, e_9] = e_{12}$$
$$[e_2, e_{10}] = e_{13} \qquad [e_2, e_{11}] = e_{14}$$
$$[e_2, e_{12}] = e_{15} \qquad [e_3, e_4] = 0$$
$$[e_3, e_5] = 0 \qquad [e_3, e_6] = 0$$
$$[e_3, e_7] = 0 \qquad [e_3, e_8] = 0$$
$$[e_3, e_9] = 0 \qquad [e_3, e_{10}] = 0$$
$$[e_3, e_{11}] = 0 \qquad [e_4, e_5] = 0$$
$$[e_4, e_6] = 0 \qquad [e_4, e_7] = 0$$
$$[e_4, e_8] = 0 \qquad [e_4, e_9] = 0$$
$$[e_4, e_{10}] = 0 \qquad [e_5, e_6] = 0$$
$$[e_5, e_7] = 0 \qquad [e_5, e_8] = 0$$
$$[e_5, e_9] = 0 \qquad [e_6, e_7] = 0$$
$$[e_6, e_8] = 0$$

Solution 3

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{6e_8}{7}$$
$$[e_2, e_6] = \frac{5e_9}{7} \qquad [e_2, e_7] = \frac{25e_{10}}{42}$$
$$[e_2, e_8] = \frac{e_{11}}{2} \qquad [e_2, e_9] = \frac{14e_{12}}{33}$$
$$[e_2, e_{10}] = \frac{4e_{13}}{11} \qquad [e_2, e_{11}] = \frac{45e_{14}}{143}$$
$$[e_2, e_{12}] = \frac{25e_{15}}{91} \qquad [e_3, e_4] = \frac{e_8}{7}$$
$$[e_3, e_5] = \frac{e_9}{7} \qquad [e_3, e_6] = \frac{5e_{10}}{42}$$
$$[e_3, e_7] = \frac{2e_{11}}{21} \qquad [e_3, e_8] = \frac{5e_{12}}{66}$$
$$[e_3, e_9] = \frac{2e_{13}}{33} \qquad [e_3, e_{10}] = \frac{7e_{14}}{143}$$
$$[e_3, e_{11}] = \frac{40e_{15}}{1001} \qquad [e_4, e_5] = \frac{e_{10}}{42}$$
$$[e_4, e_6] = \frac{e_{11}}{42} \qquad [e_4, e_7] = \frac{3e_{12}}{154}$$
$$[e_4, e_8] = \frac{e_{13}}{66} \qquad [e_4, e_9] = \frac{5e_{14}}{429}$$
$$[e_4, e_{10}] = \frac{9e_{15}}{1001} \qquad [e_5, e_6] = \frac{e_{12}}{231}$$
$$[e_5, e_7] = \frac{e_{13}}{231} \qquad [e_5, e_8] = \frac{e_{14}}{286}$$
$$[e_5, e_9] = \frac{8e_{15}}{3003} \qquad [e_6, e_7] = \frac{5e_{14}}{6006}$$
$$[e_6, e_8] = \frac{5e_{15}}{6006}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_3] = e_6 \\
[e_2, e_4] = e_7 & [e_2, e_5] = \alpha_{2,5}^{8} e_8 \\
[e_2, e_6] = \alpha_{2,6}^{9} e_9 & [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] = \alpha_{2,8}^{11} e_{11} & [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} & [e_2, e_{11}] = \alpha_{2,11}^{14} e_{14} \\
[e_2, e_{12}] = \alpha_{2,12}^{15} e_{15} & [e_3, e_4] = \alpha_{3,4}^{8} e_8 \\
[e_3, e_5] = \alpha_{3,5}^{9} e_9 & [e_3, e_6] = \alpha_{3,6}^{10} e_{10} \\
[e_3, e_7] = \alpha_{3,7}^{11} e_{11} & [e_3, e_8] = \alpha_{3,8}^{12} e_{12} \\
[e_3, e_9] = \alpha_{3,9}^{13} e_{13} & [e_3, e_{10}] = \alpha_{3,10}^{14} e_{14} \\
[e_3, e_{11}] = \alpha_{3,11}^{15} e_{15} & [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] = \alpha_{4,6}^{11} e_{11} & [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] = \alpha_{4,8}^{13} e_{13} & [e_4, e_9] = \alpha_{4,9}^{14} e_{14} \\
[e_4, e_{10}] = \alpha_{4,10}^{15} e_{15} & [e_5, e_6] = \alpha_{5,6}^{12} e_{12} \\
[e_5, e_7] = \alpha_{5,7}^{13} e_{13} & [e_5, e_8] = \alpha_{5,8}^{14} e_{14} \\
[e_5, e_9] = \alpha_{5,9}^{15} e_{15} & [e_6, e_7] = \alpha_{6,7}^{14} e_{14} \\
[e_6, e_8] = \alpha_{6,8}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{8}-\alpha_{3,4}^{8}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{8}-\alpha_{2,6}^{9}-\alpha_{3,5}^{9} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{8}-\alpha_{3,5}^{9} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{9}-\alpha_{2,7}^{10}-\alpha_{3,6}^{10} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{9}-\alpha_{3,6}^{10}-\alpha_{4,5}^{10} &&=0\\
(e_1,e_2,e_7):&\quad \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} &&=0\\
(e_1,e_3,e_6):&\quad \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} &&=0\\
(e_1,e_4,e_5):&\quad \alpha_{4,5}^{10}-\alpha_{4,6}^{11} &&=0\\
(e_2,e_3,e_4):&\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8}-\alpha_{3,7}^{11}+\alpha_{4,6}^{11} &&=0\\
(e_1,e_2,e_8):&\quad \alpha_{2,8}^{11}-\alpha_{2,9}^{12}-\alpha_{3,8}^{12} &&=0\\
(e_1,e_3,e_7):&\quad \alpha_{3,7}^{11}-\alpha_{3,8}^{12}-\alpha_{4,7}^{12} &&=0\\
(e_1,e_4,e_6):&\quad \alpha_{4,6}^{11}-\alpha_{4,7}^{12}-\alpha_{5,6}^{12} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{12}+\alpha_{2,9}^{12}\alpha_{3,5}^{9}+\alpha_{5,6}^{12} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha_{2,10}^{13}+\alpha_{2,9}^{12}-\alpha_{3,9}^{13} &&=0\\
(e_1,e_3,e_8):&\quad \alpha_{3,8}^{12}-\alpha_{3,9}^{13}-\alpha_{4,8}^{13} &&=0\\
(e_1,e_4,e_7):&\quad \alpha_{4,7}^{12}-\alpha_{4,8}^{13}-\alpha_{5,7}^{13} &&=0\\
(e_1,e_5,e_6):&\quad \alpha_{5,6}^{12}-\alpha_{5,7}^{13} &&=0\\
(e_2,e_3,e_6):&\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10}-\alpha_{2,6}^{9}\alpha_{3,9}^{13} &&=0\\
(e_2,e_4,e_5):&\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10}-\alpha_{2,5}^{8}\alpha_{4,8}^{13}+\alpha_{5,7}^{13} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha_{2,10}^{13}-\alpha_{2,11}^{14}-\alpha_{3,10}^{14} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{14}+\alpha_{3,9}^{13}-\alpha_{4,9}^{14} &&=0\\
(e_1,e_4,e_8):&\quad \alpha_{4,8}^{13}-\alpha_{4,9}^{14}-\alpha_{5,8}^{14} &&=0\\
(e_1,e_5,e_7):&\quad \alpha_{5,7}^{13}-\alpha_{5,8}^{14}-\alpha_{6,7}^{14} &&=0\\
(e_2,e_3,e_7):&\quad \alpha_{2,11}^{14}\alpha_{3,7}^{11}-\alpha_{2,7}^{10}\alpha_{3,10}^{14}-\alpha_{6,7}^{14} &&=0\\
(e_2,e_4,e_6):&\quad \alpha_{2,11}^{14}\alpha_{4,6}^{11}-\alpha_{2,6}^{9}\alpha_{4,9}^{14}+\alpha_{6,7}^{14} &&=0\\
(e_3,e_4,e_5):&\quad \alpha_{3,10}^{14}\alpha_{4,5}^{10}+\alpha_{3,4}^{8}\alpha_{5,8}^{14}-\alpha_{3,5}^{9}\alpha_{4,9}^{14} &&=0\\
(e_1,e_2,e_{11}):&\quad \alpha_{2,11}^{14}-\alpha_{2,12}^{15}-\alpha_{3,11}^{15} &&=0\\
(e_1,e_3,e_{10}):&\quad \alpha_{3,10}^{14}-\alpha_{3,11}^{15}-\alpha_{4,10}^{15} &&=0\\
(e_1,e_4,e_9):&\quad -\alpha_{4,10}^{15}+\alpha_{4,9}^{14}-\alpha_{5,9}^{15} &&=0\\
(e_1,e_5,e_8):&\quad \alpha_{5,8}^{14}-\alpha_{5,9}^{15}-\alpha_{6,8}^{15} &&=0\\
(e_1,e_6,e_7):&\quad \alpha_{6,7}^{14}-\alpha_{6,8}^{15} &&=0\\
(e_2,e_3,e_8):&\quad \alpha_{2,12}^{15}\alpha_{3,8}^{12}-\alpha_{2,8}^{11}\alpha_{3,11}^{15}-\alpha_{6,8}^{15} &&=0\\
(e_2,e_4,e_7):&\quad \alpha_{2,12}^{15}\alpha_{4,7}^{12}-\alpha_{2,7}^{10}\alpha_{4,10}^{15} &&=0\\
(e_2,e_5,e_6):&\quad \alpha_{2,12}^{15}\alpha_{5,6}^{12}+\alpha_{2,5}^{8}\alpha_{6,8}^{15}-\alpha_{2,6}^{9}\alpha_{5,9}^{15} &&=0\\
(e_3,e_4,e_6):&\quad \alpha_{3,11}^{15}\alpha_{4,6}^{11}+\alpha_{3,4}^{8}\alpha_{6,8}^{15}-\alpha_{3,6}^{10}\alpha_{4,10}^{15} &&=0
\end{aligned}$$

Solution 1:

$$\begin{aligned}
\alpha^{12}_{3,8} &= -3/10 \\
\alpha^{13}_{2,10} &= 2/5 \\
\alpha^{10}_{3,6} &= 3/10 \\
\alpha^{13}_{3,9} &= -3/5 \\
\alpha^{12}_{2,9} &= -1/5 \\
\alpha^{8}_{3,4} &= 3/5 \\
\alpha^{13}_{4,8} &= 3/10 \\
\alpha^{14}_{3,10} &= -3/5 \\
\alpha^{11}_{3,7} &= 0 \\
\alpha^{12}_{4,7} &= 3/10 \\
\alpha^{11}_{2,8} &= -1/2 \\
\alpha^{12}_{5,6} &= 0 \\
\alpha^{14}_{2,11} &= 1 \\
\alpha^{10}_{4,5} &= 3/10 \\
\alpha^{9}_{3,5} &= 3/5 \\
\alpha^{14}_{5,8} &= 3/10 \\
\alpha^{15}_{3,11} &= 0 \\
\alpha^{15}_{6,8} &= -3/10 \\
\alpha^{14}_{6,7} &= -3/10 \\
\alpha^{10}_{2,7} &= -1/2 \\
\alpha^{8}_{2,5} &= 2/5 \\
\alpha^{15}_{5,9} &= 3/5 \\
\alpha^{15}_{2,12} &= 1 \\
\alpha^{13}_{5,7} &= 0 \\
\alpha^{9}_{2,6} &= -1/5 \\
\alpha^{15}_{4,10} &= -3/5 \\
\alpha^{11}_{4,6} &= 3/10 \\
\alpha^{14}_{4,9} &= 0
\end{aligned}$$

Solution 2:

$$\alpha^{12}_{3,8} = 0$$
$$\alpha^{13}_{2,10} = 1$$
$$\alpha^{10}_{3,6} = 0$$
$$\alpha^{13}_{3,9} = 0$$
$$\alpha^{12}_{2,9} = 1$$
$$\alpha^{8}_{3,4} = 0$$
$$\alpha^{13}_{4,8} = 0$$
$$\alpha^{14}_{3,10} = 0$$
$$\alpha^{11}_{3,7} = 0$$
$$\alpha^{12}_{4,7} = 0$$
$$\alpha^{11}_{2,8} = 1$$
$$\alpha^{12}_{5,6} = 0$$
$$\alpha^{14}_{2,11} = 1$$
$$\alpha^{10}_{4,5} = 0$$
$$\alpha^{9}_{3,5} = 0$$
$$\alpha^{14}_{5,8} = 0$$
$$\alpha^{15}_{3,11} = 0$$
$$\alpha^{15}_{6,8} = 0$$
$$\alpha^{14}_{6,7} = 0$$
$$\alpha^{10}_{2,7} = 1$$
$$\alpha^{8}_{2,5} = 1$$
$$\alpha^{15}_{5,9} = 0$$
$$\alpha^{15}_{2,12} = 1$$
$$\alpha^{13}_{5,7} = 0$$
$$\alpha^{9}_{2,6} = 1$$
$$\alpha^{15}_{4,10} = 0$$
$$\alpha^{11}_{4,6} = 0$$
$$\alpha^{14}_{4,9} = 0$$

Solution 3:

$$\alpha_{3,8}^{12} = 5/66$$
$$\alpha_{2,10}^{13} = 4/11$$
$$\alpha_{3,6}^{10} = 5/42$$
$$\alpha_{3,9}^{13} = 2/33$$
$$\alpha_{2,9}^{12} = 14/33$$
$$\alpha_{3,4}^{8} = 1/7$$
$$\alpha_{4,8}^{13} = 1/66$$
$$\alpha_{3,10}^{14} = 7/143$$
$$\alpha_{3,7}^{11} = 2/21$$
$$\alpha_{4,7}^{12} = 3/154$$
$$\alpha_{2,8}^{11} = 1/2$$
$$\alpha_{5,6}^{12} = 1/231$$
$$\alpha_{2,11}^{14} = 45/143$$
$$\alpha_{4,5}^{10} = 1/42$$
$$\alpha_{3,5}^{9} = 1/7$$
$$\alpha_{5,8}^{14} = 1/286$$
$$\alpha_{3,11}^{15} = 40/1001$$
$$\alpha_{6,8}^{15} = 5/6006$$
$$\alpha_{6,7}^{14} = 5/6006$$
$$\alpha_{2,7}^{10} = 25/42$$
$$\alpha_{2,5}^{8} = 6/7$$
$$\alpha_{5,9}^{15} = 8/3003$$
$$\alpha_{2,12}^{15} = 25/91$$
$$\alpha_{5,7}^{13} = 1/231$$
$$\alpha_{2,6}^{9} = 5/7$$
$$\alpha_{4,10}^{15} = 9/1001$$
$$\alpha_{4,6}^{11} = 1/42$$
$$\alpha_{4,9}^{14} = 5/429$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,8}^{12} \to x_1$$

$$\alpha_{2,10}^{13} \to x_2$$

$$\alpha_{3,6}^{10} \to x_3$$

$$\alpha_{3,9}^{13} \to x_4$$

$$\alpha_{2,9}^{12} \to x_5$$

$$\alpha_{3,4}^{8} \to x_6$$

$$\alpha_{4,8}^{13} \to x_7$$

$$\alpha_{3,10}^{14} \to x_8$$

$$\alpha_{3,7}^{11} \to x_9$$

$$\alpha_{4,7}^{12} \to x_{10}$$

$$\alpha_{2,8}^{11} \to x_{11}$$

$$\alpha_{5,6}^{12} \to x_{12}$$

$$\alpha_{2,11}^{14} \to x_{13}$$

$$\alpha_{4,5}^{10} \to x_{14}$$

$$\alpha_{3,5}^{9} \to x_{15}$$

$$\alpha_{5,8}^{14} \to x_{16}$$

$$\alpha_{3,11}^{15} \to x_{17}$$

$$\alpha_{6,8}^{15} \to x_{18}$$

$$\alpha_{6,7}^{14} \to x_{19}$$

$$\alpha_{2,7}^{10} \to x_{20}$$

$$\alpha_{2,5}^{8} \to x_{21}$$

$$\alpha_{5,9}^{15} \to x_{22}$$

$$\alpha_{2,12}^{15} \to x_{23}$$

$$\alpha_{5,7}^{13} \to x_{24}$$

$$\alpha_{2,6}^{9} \to x_{25}$$

$$\alpha_{4,10}^{15} \to x_{26}$$

$$\alpha_{4,6}^{11} \to x_{27}$$

$$\alpha_{4,9}^{14} \to x_{28}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{21}-x_6+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{15}+x_{21}-x_{25} &&=0\\
(e_1,e_3,e_4):&\quad -x_{15}+x_6 &&=0\\
(e_1,e_2,e_6):&\quad -x_{20}+x_{25}-x_3 &&=0\\
(e_1,e_3,e_5):&\quad -x_{14}+x_{15}-x_3 &&=0\\
(e_1,e_2,e_7):&\quad -x_{11}+x_{20}-x_9 &&=0\\
(e_1,e_3,e_6):&\quad -x_{27}+x_3-x_9 &&=0\\
(e_1,e_4,e_5):&\quad x_{14}-x_{27} &&=0\\
(e_2,e_3,e_4):&\quad x_{11}x_6+x_{27}-x_9 &&=0\\
(e_1,e_2,e_8):&\quad -x_1+x_{11}-x_5 &&=0\\
(e_1,e_3,e_7):&\quad -x_1-x_{10}+x_9 &&=0\\
(e_1,e_4,e_6):&\quad -x_{10}-x_{12}+x_{27} &&=0\\
(e_2,e_3,e_5):&\quad -x_1x_{21}+x_{12}+x_{15}x_5 &&=0\\
(e_1,e_2,e_9):&\quad -x_2-x_4+x_5 &&=0\\
(e_1,e_3,e_8):&\quad x_1-x_4-x_7 &&=0\\
(e_1,e_4,e_7):&\quad x_{10}-x_{24}-x_7 &&=0\\
(e_1,e_5,e_6):&\quad x_{12}-x_{24} &&=0\\
(e_2,e_3,e_6):&\quad x_2x_3-x_{25}x_4 &&=0\\
(e_2,e_4,e_5):&\quad x_{14}x_2-x_{21}x_7+x_{24} &&=0\\
(e_1,e_2,e_{10}):&\quad -x_{13}+x_2-x_8 &&=0\\
(e_1,e_3,e_9):&\quad -x_{28}+x_4-x_8 &&=0\\
(e_1,e_4,e_8):&\quad -x_{16}-x_{28}+x_7 &&=0\\
(e_1,e_5,e_7):&\quad -x_{16}-x_{19}+x_{24} &&=0\\
(e_2,e_3,e_7):&\quad x_{13}x_9-x_{19}-x_{20}x_8 &&=0\\
(e_2,e_4,e_6):&\quad x_{13}x_{27}+x_{19}-x_{25}x_{28} &&=0\\
(e_3,e_4,e_5):&\quad x_{14}x_8-x_{15}x_{28}+x_{16}x_6 &&=0\\
(e_1,e_2,e_{11}):&\quad x_{13}-x_{17}-x_{23} &&=0\\
(e_1,e_3,e_{10}):&\quad -x_{17}-x_{26}+x_8 &&=0\\
(e_1,e_4,e_9):&\quad -x_{22}-x_{26}+x_{28} &&=0\\
(e_1,e_5,e_8):&\quad x_{16}-x_{18}-x_{22} &&=0\\
(e_1,e_6,e_7):&\quad -x_{18}+x_{19} &&=0\\
(e_2,e_3,e_8):&\quad x_1x_{23}-x_{11}x_{17}-x_{18} &&=0\\
(e_2,e_4,e_7):&\quad x_{10}x_{23}-x_{20}x_{26} &&=0\\
(e_2,e_5,e_6):&\quad x_{12}x_{23}+x_{18}x_{21}-x_{22}x_{25} &&=0\\
(e_3,e_4,e_6):&\quad x_{17}x_{27}+x_{18}x_6-x_{26}x_3 &&=0
\end{aligned}
$$

Groebner basis (28 variables, 8 linear, 24 nonlinear)

$$x_1+\frac{x_{25}}{2}+\frac{33859x_{27}^4}{14418}+\frac{584801x_{27}^3}{48060}-\frac{22033x_{27}^2}{8010}+\frac{18028x_{27}x_{28}}{4005}+\frac{8x_{27}}{3}+\frac{292816168073x_{28}^3}{2119446000}-\frac{10349x_{28}^2}{4005}+\frac{x_{28}}{3}-\frac{1}{2}=0$$

$$x_2-3x_{25}-\frac{67718x_{27}^4}{7209}-\frac{584801x_{27}^3}{12015}+\frac{44066x_{27}^2}{4005}-\frac{72112x_{27}x_{28}}{4005}-\frac{26x_{27}}{3}-\frac{292816168073x_{28}^3}{529861500}+\frac{41396x_{28}^2}{4005}-\frac{4x_{28}}{3}+2=0$$

$$\frac{x_{25}}{2}+x_{27}+x_3-\frac{1}{2}=0$$

$$\frac{x_{25}}{2}+\frac{33859x_{27}^4}{4806}+\frac{584801x_{27}^3}{16020}-\frac{22033x_{27}^2}{2670}+\frac{18028x_{27}x_{28}}{1335}+3x_{27}+\frac{292816168073x_{28}^3}{706482000}-\frac{10349x_{28}^2}{1335}+x_{28}+x_4-\frac{1}{2}=0$$

$$-\frac{5x_{25}}{2}-\frac{33859x_{27}^4}{14418}-\frac{584801x_{27}^3}{48060}+\frac{22033x_{27}^2}{8010}-\frac{18028x_{27}x_{28}}{4005}-\frac{17x_{27}}{3}-\frac{292816168073x_{28}^3}{2119446000}+\frac{10349x_{28}^2}{4005}-\frac{x_{28}}{3}+x_5+\frac{3}{2}=$$

$$\frac{x_{25}}{2}+x_6-\frac{1}{2}=0$$

$$-\frac{33859x_{27}^4}{7209}-\frac{584801x_{27}^3}{24030}+\frac{22033x_{27}^2}{4005}-\frac{36056x_{27}x_{28}}{4005}-\frac{x_{27}}{3}-\frac{292816168073x_{28}^3}{1059723000}+\frac{20698x_{28}^2}{4005}-\frac{2x_{28}}{3}+x_7=0$$

$$\frac{x_{25}}{2}+\frac{33859x_{27}^4}{4806}+\frac{584801x_{27}^3}{16020}-\frac{22033x_{27}^2}{2670}+\frac{18028x_{27}x_{28}}{1335}+3x_{27}+\frac{292816168073x_{28}^3}{706482000}-\frac{10349x_{28}^2}{1335}+2x_{28}+x_8-\frac{1}{2}=0$$

$$\frac{x_{25}}{2}+2x_{27}+x_9-\frac{1}{2}=0$$

$$x_{10}-\frac{33859x_{27}^4}{14418}-\frac{584801x_{27}^3}{48060}+\frac{22033x_{27}^2}{8010}-\frac{18028x_{27}x_{28}}{4005}-\frac{2x_{27}}{3}-\frac{292816168073x_{28}^3}{2119446000}+\frac{10349x_{28}^2}{4005}-\frac{x_{28}}{3}=0$$

$$x_{11}-2x_{25}-3x_{27}+1=0$$

$$x_{12}+\frac{33859x_{27}^4}{14418}+\frac{584801x_{27}^3}{48060}-\frac{22033x_{27}^2}{8010}+\frac{18028x_{27}x_{28}}{4005}-\frac{x_{27}}{3}+\frac{292816168073x_{28}^3}{2119446000}-\frac{10349x_{28}^2}{4005}+\frac{x_{28}}{3}=0$$

$$x_{13}-\frac{7x_{25}}{2}-\frac{237013x_{27}^4}{14418}-\frac{4093607x_{27}^3}{48060}+\frac{154231x_{27}^2}{8010}-\frac{126196x_{27}x_{28}}{4005}-\frac{35x_{27}}{3}-\frac{292816168073x_{28}^3}{302778000}+\frac{72443x_{28}^2}{4005}-\frac{10x_{28}}{3}$$

$$x_{14}-x_{27}=0$$

$$x_{15}+\frac{x_{25}}{2}-\frac{1}{2}=0$$

$$x_{16}-\frac{33859x_{27}^4}{7209}-\frac{584801x_{27}^3}{24030}+\frac{22033x_{27}^2}{4005}-\frac{36056x_{27}x_{28}}{4005}-\frac{x_{27}}{3}-\frac{292816168073x_{28}^3}{1059723000}+\frac{20698x_{28}^2}{4005}+\frac{x_{28}}{3}=0$$

$$x_{17}+\frac{x_{25}}{2}-\frac{33859x_{27}^4}{7209}-\frac{584801x_{27}^3}{24030}+\frac{22033x_{27}^2}{4005}-\frac{36056x_{27}x_{28}}{4005}+\frac{8x_{27}}{3}-\frac{292816168073x_{28}^3}{1059723000}+\frac{20698x_{28}^2}{4005}+\frac{10x_{28}}{3}-\frac{1}{2}=$$

$$x_{18}+\frac{33859x_{27}^4}{4806}+\frac{584801x_{27}^3}{16020}-\frac{22033x_{27}^2}{2670}+\frac{18028x_{27}x_{28}}{1335}+\frac{292816168073x_{28}^3}{706482000}-\frac{10349x_{28}^2}{1335}=0$$

$$x_{19}+\frac{33859x_{27}^4}{4806}+\frac{584801x_{27}^3}{16020}-\frac{22033x_{27}^2}{2670}+\frac{18028x_{27}x_{28}}{1335}+\frac{292816168073x_{28}^3}{706482000}-\frac{10349x_{28}^2}{1335}=0$$

$$x_{20}-\frac{3x_{25}}{2}-x_{27}+\frac{1}{2}=0$$

$$x_{21} - \frac{x_{25}}{2} - \frac{1}{2} = 0$$

$$x_{22} - \frac{169295x_{27}^4}{14418} - \frac{584801x_{27}^3}{9612} + \frac{22033x_{27}^2}{1602} - \frac{18028x_{27}x_{28}}{801} - \frac{x_{27}}{3} - \frac{292816168073x_{28}^3}{423889200} + \frac{10349x_{28}^2}{801} + \frac{x_{28}}{3} = 0$$

$$x_{23} - 4x_{25} - \frac{169295x_{27}^4}{14418} - \frac{584801x_{27}^3}{9612} + \frac{22033x_{27}^2}{1602} - \frac{18028x_{27}x_{28}}{801} - \frac{43x_{27}}{3} - \frac{292816168073x_{28}^3}{423889200} + \frac{10349x_{28}^2}{801} - \frac{20x_{28}}{3} + 3$$

$$x_{24} + \frac{33859x_{27}^4}{14418} + \frac{584801x_{27}^3}{48060} - \frac{22033x_{27}^2}{8010} + \frac{18028x_{27}x_{28}}{4005} - \frac{x_{27}}{3} + \frac{292816168073x_{28}^3}{2119446000} - \frac{10349x_{28}^2}{4005} + \frac{x_{28}}{3} = 0$$

$$x_{25}^2 - 2x_{25} + \frac{196x_{27}^4}{801} + \frac{1057x_{27}^3}{1335} - \frac{3791x_{27}^2}{890} - \frac{317x_{27}x_{28}}{178} - \frac{18x_{27}}{5} - \frac{174061316x_{28}^3}{14718375} + \frac{18x_{28}^2}{445} + \frac{3x_{28}}{5} + 1 = 0$$

$$x_{25}x_{27} - \frac{392x_{27}^4}{2403} - \frac{2114x_{27}^3}{4005} + \frac{3791x_{27}^2}{1335} + \frac{317x_{27}x_{28}}{267} - \frac{3x_{27}}{5} + \frac{348122632x_{28}^3}{44155125} - \frac{12x_{28}^2}{445} - \frac{2x_{28}}{5} = 0$$

$$x_{25}x_{28} + \frac{49x_{27}^4}{54} + \frac{791x_{27}^3}{180} - \frac{41x_{27}^2}{15} - \frac{x_{27}x_{28}}{6} + \frac{2x_{27}}{5} + \frac{430467323x_{28}^3}{7938000} - \frac{3x_{28}^2}{5} - \frac{7x_{28}}{5} = 0$$

$$x_{26} + \frac{169295x_{27}^4}{14418} + \frac{584801x_{27}^3}{9612} - \frac{22033x_{27}^2}{1602} + \frac{18028x_{27}x_{28}}{801} + \frac{x_{27}}{3} + \frac{292816168073x_{28}^3}{423889200} - \frac{10349x_{28}^2}{801} - \frac{4x_{28}}{3} = 0$$

$$x_{27}^5 + \frac{339x_{27}^4}{70} - \frac{738x_{27}^3}{245} + \frac{108x_{27}^2}{245} - \frac{216x_{27}x_{28}}{245} - \frac{4973743489x_{28}^3}{302526000} + \frac{108x_{28}^2}{245} = 0$$

$$x_{27}^2x_{28} - \frac{20449x_{28}^3}{4900} = 0$$

$$x_{27}x_{28}^2 - \frac{143x_{28}^3}{70} = 0$$

$$x_{28}^4 - \frac{5x_{28}^3}{429} = 0$$

Solution 1:

$$x_1 = -3/10$$

$$x_2 = 2/5$$

$$x_3 = 3/10$$

$$x_4 = -3/5$$

$$x_5 = -1/5$$

$$x_6 = 3/5$$

$$x_7 = 3/10$$

$$x_8 = -3/5$$

$$x_9 = 0$$

$$x_1 0 = 3/10$$

$$x_1 1 = -1/2$$

$$x_1 2 = 0$$

$$x_13 = 1$$
$$x_14 = 3/10$$
$$x_15 = 3/5$$
$$x_16 = 3/10$$
$$x_17 = 0$$
$$x_18 = -3/10$$
$$x_19 = -3/10$$
$$x_20 = -1/2$$
$$x_21 = 2/5$$
$$x_22 = 3/5$$
$$x_23 = 1$$
$$x_24 = 0$$
$$x_25 = -1/5$$
$$x_26 = -3/5$$
$$x_27 = 3/10$$
$$x_28 = 0$$

Solution 2:

$$x_1 = 0$$
$$x_2 = 1$$
$$x_3 = 0$$
$$x_4 = 0$$
$$x_5 = 1$$
$$x_6 = 0$$
$$x_7 = 0$$
$$x_8 = 0$$
$$x_9 = 0$$
$$x_10 = 0$$
$$x_11 = 1$$
$$x_12 = 0$$
$$x_13 = 1$$
$$x_14 = 0$$
$$x_15 = 0$$

$$x_16 = 0$$
$$x_17 = 0$$
$$x_18 = 0$$
$$x_19 = 0$$
$$x_20 = 1$$
$$x_21 = 1$$
$$x_22 = 0$$
$$x_23 = 1$$
$$x_24 = 0$$
$$x_25 = 1$$
$$x_26 = 0$$
$$x_27 = 0$$
$$x_28 = 0$$

Solution 3:

$$x_1 = 5/66$$
$$x_2 = 4/11$$
$$x_3 = 5/42$$
$$x_4 = 2/33$$
$$x_5 = 14/33$$
$$x_6 = 1/7$$
$$x_7 = 1/66$$
$$x_8 = 7/143$$
$$x_9 = 2/21$$
$$x_10 = 3/154$$
$$x_11 = 1/2$$
$$x_12 = 1/231$$
$$x_13 = 45/143$$
$$x_14 = 1/42$$
$$x_15 = 1/7$$
$$x_16 = 1/286$$
$$x_17 = 40/1001$$
$$x_18 = 5/6006$$

$$x_19 = 5/6006$$

$$x_20 = 25/42$$

$$x_21 = 6/7$$

$$x_22 = 8/3003$$

$$x_23 = 25/91$$

$$x_24 = 1/231$$

$$x_25 = 5/7$$

$$x_26 = 9/1001$$

$$x_27 = 1/42$$

$$x_28 = 5/429$$

$\mathfrak{m}_{1A}(4, 15)$

m1A415 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_{11}] &= e_{15} \\
[e_3, e_{10}] &= -e_{15} & [e_4, e_9] &= e_{15} \\
[e_5, e_8] &= -e_{15} & [e_6, e_7] &= e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(4, 15)$

m3A415 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_9] &= e_{13} \\
[e_2, e_{10}] &= 4e_{14} & [e_2, e_{11}] &= \alpha_{2,11}^{15} e_{15} \\
[e_3, e_8] &= -e_{13} & [e_3, e_9] &= -3e_{14} \\
[e_3, e_{10}] &= \alpha_{3,10}^{15} e_{15} & [e_4, e_7] &= e_{13} \\
[e_4, e_8] &= 2e_{14} & [e_4, e_9] &= \alpha_{4,9}^{15} e_{15} \\
[e_5, e_6] &= -e_{13} & [e_5, e_7] &= -e_{14} \\
[e_5, e_8] &= \alpha_{5,8}^{15} e_{15} & [e_6, e_7] &= \alpha_{6,7}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}) :& \quad -\alpha_{2,11}^{15} - \alpha_{3,10}^{15} + 4 &= 0 \\
(e_1, e_3, e_9) :& \quad -\alpha_{3,10}^{15} - \alpha_{4,9}^{15} - 3 &= 0 \\
(e_1, e_4, e_8) :& \quad -\alpha_{4,9}^{15} - \alpha_{5,8}^{15} + 2 &= 0 \\
(e_1, e_5, e_7) :& \quad -\alpha_{5,8}^{15} - \alpha_{6,7}^{15} - 1 &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{5,8}^{15} &\to x_1 \\
\alpha_{2,11}^{15} &\to x_2 \\
\alpha_{6,7}^{15} &\to x_3 \\
\alpha_{3,10}^{15} &\to x_4 \\
\alpha_{4,9}^{15} &\to x_5
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_2 - x_4 + 4 &= 0\\
(e_1, e_3, e_9): &\quad -x_4 - x_5 - 3 &= 0\\
(e_1, e_4, e_8): &\quad -x_1 - x_5 + 2 &= 0\\
(e_1, e_5, e_7): &\quad -x_1 - x_3 - 1 &= 0
\end{aligned}
$$

Groebner basis (5 variables, 4 linear, 0 nonlinear)

$$x_1 + x_5 - 2 = 0$$
$$x_2 - x_5 - 7 = 0$$
$$x_3 - x_5 + 3 = 0$$
$$x_4 + x_5 + 3 = 0$$

$\mathfrak{m}_{5A}(4, 15)$

m5A415 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_1, e_{14}] &= e_{15} & [e_2, e_7] &= e_{11}\\
[e_2, e_8] &= 3e_{12} & [e_2, e_9] &= \alpha_{2,9}^{13} e_{13}\\
[e_2, e_{10}] &= \alpha_{2,10}^{14} e_{14} & [e_2, e_{11}] &= \alpha_{2,11}^{15} e_{15}\\
[e_3, e_6] &= -e_{11} & [e_3, e_7] &= -2e_{12}\\
[e_3, e_8] &= \alpha_{3,8}^{13} e_{13} & [e_3, e_9] &= \alpha_{3,9}^{14} e_{14}\\
[e_3, e_{10}] &= \alpha_{3,10}^{15} e_{15} & [e_4, e_5] &= e_{11}\\
[e_4, e_6] &= e_{12} & [e_4, e_7] &= \alpha_{4,7}^{13} e_{13}\\
[e_4, e_8] &= \alpha_{4,8}^{14} e_{14} & [e_4, e_9] &= \alpha_{4,9}^{15} e_{15}\\
[e_5, e_6] &= \alpha_{5,6}^{13} e_{13} & [e_5, e_7] &= \alpha_{5,7}^{14} e_{14}\\
[e_5, e_8] &= \alpha_{5,8}^{15} e_{15} & [e_6, e_7] &= \alpha_{6,7}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{13} - \alpha_{3,8}^{13} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{13} - \alpha_{4,7}^{13} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{13} - \alpha_{5,6}^{13} + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &&= 0\\
(e_1, e_2, e_{10}): &\quad \alpha_{2,10}^{14} - \alpha_{2,11}^{15} - \alpha_{3,10}^{15} &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{15} + \alpha_{3,9}^{14} - \alpha_{4,9}^{15} &&= 0\\
(e_1, e_4, e_8): &\quad \alpha_{4,8}^{14} - \alpha_{4,9}^{15} - \alpha_{5,8}^{15} &&= 0\\
(e_1, e_5, e_7): &\quad \alpha_{5,7}^{14} - \alpha_{5,8}^{15} - \alpha_{6,7}^{15} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,11}^{15} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,11}^{15} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{5,8}^{15} \to x_1$$

$$\alpha_{3,9}^{14} \to x_2$$

$$\alpha_{2,10}^{14} \to x_3$$

$$\alpha_{2,11}^{15} \to x_4$$

$$\alpha_{4,9}^{15} \to x_5$$

$$\alpha_{6,7}^{15} \to x_6$$

$$\alpha_{5,6}^{13} \to x_7$$

$$\alpha_{4,8}^{14} \to x_8$$

$$\alpha_{2,9}^{13} \to x_9$$

$$\alpha_{3,10}^{15} \to x_{10}$$

$$\alpha_{4,7}^{13} \to x_{11}$$

$$\alpha_{5,7}^{14} \to x_{12}$$

$$\alpha_{3,8}^{13} \to x_{13}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_8): &\quad -x_{13}-x_9+3 &&= 0\\
(e_1,e_3,e_7): &\quad -x_{11}-x_{13}-2 &&= 0\\
(e_1,e_4,e_6): &\quad -x_{11}-x_7+1 &&= 0\\
(e_1,e_2,e_9): &\quad -x_2-x_3+x_9 &&= 0\\
(e_1,e_3,e_8): &\quad x_{13}-x_2-x_8 &&= 0\\
(e_1,e_4,e_7): &\quad x_{11}-x_{12}-x_8 &&= 0\\
(e_1,e_5,e_6): &\quad -x_{12}+x_7 &&= 0\\
(e_1,e_2,e_{10}): &\quad -x_{10}+x_3-x_4 &&= 0\\
(e_1,e_3,e_9): &\quad -x_{10}+x_2-x_5 &&= 0\\
(e_1,e_4,e_8): &\quad -x_1-x_5+x_8 &&= 0\\
(e_1,e_5,e_7): &\quad -x_1+x_{12}-x_6 &&= 0\\
(e_2,e_3,e_6): &\quad -x_4 &&= 0\\
(e_2,e_4,e_5): &\quad x_4 &&= 0
\end{aligned}
$$

Groebner basis (13 variables, 12 linear, 0 nonlinear)

$$x_1+9x_{13}+12=0$$

$$-3x_{13}+x_2-5=0$$

$$4x_{13}+x_3+2=0$$

$$x_4=0$$

$$-7x_{13}+x_5-7=0$$

$$-10x_{13}+x_6-15=0$$

$$-x_{13}+x_7-3=0$$

$$2x_{13}+x_8+5=0$$

$$x_{13}+x_9-3=0$$

$$x_{10}+4x_{13}+2=0$$

$$x_{11}+x_{13}+2=0$$

$$x_{12}-x_{13}-3=0$$

$\mathfrak{m}_{7A}(4,15)$

m7A415 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14}\\
[e_1,e_{14}] &= e_{15} & [e_2,e_5] &= e_9\\
[e_2,e_6] &= 2e_{10} & [e_2,e_7] &= \alpha_{2,7}^{11}e_{11}\\
[e_2,e_8] &= \alpha_{2,8}^{12}e_{12} & [e_2,e_9] &= \alpha_{2,9}^{13}e_{13}\\
[e_2,e_{10}] &= \alpha_{2,10}^{14}e_{14} & [e_2,e_{11}] &= \alpha_{2,11}^{15}e_{15}\\
[e_3,e_4] &= -e_9 & [e_3,e_5] &= -e_{10}\\
[e_3,e_6] &= \alpha_{3,6}^{11}e_{11} & [e_3,e_7] &= \alpha_{3,7}^{12}e_{12}\\
[e_3,e_8] &= \alpha_{3,8}^{13}e_{13} & [e_3,e_9] &= \alpha_{3,9}^{14}e_{14}\\
[e_3,e_{10}] &= \alpha_{3,10}^{15}e_{15} & [e_4,e_5] &= \alpha_{4,5}^{11}e_{11}\\
[e_4,e_6] &= \alpha_{4,6}^{12}e_{12} & [e_4,e_7] &= \alpha_{4,7}^{13}e_{13}\\
[e_4,e_8] &= \alpha_{4,8}^{14}e_{14} & [e_4,e_9] &= \alpha_{4,9}^{15}e_{15}\\
[e_5,e_6] &= \alpha_{5,6}^{13}e_{13} & [e_5,e_7] &= \alpha_{5,7}^{14}e_{14}\\
[e_5,e_8] &= \alpha_{5,8}^{15}e_{15} & [e_6,e_7] &= \alpha_{6,7}^{15}e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &= 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &= 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &= 0 \\
(e_2, e_3, e_4): &\quad -\alpha_{2,9}^{13} &= 0 \\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &= 0 \\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &= 0 \\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &= 0 \\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{2,10}^{14} - \alpha_{3,9}^{14} &= 0 \\
(e_1, e_2, e_{10}): &\quad \alpha_{2,10}^{14} - \alpha_{2,11}^{15} - \alpha_{3,10}^{15} &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{15} + \alpha_{3,9}^{14} - \alpha_{4,9}^{15} &= 0 \\
(e_1, e_4, e_8): &\quad \alpha_{4,8}^{14} - \alpha_{4,9}^{15} - \alpha_{5,8}^{15} &= 0 \\
(e_1, e_5, e_7): &\quad \alpha_{5,7}^{14} - \alpha_{5,8}^{15} - \alpha_{6,7}^{15} &= 0 \\
(e_2, e_3, e_6): &\quad \alpha_{2,11}^{15}\alpha_{3,6}^{11} - 2\alpha_{3,10}^{15} &= 0 \\
(e_2, e_4, e_5): &\quad \alpha_{2,11}^{15}\alpha_{4,5}^{11} - \alpha_{4,9}^{15} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{5,8}^{15} \to x_1$$

$$\alpha_{3,9}^{14} \to x_2$$

$$\alpha_{2,10}^{14} \to x_3$$

$$\alpha_{2,11}^{15} \to x_4$$

$$\alpha_{4,5}^{11} \to x_5$$

$$\alpha_{4,9}^{15} \to x_6$$

$$\alpha_{2,7}^{11} \to x_7$$

$$\alpha^{12}_{2,8} \to x_8$$
$$\alpha^{13}_{5,6} \to x_9$$
$$\alpha^{15}_{6,7} \to x_{10}$$
$$\alpha^{12}_{3,7} \to x_{11}$$
$$\alpha^{14}_{4,8} \to x_{12}$$
$$\alpha^{13}_{3,8} \to x_{13}$$
$$\alpha^{13}_{2,9} \to x_{14}$$
$$\alpha^{15}_{3,10} \to x_{15}$$
$$\alpha^{11}_{3,6} \to x_{16}$$
$$\alpha^{13}_{4,7} \to x_{17}$$
$$\alpha^{14}_{5,7} \to x_{18}$$
$$\alpha^{12}_{4,6} \to x_{19}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_6): &\quad -x_{16}-x_7+2 &&= 0\\
(e_1,e_3,e_5): &\quad -x_{16}-x_5-1 &&= 0\\
(e_1,e_2,e_7): &\quad -x_{11}+x_7-x_8 &&= 0\\
(e_1,e_3,e_6): &\quad -x_{11}+x_{16}-x_{19} &&= 0\\
(e_1,e_4,e_5): &\quad -x_{19}+x_5 &&= 0\\
(e_1,e_2,e_8): &\quad -x_{13}-x_{14}+x_8 &&= 0\\
(e_1,e_3,e_7): &\quad x_{11}-x_{13}-x_{17} &&= 0\\
(e_1,e_4,e_6): &\quad -x_{17}+x_{19}-x_9 &&= 0\\
(e_2,e_3,e_4): &\quad -x_{14} &&= 0\\
(e_1,e_2,e_9): &\quad x_{14}-x_2-x_3 &&= 0\\
(e_1,e_3,e_8): &\quad -x_{12}+x_{13}-x_2 &&= 0\\
(e_1,e_4,e_7): &\quad -x_{12}+x_{17}-x_{18} &&= 0\\
(e_1,e_5,e_6): &\quad -x_{18}+x_9 &&= 0\\
(e_2,e_3,e_5): &\quad -x_2-x_3 &&= 0\\
(e_1,e_2,e_{10}): &\quad -x_{15}+x_3-x_4 &&= 0\\
(e_1,e_3,e_9): &\quad -x_{15}+x_2-x_6 &&= 0\\
(e_1,e_4,e_8): &\quad -x_1+x_{12}-x_6 &&= 0\\
(e_1,e_5,e_7): &\quad -x_1-x_{10}+x_{18} &&= 0\\
(e_2,e_3,e_6): &\quad -2x_{15}+x_{16}x_4 &&= 0\\
(e_2,e_4,e_5): &\quad x_4x_5-x_6 &&= 0
\end{aligned}$$

Groebner basis (19 variables, 17 linear, 1 nonlinear)

$$x_1 - x_{15} + 25x_{19} + 24 = 0$$
$$-14x_{19} + x_2 - 14 = 0$$
$$14x_{19} + x_3 + 14 = 0$$
$$x_{15} + 14x_{19} + x_4 + 14 = 0$$
$$-x_{19} + x_5 = 0$$
$$x_{15} - 14x_{19} + x_6 - 14 = 0$$
$$-x_{19} + x_7 - 3 = 0$$
$$-3x_{19} + x_8 - 4 = 0$$
$$-6x_{19} + x_9 - 5 = 0$$
$$x_{10} + x_{15} - 31x_{19} - 29 = 0$$
$$x_{11} + 2x_{19} + 1 = 0$$
$$x_{12} + 11x_{19} + 10 = 0$$
$$x_{13} - 3x_{19} - 4 = 0$$
$$x_{14} = 0$$
$$x_{15}x_{19} - x_{15} + 14x_{19}^2 + 28x_{19} + 14 = 0$$
$$x_{16} + x_{19} + 1 = 0$$
$$x_{17} + 5x_{19} + 5 = 0$$
$$x_{18} - 6x_{19} - 5 = 0$$

$\mathfrak{m}_{9A}(4, 15)$

m9A415 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_3] = e_7 \\
[e_2, e_4] = e_8 & [e_2, e_5] = \alpha_{2,5}^{9} e_9 \\
[e_2, e_6] = \alpha_{2,6}^{10} e_{10} & [e_2, e_7] = \alpha_{2,7}^{11} e_{11} \\
[e_2, e_8] = \alpha_{2,8}^{12} e_{12} & [e_2, e_9] = \alpha_{2,9}^{13} e_{13} \\
[e_2, e_{10}] = \alpha_{2,10}^{14} e_{14} & [e_2, e_{11}] = \alpha_{2,11}^{15} e_{15} \\
[e_3, e_4] = \alpha_{3,4}^{9} e_9 & [e_3, e_5] = \alpha_{3,5}^{10} e_{10} \\
[e_3, e_6] = \alpha_{3,6}^{11} e_{11} & [e_3, e_7] = \alpha_{3,7}^{12} e_{12} \\
[e_3, e_8] = \alpha_{3,8}^{13} e_{13} & [e_3, e_9] = \alpha_{3,9}^{14} e_{14} \\
[e_3, e_{10}] = \alpha_{3,10}^{15} e_{15} & [e_4, e_5] = \alpha_{4,5}^{11} e_{11} \\
[e_4, e_6] = \alpha_{4,6}^{12} e_{12} & [e_4, e_7] = \alpha_{4,7}^{13} e_{13} \\
[e_4, e_8] = \alpha_{4,8}^{14} e_{14} & [e_4, e_9] = \alpha_{4,9}^{15} e_{15} \\
[e_5, e_6] = \alpha_{5,6}^{13} e_{13} & [e_5, e_7] = \alpha_{5,7}^{14} e_{14} \\
[e_5, e_8] = \alpha_{5,8}^{15} e_{15} & [e_6, e_7] = \alpha_{6,7}^{15} e_{15}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 &&= 0\\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{9} - \alpha_{2,6}^{10} - \alpha_{3,5}^{10} &&= 0\\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{9} - \alpha_{3,5}^{10} &&= 0\\
(e_1, e_2, e_6):&\quad \alpha_{2,6}^{10} - \alpha_{2,7}^{11} - \alpha_{3,6}^{11} &&= 0\\
(e_1, e_3, e_5):&\quad \alpha_{3,5}^{10} - \alpha_{3,6}^{11} - \alpha_{4,5}^{11} &&= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &&= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &&= 0\\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &&= 0\\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &&= 0\\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &&= 0\\
(e_2, e_3, e_4):&\quad \alpha_{2,9}^{13}\alpha_{3,4}^{9} - \alpha_{3,8}^{13} + \alpha_{4,7}^{13} &&= 0\\
(e_1, e_2, e_9):&\quad -\alpha_{2,10}^{14} + \alpha_{2,9}^{13} - \alpha_{3,9}^{14} &&= 0\\
(e_1, e_3, e_8):&\quad \alpha_{3,8}^{13} - \alpha_{3,9}^{14} - \alpha_{4,8}^{14} &&= 0\\
(e_1, e_4, e_7):&\quad \alpha_{4,7}^{13} - \alpha_{4,8}^{14} - \alpha_{5,7}^{14} &&= 0\\
(e_1, e_5, e_6):&\quad \alpha_{5,6}^{13} - \alpha_{5,7}^{14} &&= 0\\
(e_2, e_3, e_5):&\quad \alpha_{2,10}^{14}\alpha_{3,5}^{10} - \alpha_{2,5}^{9}\alpha_{3,9}^{14} + \alpha_{5,7}^{14} &&= 0\\
(e_1, e_2, e_{10}):&\quad \alpha_{2,10}^{14} - \alpha_{2,11}^{15} - \alpha_{3,10}^{15} &&= 0\\
(e_1, e_3, e_9):&\quad -\alpha_{3,10}^{15} + \alpha_{3,9}^{14} - \alpha_{4,9}^{15} &&= 0\\
(e_1, e_4, e_8):&\quad \alpha_{4,8}^{14} - \alpha_{4,9}^{15} - \alpha_{5,8}^{15} &&= 0\\
(e_1, e_5, e_7):&\quad \alpha_{5,7}^{14} - \alpha_{5,8}^{15} - \alpha_{6,7}^{15} &&= 0\\
(e_2, e_3, e_6):&\quad \alpha_{2,11}^{15}\alpha_{3,6}^{11} - \alpha_{2,6}^{10}\alpha_{3,10}^{15} + \alpha_{6,7}^{15} &&= 0\\
(e_2, e_4, e_5):&\quad \alpha_{2,11}^{15}\alpha_{4,5}^{11} - \alpha_{2,5}^{9}\alpha_{4,9}^{15} + \alpha_{5,8}^{15} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,10}^{14} \to x_1$$

$$\alpha_{2,5}^{9} \to x_2$$

$$\alpha_{3,4}^{9} \to x_3$$

$$\alpha_{2,7}^{11} \to x_4$$

$$\alpha_{6,7}^{15} \to x_5$$

$$\alpha_{4,9}^{15} \to x_6$$

$$\alpha_{4,7}^{13} \to x_7$$

$$\alpha_{4,8}^{14} \to x_8$$

$$\alpha_{4,6}^{12} \to x_9$$

$$\alpha_{5,8}^{15} \to x_{10}$$

$$\alpha_{2,11}^{15} \to x_{11}$$

$$\alpha_{3,7}^{12} \to x_{12}$$

$$\alpha_{3,5}^{10} \to x_{13}$$

$$\alpha_{3,10}^{15} \to x_{14}$$

$$\alpha_{5,7}^{14} \to x_{15}$$

$$\alpha_{3,8}^{13} \to x_{16}$$

$$\alpha_{2,6}^{10} \to x_{17}$$

$$\alpha_{3,9}^{14} \to x_{18}$$

$$\alpha_{4,5}^{11} \to x_{19}$$

$$\alpha_{2,8}^{12} \to x_{20}$$

$$\alpha_{5,6}^{13} \to x_{21}$$

$$\alpha_{2,9}^{13} \to x_{22}$$

$$\alpha_{3,6}^{11} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_2-x_3+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{13}-x_{17}+x_2 &&=0\\
(e_1,e_3,e_4):&\quad -x_{13}+x_3 &&=0\\
(e_1,e_2,e_6):&\quad x_{17}-x_{23}-x_4 &&=0\\
(e_1,e_3,e_5):&\quad x_{13}-x_{19}-x_{23} &&=0\\
(e_1,e_2,e_7):&\quad -x_{12}-x_{20}+x_4 &&=0\\
(e_1,e_3,e_6):&\quad -x_{12}+x_{23}-x_9 &&=0\\
(e_1,e_4,e_5):&\quad x_{19}-x_9 &&=0\\
(e_1,e_2,e_8):&\quad -x_{16}+x_{20}-x_{22} &&=0\\
(e_1,e_3,e_7):&\quad x_{12}-x_{16}-x_7 &&=0\\
(e_1,e_4,e_6):&\quad -x_{21}-x_7+x_9 &&=0\\
(e_2,e_3,e_4):&\quad -x_{16}+x_{22}x_3+x_7 &&=0\\
(e_1,e_2,e_9):&\quad -x_1-x_{18}+x_{22} &&=0\\
(e_1,e_3,e_8):&\quad x_{16}-x_{18}-x_8 &&=0\\
(e_1,e_4,e_7):&\quad -x_{15}+x_7-x_8 &&=0\\
(e_1,e_5,e_6):&\quad -x_{15}+x_{21} &&=0\\
(e_2,e_3,e_5):&\quad x_1x_{13}+x_{15}-x_{18}x_2 &&=0\\
(e_1,e_2,e_{10}):&\quad x_1-x_{11}-x_{14} &&=0\\
(e_1,e_3,e_9):&\quad -x_{14}+x_{18}-x_6 &&=0\\
(e_1,e_4,e_8):&\quad -x_{10}-x_6+x_8 &&=0\\
(e_1,e_5,e_7):&\quad -x_{10}+x_{15}-x_5 &&=0\\
(e_2,e_3,e_6):&\quad x_{11}x_{23}-x_{14}x_{17}+x_5 &&=0\\
(e_2,e_4,e_5):&\quad x_{10}+x_{11}x_{19}-x_2x_6 &&=0
\end{aligned}
$$

Groebner basis (23 variables, 19 linear, 3 nonlinear)

$$x_1-4x_{22}-14x_{23}+3=0$$

$$x_2+x_{21}+x_{22}+6x_{23}-2=0$$

$$-x_{21}-x_{22}-6x_{23}+x_3+1=0$$

$$2x_{21}+2x_{22}+13x_{23}+x_4-3=0$$

$$x_{14}-2x_{21}+4x_{22}+19x_{23}+x_5-4=0$$

$$x_{14}+3x_{22}+14x_{23}+x_6-3=0$$

$$-x_{22}-5x_{23}+x_7+1=0$$

$$x_{21}-x_{22}-5x_{23}+x_8+1=0$$

$$-x_{21} - x_{22} - 5x_{23} + x_9 + 1 = 0$$
$$x_{10} - x_{14} + x_{21} - 4x_{22} - 19x_{23} + 4 = 0$$
$$x_{11} + x_{14} - 4x_{22} - 14x_{23} + 3 = 0$$
$$x_{12} + x_{21} + x_{22} + 4x_{23} - 1 = 0$$
$$x_{13} - x_{21} - x_{22} - 6x_{23} + 1 = 0$$
$$x_{14}x_{21} + x_{14}x_{22} + \frac{11x_{14}x_{23}}{2} - 2x_{14} + x_{21} + 2x_{22}x_{23} - 2x_{22} + 7x_{23}^2 - 11x_{23} + 2 = 0$$
$$x_{14}x_{22}x_{23} + 6x_{14}x_{22} + 17x_{14}x_{23} - 2x_{14} - 4x_{22}^2x_{23} + 6x_{22}^2 - 14x_{22}x_{23}^2 + 30x_{22}x_{23} + 4x_{22} - 14x_{23}^2 + 50x_{23} - 10 = 0$$
$$x_{15} - x_{21} = 0$$
$$x_{16} + x_{21} + 2x_{22} + 9x_{23} - 2 = 0$$
$$x_{17} + 2x_{21} + 2x_{22} + 12x_{23} - 3 = 0$$
$$x_{18} + 3x_{22} + 14x_{23} - 3 = 0$$
$$x_{19} - x_{21} - x_{22} - 5x_{23} + 1 = 0$$
$$x_{20} + x_{21} + x_{22} + 9x_{23} - 2 = 0$$
$$x_{21}x_{22} + x_{21} + x_{22}^2 + 6x_{22}x_{23} + 2x_{22} + 14x_{23} - 3 = 0$$

$\mathfrak{m}_{2A}(5, 15)$

m2A515 (this line included for string searching purposes)

Original brackets:

$[e_1, e_2] = e_3$	$[e_1, e_3] = e_4$
$[e_1, e_4] = e_5$	$[e_1, e_5] = e_6$
$[e_1, e_6] = e_7$	$[e_1, e_7] = e_8$
$[e_1, e_8] = e_9$	$[e_1, e_9] = e_{10}$
$[e_1, e_{10}] = e_{11}$	$[e_1, e_{11}] = e_{12}$
$[e_1, e_{12}] = e_{13}$	$[e_1, e_{13}] = e_{14}$
$[e_1, e_{14}] = e_{15}$	$[e_2, e_9] = e_{14}$
$[e_2, e_{10}] = 4e_{15}$	$[e_3, e_8] = -e_{14}$
$[e_3, e_9] = -3e_{15}$	$[e_4, e_7] = e_{14}$
$[e_4, e_8] = 2e_{15}$	$[e_5, e_6] = -e_{14}$
$[e_5, e_7] = -e_{15}$	

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(5, 15)$

m4A515 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_7] = e_{12}$$
$$[e_2, e_8] = 3e_{13} \qquad [e_2, e_9] = \alpha_{2,9}^{14} e_{14}$$
$$[e_2, e_{10}] = \alpha_{2,10}^{15} e_{15} \qquad [e_3, e_6] = -e_{12}$$
$$[e_3, e_7] = -2e_{13} \qquad [e_3, e_8] = \alpha_{3,8}^{14} e_{14}$$
$$[e_3, e_9] = \alpha_{3,9}^{15} e_{15} \qquad [e_4, e_5] = e_{12}$$
$$[e_4, e_6] = e_{13} \qquad [e_4, e_7] = \alpha_{4,7}^{14} e_{14}$$
$$[e_4, e_8] = \alpha_{4,8}^{15} e_{15} \qquad [e_5, e_6] = \alpha_{5,6}^{14} e_{14}$$
$$[e_5, e_7] = \alpha_{5,7}^{15} e_{15}$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_8): \quad -\alpha_{2,9}^{14} - \alpha_{3,8}^{14} + 3 = 0$$
$$(e_1, e_3, e_7): \quad -\alpha_{3,8}^{14} - \alpha_{4,7}^{14} - 2 = 0$$
$$(e_1, e_4, e_6): \quad -\alpha_{4,7}^{14} - \alpha_{5,6}^{14} + 1 = 0$$
$$(e_1, e_2, e_9): \quad -\alpha_{2,10}^{15} + \alpha_{2,9}^{14} - \alpha_{3,9}^{15} = 0$$
$$(e_1, e_3, e_8): \quad \alpha_{3,8}^{14} - \alpha_{3,9}^{15} - \alpha_{4,8}^{15} = 0$$
$$(e_1, e_4, e_7): \quad \alpha_{4,7}^{14} - \alpha_{4,8}^{15} - \alpha_{5,7}^{15} = 0$$
$$(e_1, e_5, e_6): \quad \alpha_{5,6}^{14} - \alpha_{5,7}^{15} = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{4,7}^{14} \to x_1$$
$$\alpha_{3,8}^{14} \to x_2$$
$$\alpha_{5,7}^{15} \to x_3$$
$$\alpha_{3,9}^{15} \to x_4$$

$$\alpha_{5,6}^{14} \to x_5$$

$$\alpha_{2,10}^{15} \to x_6$$

$$\alpha_{4,8}^{15} \to x_7$$

$$\alpha_{2,9}^{14} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8) &: \quad -x_2 - x_8 + 3 &= 0 \\
(e_1, e_3, e_7) &: \quad -x_1 - x_2 - 2 &= 0 \\
(e_1, e_4, e_6) &: \quad -x_1 - x_5 + 1 &= 0 \\
(e_1, e_2, e_9) &: \quad -x_4 - x_6 + x_8 &= 0 \\
(e_1, e_3, e_8) &: \quad x_2 - x_4 - x_7 &= 0 \\
(e_1, e_4, e_7) &: \quad x_1 - x_3 - x_7 &= 0 \\
(e_1, e_5, e_6) &: \quad -x_3 + x_5 &= 0
\end{aligned}$$

Groebner basis (8 variables, 7 linear, 0 nonlinear)

$$x_1 - x_8 + 5 = 0$$

$$x_2 + x_8 - 3 = 0$$

$$x_3 + x_8 - 6 = 0$$

$$x_4 + 3x_8 - 14 = 0$$

$$x_5 + x_8 - 6 = 0$$

$$x_6 - 4x_8 + 14 = 0$$

$$x_7 - 2x_8 + 11 = 0$$

$\mathfrak{m}_{6A}(5, 15)$

m6A515 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_5] = e_{10} \\
[e_2, e_6] = 2e_{11} & [e_2, e_7] = \alpha_{2,7}^{12} e_{12} \\
[e_2, e_8] = \alpha_{2,8}^{13} e_{13} & [e_2, e_9] = \alpha_{2,9}^{14} e_{14} \\
[e_2, e_{10}] = \alpha_{2,10}^{15} e_{15} & [e_3, e_4] = -e_{10} \\
[e_3, e_5] = -e_{11} & [e_3, e_6] = \alpha_{3,6}^{12} e_{12} \\
[e_3, e_7] = \alpha_{3,7}^{13} e_{13} & [e_3, e_8] = \alpha_{3,8}^{14} e_{14} \\
[e_3, e_9] = \alpha_{3,9}^{15} e_{15} & [e_4, e_5] = \alpha_{4,5}^{12} e_{12} \\
[e_4, e_6] = \alpha_{4,6}^{13} e_{13} & [e_4, e_7] = \alpha_{4,7}^{14} e_{14} \\
[e_4, e_8] = \alpha_{4,8}^{15} e_{15} & [e_5, e_6] = \alpha_{5,6}^{14} e_{14} \\
[e_5, e_7] = \alpha_{5,7}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_6): & -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 & = 0 \\
(e_1, e_3, e_5): & -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 & = 0 \\
(e_1, e_2, e_7): & \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} & = 0 \\
(e_1, e_3, e_6): & \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} & = 0 \\
(e_1, e_4, e_5): & \alpha_{4,5}^{12} - \alpha_{4,6}^{13} & = 0 \\
(e_1, e_2, e_8): & \alpha_{2,8}^{13} - \alpha_{2,9}^{14} - \alpha_{3,8}^{14} & = 0 \\
(e_1, e_3, e_7): & \alpha_{3,7}^{13} - \alpha_{3,8}^{14} - \alpha_{4,7}^{14} & = 0 \\
(e_1, e_4, e_6): & \alpha_{4,6}^{13} - \alpha_{4,7}^{14} - \alpha_{5,6}^{14} & = 0 \\
(e_1, e_2, e_9): & -\alpha_{2,10}^{15} + \alpha_{2,9}^{14} - \alpha_{3,9}^{15} & = 0 \\
(e_1, e_3, e_8): & \alpha_{3,8}^{14} - \alpha_{3,9}^{15} - \alpha_{4,8}^{15} & = 0 \\
(e_1, e_4, e_7): & \alpha_{4,7}^{14} - \alpha_{4,8}^{15} - \alpha_{5,7}^{15} & = 0 \\
(e_1, e_5, e_6): & \alpha_{5,6}^{14} - \alpha_{5,7}^{15} & = 0 \\
(e_2, e_3, e_4): & -\alpha_{2,10}^{15} & = 0
\end{array}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{4,6}^{13} \to x_1$$
$$\alpha_{4,5}^{12} \to x_2$$
$$\alpha_{4,7}^{14} \to x_3$$
$$\alpha_{3,8}^{14} \to x_4$$
$$\alpha_{2,8}^{13} \to x_5$$
$$\alpha_{3,9}^{15} \to x_6$$
$$\alpha_{5,7}^{15} \to x_7$$
$$\alpha_{3,7}^{13} \to x_8$$
$$\alpha_{2,7}^{12} \to x_9$$
$$\alpha_{5,6}^{14} \to x_{10}$$
$$\alpha_{2,10}^{15} \to x_{11}$$
$$\alpha_{4,8}^{15} \to x_{12}$$
$$\alpha_{2,9}^{14} \to x_{13}$$
$$\alpha_{3,6}^{12} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{14} - x_9 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_{14} - x_2 - 1 &= 0\\
(e_1, e_2, e_7): &\quad -x_5 - x_8 + x_9 &= 0\\
(e_1, e_3, e_6): &\quad -x_1 + x_{14} - x_8 &= 0\\
(e_1, e_4, e_5): &\quad -x_1 + x_2 &= 0\\
(e_1, e_2, e_8): &\quad -x_{13} - x_4 + x_5 &= 0\\
(e_1, e_3, e_7): &\quad -x_3 - x_4 + x_8 &= 0\\
(e_1, e_4, e_6): &\quad x_1 - x_{10} - x_3 &= 0\\
(e_1, e_2, e_9): &\quad -x_{11} + x_{13} - x_6 &= 0\\
(e_1, e_3, e_8): &\quad -x_{12} + x_4 - x_6 &= 0\\
(e_1, e_4, e_7): &\quad -x_{12} + x_3 - x_7 &= 0\\
(e_1, e_5, e_6): &\quad x_{10} - x_7 &= 0\\
(e_2, e_3, e_4): &\quad -x_{11} &= 0
\end{aligned}$$

Groebner basis (14 variables, 13 linear, 0 nonlinear)

$$x_1 + x_{14} + 1 = 0$$
$$x_{14} + x_2 + 1 = 0$$
$$-\frac{3x_{14}}{2} + x_3 = 0$$
$$-\frac{x_{14}}{2} + x_4 - 1 = 0$$
$$3x_{14} + x_5 - 1 = 0$$
$$\frac{7x_{14}}{2} + x_6 = 0$$
$$\frac{5x_{14}}{2} + x_7 + 1 = 0$$
$$-2x_{14} + x_8 - 1 = 0$$
$$x_{14} + x_9 - 2 = 0$$
$$x_{10} + \frac{5x_{14}}{2} + 1 = 0$$
$$x_{11} = 0$$
$$x_{12} - 4x_{14} - 1 = 0$$
$$x_{13} + \frac{7x_{14}}{2} = 0$$

$\mathfrak{m}_{8A}(5, 15)$

m8A515 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_3] = e_8 \\
[e_2, e_4] = e_9 & [e_2, e_5] = \alpha_{2,5}^{10} e_{10} \\
[e_2, e_6] = \alpha_{2,6}^{11} e_{11} & [e_2, e_7] = \alpha_{2,7}^{12} e_{12} \\
[e_2, e_8] = \alpha_{2,8}^{13} e_{13} & [e_2, e_9] = \alpha_{2,9}^{14} e_{14} \\
[e_2, e_{10}] = \alpha_{2,10}^{15} e_{15} & [e_3, e_4] = \alpha_{3,4}^{10} e_{10} \\
[e_3, e_5] = \alpha_{3,5}^{11} e_{11} & [e_3, e_6] = \alpha_{3,6}^{12} e_{12} \\
[e_3, e_7] = \alpha_{3,7}^{13} e_{13} & [e_3, e_8] = \alpha_{3,8}^{14} e_{14} \\
[e_3, e_9] = \alpha_{3,9}^{15} e_{15} & [e_4, e_5] = \alpha_{4,5}^{12} e_{12} \\
[e_4, e_6] = \alpha_{4,6}^{13} e_{13} & [e_4, e_7] = \alpha_{4,7}^{14} e_{14} \\
[e_4, e_8] = \alpha_{4,8}^{15} e_{15} & [e_5, e_6] = \alpha_{5,6}^{14} e_{14} \\
[e_5, e_7] = \alpha_{5,7}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{13} - \alpha_{2,9}^{14} - \alpha_{3,8}^{14} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{13} - \alpha_{3,8}^{14} - \alpha_{4,7}^{14} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{13} - \alpha_{4,7}^{14} - \alpha_{5,6}^{14} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{15} + \alpha_{2,9}^{14} - \alpha_{3,9}^{15} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{14} - \alpha_{3,9}^{15} - \alpha_{4,8}^{15} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{14} - \alpha_{4,8}^{15} - \alpha_{5,7}^{15} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{14} - \alpha_{5,7}^{15} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,10}^{15}\alpha_{3,4}^{10} - \alpha_{3,9}^{15} + \alpha_{4,8}^{15} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,6}^{13} \to x_1$$
$$\alpha_{4,5}^{12} \to x_2$$
$$\alpha_{4,7}^{14} \to x_3$$
$$\alpha_{3,8}^{14} \to x_4$$
$$\alpha_{3,5}^{11} \to x_5$$
$$\alpha_{2,8}^{13} \to x_6$$
$$\alpha_{3,9}^{15} \to x_7$$
$$\alpha_{2,5}^{10} \to x_8$$
$$\alpha_{5,7}^{15} \to x_9$$
$$\alpha_{3,7}^{13} \to x_{10}$$

$$\alpha^{12}_{3,6} \to x_{11}$$
$$\alpha^{12}_{2,7} \to x_{12}$$
$$\alpha^{14}_{5,6} \to x_{13}$$
$$\alpha^{15}_{2,10} \to x_{14}$$
$$\alpha^{10}_{3,4} \to x_{15}$$
$$\alpha^{15}_{4,8} \to x_{16}$$
$$\alpha^{14}_{2,9} \to x_{17}$$
$$\alpha^{11}_{2,6} \to x_{18}$$

Jacobi Tests

$$\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{15}-x_8+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_{18}-x_5+x_8 &&=0\\
(e_1,e_3,e_4):&\quad x_{15}-x_5 &&=0\\
(e_1,e_2,e_6):&\quad -x_{11}-x_{12}+x_{18} &&=0\\
(e_1,e_3,e_5):&\quad -x_{11}-x_2+x_5 &&=0\\
(e_1,e_2,e_7):&\quad -x_{10}+x_{12}-x_6 &&=0\\
(e_1,e_3,e_6):&\quad -x_1-x_{10}+x_{11} &&=0\\
(e_1,e_4,e_5):&\quad -x_1+x_2 &&=0\\
(e_1,e_2,e_8):&\quad -x_{17}-x_4+x_6 &&=0\\
(e_1,e_3,e_7):&\quad x_{10}-x_3-x_4 &&=0\\
(e_1,e_4,e_6):&\quad x_1-x_{13}-x_3 &&=0\\
(e_1,e_2,e_9):&\quad -x_{14}+x_{17}-x_7 &&=0\\
(e_1,e_3,e_8):&\quad -x_{16}+x_4-x_7 &&=0\\
(e_1,e_4,e_7):&\quad -x_{16}+x_3-x_9 &&=0\\
(e_1,e_5,e_6):&\quad x_{13}-x_9 &&=0\\
(e_2,e_3,e_4):&\quad x_{14}x_{15}+x_{16}-x_7 &&=0
\end{aligned}$$

Groebner basis (18 variables, 15 linear, 1 nonlinear)

$$x_1+\frac{x_{16}}{11}-\frac{2x_{17}}{11}+\frac{5x_{18}}{11}-\frac{3}{11}=0$$
$$\frac{x_{16}}{11}-\frac{2x_{17}}{11}+\frac{5x_{18}}{11}+x_2-\frac{3}{11}=0$$
$$-\frac{5x_{16}}{11}-\frac{x_{17}}{11}+\frac{5x_{18}}{22}+x_3-\frac{3}{22}=0$$

$$\frac{3x_{16}}{11} + \frac{5x_{17}}{11} - \frac{7x_{18}}{11} + x_4 + \frac{2}{11} = 0$$
$$\frac{x_{18}}{2} + x_5 - \frac{1}{2} = 0$$
$$\frac{3x_{16}}{11} - \frac{6x_{17}}{11} - \frac{7x_{18}}{11} + x_6 + \frac{2}{11} = 0$$
$$\frac{14x_{16}}{11} + \frac{5x_{17}}{11} - \frac{7x_{18}}{11} + x_7 + \frac{2}{11} = 0$$
$$-\frac{x_{18}}{2} + x_8 - \frac{1}{2} = 0$$
$$\frac{6x_{16}}{11} - \frac{x_{17}}{11} + \frac{5x_{18}}{22} + x_9 - \frac{3}{22} = 0$$
$$x_{10} - \frac{2x_{16}}{11} + \frac{4x_{17}}{11} - \frac{9x_{18}}{22} + \frac{1}{22} = 0$$
$$x_{11} - \frac{x_{16}}{11} + \frac{2x_{17}}{11} + \frac{x_{18}}{22} - \frac{5}{22} = 0$$
$$x_{12} + \frac{x_{16}}{11} - \frac{2x_{17}}{11} - \frac{23x_{18}}{22} + \frac{5}{22} = 0$$
$$x_{13} + \frac{6x_{16}}{11} - \frac{x_{17}}{11} + \frac{5x_{18}}{22} - \frac{3}{22} = 0$$
$$x_{14} - \frac{14x_{16}}{11} - \frac{16x_{17}}{11} + \frac{7x_{18}}{11} - \frac{2}{11} = 0$$
$$x_{15} + \frac{x_{18}}{2} - \frac{1}{2} = 0$$
$$x_{16}x_{18} - \frac{32x_{16}}{7} + \frac{8x_{17}x_{18}}{7} - \frac{13x_{17}}{7} - \frac{x_{18}^2}{2} + \frac{23x_{18}}{14} - \frac{3}{7} = 0$$

$\mathfrak{m}_{1A}(6, 15)$

m1A615 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_1, e_{13}] = e_{14}$$
$$[e_1, e_{14}] = e_{15} \qquad [e_2, e_9] = e_{15}$$
$$[e_3, e_8] = -e_{15} \qquad [e_4, e_7] = e_{15}$$
$$[e_5, e_6] = -e_{15}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(6, 15)$

m3A615 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_7] = e_{13} \\
[e_2, e_8] = 3e_{14} & [e_2, e_9] = \alpha_{2,9}^{15} e_{15} \\
[e_3, e_6] = -e_{13} & [e_3, e_7] = -2e_{14} \\
[e_3, e_8] = \alpha_{3,8}^{15} e_{15} & [e_4, e_5] = e_{13} \\
[e_4, e_6] = e_{14} & [e_4, e_7] = \alpha_{4,7}^{15} e_{15} \\
[e_5, e_6] = \alpha_{5,6}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_8): & -\alpha_{2,9}^{15} - \alpha_{3,8}^{15} + 3 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{15} - \alpha_{4,7}^{15} - 2 & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{15} - \alpha_{5,6}^{15} + 1 & = 0
\end{array}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{array}{c}
\alpha_{5,6}^{15} \to x_1 \\
\alpha_{4,7}^{15} \to x_2 \\
\alpha_{2,9}^{15} \to x_3 \\
\alpha_{3,8}^{15} \to x_4
\end{array}
$$

Jacobi Tests

$$
\begin{array}{lll}
(e_1, e_2, e_8): & -x_3 - x_4 + 3 & = 0 \\
(e_1, e_3, e_7): & -x_2 - x_4 - 2 & = 0 \\
(e_1, e_4, e_6): & -x_1 - x_2 + 1 & = 0
\end{array}
$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 - 3 = 0$$
$$x_2 + x_4 + 2 = 0$$
$$x_3 + x_4 - 3 = 0$$

$\mathfrak{m}_{5A}(6, 15)$

m5A615 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_5] &= e_{11} \\
[e_2, e_6] &= 2e_{12} & [e_2, e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_2, e_8] &= \alpha_{2,8}^{14} e_{14} & [e_2, e_9] &= \alpha_{2,9}^{15} e_{15} \\
[e_3, e_4] &= -e_{11} & [e_3, e_5] &= -e_{12} \\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_3, e_7] &= \alpha_{3,7}^{14} e_{14} \\
[e_3, e_8] &= \alpha_{3,8}^{15} e_{15} & [e_4, e_5] &= \alpha_{4,5}^{13} e_{13} \\
[e_4, e_6] &= \alpha_{4,6}^{14} e_{14} & [e_4, e_7] &= \alpha_{4,7}^{15} e_{15} \\
[e_5, e_6] &= \alpha_{5,6}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{13} - \alpha_{3,6}^{13} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{13} - \alpha_{4,5}^{13} - 1 &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{13} - \alpha_{2,8}^{14} - \alpha_{3,7}^{14} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{13} - \alpha_{3,7}^{14} - \alpha_{4,6}^{14} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{13} - \alpha_{4,6}^{14} &= 0 \\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{14} - \alpha_{2,9}^{15} - \alpha_{3,8}^{15} &= 0 \\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{14} - \alpha_{3,8}^{15} - \alpha_{4,7}^{15} &= 0 \\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{14} - \alpha_{4,7}^{15} - \alpha_{5,6}^{15} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{15}_{2,9} \to x_1$$
$$\alpha^{15}_{5,6} \to x_2$$
$$\alpha^{13}_{3,6} \to x_3$$
$$\alpha^{15}_{3,8} \to x_4$$
$$\alpha^{13}_{2,7} \to x_5$$
$$\alpha^{14}_{4,6} \to x_6$$
$$\alpha^{14}_{2,8} \to x_7$$
$$\alpha^{15}_{4,7} \to x_8$$
$$\alpha^{13}_{4,5} \to x_9$$
$$\alpha^{14}_{3,7} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):& \quad -x_3 - x_5 + 2 &&= 0\\
(e_1, e_3, e_5):& \quad -x_3 - x_9 - 1 &&= 0\\
(e_1, e_2, e_7):& \quad -x_{10} + x_5 - x_7 &&= 0\\
(e_1, e_3, e_6):& \quad -x_{10} + x_3 - x_6 &&= 0\\
(e_1, e_4, e_5):& \quad -x_6 + x_9 &&= 0\\
(e_1, e_2, e_8):& \quad -x_1 - x_4 + x_7 &&= 0\\
(e_1, e_3, e_7):& \quad x_{10} - x_4 - x_8 &&= 0\\
(e_1, e_4, e_6):& \quad -x_2 + x_6 - x_8 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 + \frac{5x_{10}}{2} - x_8 - \frac{5}{2} = 0$$
$$\frac{x_{10}}{2} + x_2 + x_8 + \frac{1}{2} = 0$$
$$-\frac{x_{10}}{2} + x_3 + \frac{1}{2} = 0$$
$$-x_{10} + x_4 + x_8 = 0$$
$$\frac{x_{10}}{2} + x_5 - \frac{5}{2} = 0$$

$$\frac{x_{10}}{2} + x_6 + \frac{1}{2} = 0$$
$$\frac{3x_{10}}{2} + x_7 - \frac{5}{2} = 0$$
$$\frac{x_{10}}{2} + x_9 + \frac{1}{2} = 0$$

$\mathfrak{m}_{7A}(6, 15)$

m7A615 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_9 \\
[e_2, e_4] &= e_{10} & [e_2, e_5] &= \alpha_{2,5}^{11} e_{11} \\
[e_2, e_6] &= \alpha_{2,6}^{12} e_{12} & [e_2, e_7] &= \alpha_{2,7}^{13} e_{13} \\
[e_2, e_8] &= \alpha_{2,8}^{14} e_{14} & [e_2, e_9] &= \alpha_{2,9}^{15} e_{15} \\
[e_3, e_4] &= \alpha_{3,4}^{11} e_{11} & [e_3, e_5] &= \alpha_{3,5}^{12} e_{12} \\
[e_3, e_6] &= \alpha_{3,6}^{13} e_{13} & [e_3, e_7] &= \alpha_{3,7}^{14} e_{14} \\
[e_3, e_8] &= \alpha_{3,8}^{15} e_{15} & [e_4, e_5] &= \alpha_{4,5}^{13} e_{13} \\
[e_4, e_6] &= \alpha_{4,6}^{14} e_{14} & [e_4, e_7] &= \alpha_{4,7}^{15} e_{15} \\
[e_5, e_6] &= \alpha_{5,6}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{11} - \alpha_{3,4}^{11} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{11} - \alpha_{2,6}^{12} - \alpha_{3,5}^{12} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{11} - \alpha_{3,5}^{12} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{12} - \alpha_{2,7}^{13} - \alpha_{3,6}^{13} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{12} - \alpha_{3,6}^{13} - \alpha_{4,5}^{13} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{13} - \alpha_{2,8}^{14} - \alpha_{3,7}^{14} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{13} - \alpha_{3,7}^{14} - \alpha_{4,6}^{14} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{13} - \alpha_{4,6}^{14} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{14} - \alpha_{2,9}^{15} - \alpha_{3,8}^{15} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{14} - \alpha_{3,8}^{15} - \alpha_{4,7}^{15} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{14} - \alpha_{4,7}^{15} - \alpha_{5,6}^{15} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\begin{gathered}
\alpha_{2,6}^{12} \to x_1\\
\alpha_{4,5}^{13} \to x_2\\
\alpha_{2,9}^{15} \to x_3\\
\alpha_{5,6}^{15} \to x_4\\
\alpha_{3,6}^{13} \to x_5\\
\alpha_{3,8}^{15} \to x_6\\
\alpha_{2,7}^{13} \to x_7\\
\alpha_{3,4}^{11} \to x_8\\
\alpha_{4,6}^{14} \to x_9\\
\alpha_{2,5}^{11} \to x_{10}\\
\alpha_{2,8}^{14} \to x_{11}\\
\alpha_{4,7}^{15} \to x_{12}\\
\alpha_{3,5}^{12} \to x_{13}\\
\alpha_{3,7}^{14} \to x_{14}
\end{gathered}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -x_{10} - x_8 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_1 + x_{10} - x_{13} &&= 0\\
(e_1, e_3, e_4):&\quad -x_{13} + x_8 &&= 0\\
(e_1, e_2, e_6):&\quad x_1 - x_5 - x_7 &&= 0\\
(e_1, e_3, e_5):&\quad x_{13} - x_2 - x_5 &&= 0\\
(e_1, e_2, e_7):&\quad -x_{11} - x_{14} + x_7 &&= 0\\
(e_1, e_3, e_6):&\quad -x_{14} + x_5 - x_9 &&= 0\\
(e_1, e_4, e_5):&\quad x_2 - x_9 &&= 0\\
(e_1, e_2, e_8):&\quad x_{11} - x_3 - x_6 &&= 0\\
(e_1, e_3, e_7):&\quad -x_{12} + x_{14} - x_6 &&= 0\\
(e_1, e_4, e_6):&\quad -x_{12} - x_4 + x_9 &&= 0
\end{aligned}
$$

Groebner basis (14 variables, 11 linear, 0 nonlinear)

$$
\begin{gathered}
x_1 + 2x_{13} - 1 = 0\\
-\frac{x_{13}}{2} + \frac{x_{14}}{2} + x_2 = 0\\
-x_{12} + \frac{5x_{13}}{2} + \frac{5x_{14}}{2} + x_3 - 1 = 0\\
x_{12} - \frac{x_{13}}{2} + \frac{x_{14}}{2} + x_4 = 0\\
-\frac{x_{13}}{2} - \frac{x_{14}}{2} + x_5 = 0\\
x_{12} - x_{14} + x_6 = 0\\
\frac{5x_{13}}{2} + \frac{x_{14}}{2} + x_7 - 1 = 0\\
-x_{13} + x_8 = 0\\
-\frac{x_{13}}{2} + \frac{x_{14}}{2} + x_9 = 0\\
x_{10} + x_{13} - 1 = 0\\
x_{11} + \frac{5x_{13}}{2} + \frac{3x_{14}}{2} - 1 = 0
\end{gathered}
$$

$\mathfrak{m}_{2A}(7,15)$

m2A715 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_1,e_{13}]=e_{14} \\
[e_1,e_{14}]=e_{15} & [e_2,e_7]=e_{14} \\
[e_2,e_8]=3e_{15} & [e_3,e_6]=-e_{14} \\
[e_3,e_7]=-2e_{15} & [e_4,e_5]=e_{14} \\
[e_4,e_6]=e_{15} &
\end{array}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(7,15)$

m4A715 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_1,e_{13}]=e_{14} \\
[e_1,e_{14}]=e_{15} & [e_2,e_5]=e_{12} \\
[e_2,e_6]=2e_{13} & [e_2,e_7]=\alpha_{2,7}^{14}e_{14} \\
[e_2,e_8]=\alpha_{2,8}^{15}e_{15} & [e_3,e_4]=-e_{12} \\
[e_3,e_5]=-e_{13} & [e_3,e_6]=\alpha_{3,6}^{14}e_{14} \\
[e_3,e_7]=\alpha_{3,7}^{15}e_{15} & [e_4,e_5]=\alpha_{4,5}^{14}e_{14} \\
[e_4,e_6]=\alpha_{4,6}^{15}e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -\alpha^{14}_{2,7} - \alpha^{14}_{3,6} + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -\alpha^{14}_{3,6} - \alpha^{14}_{4,5} - 1 &&= 0\\
(e_1, e_2, e_7):&\quad \alpha^{14}_{2,7} - \alpha^{15}_{2,8} - \alpha^{15}_{3,7} &&= 0\\
(e_1, e_3, e_6):&\quad \alpha^{14}_{3,6} - \alpha^{15}_{3,7} - \alpha^{15}_{4,6} &&= 0\\
(e_1, e_4, e_5):&\quad \alpha^{14}_{4,5} - \alpha^{15}_{4,6} &&= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{15}_{3,7} \to x_1$$

$$\alpha^{14}_{2,7} \to x_2$$

$$\alpha^{15}_{2,8} \to x_3$$

$$\alpha^{14}_{3,6} \to x_4$$

$$\alpha^{14}_{4,5} \to x_5$$

$$\alpha^{15}_{4,6} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6):&\quad -x_2 - x_4 + 2 &&= 0\\
(e_1, e_3, e_5):&\quad -x_4 - x_5 - 1 &&= 0\\
(e_1, e_2, e_7):&\quad -x_1 + x_2 - x_3 &&= 0\\
(e_1, e_3, e_6):&\quad -x_1 + x_4 - x_6 &&= 0\\
(e_1, e_4, e_5):&\quad x_5 - x_6 &&= 0
\end{aligned}$$

Groebner basis (6 variables, 5 linear, 0 nonlinear)

$$x_1 + 2x_6 + 1 = 0$$

$$x_2 - x_6 - 3 = 0$$

$$x_3 - 3x_6 - 4 = 0$$

$$x_4 + x_6 + 1 = 0$$

$$x_5 - x_6 = 0$$

$\mathfrak{m}_{6A}(7, 15)$

m6A715 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_5] &= \alpha_{2,5}^{12} e_{12} \\
[e_2, e_6] &= \alpha_{2,6}^{13} e_{13} & [e_2, e_7] &= \alpha_{2,7}^{14} e_{14} \\
[e_2, e_8] &= \alpha_{2,8}^{15} e_{15} & [e_3, e_4] &= \alpha_{3,4}^{12} e_{12} \\
[e_3, e_5] &= \alpha_{3,5}^{13} e_{13} & [e_3, e_6] &= \alpha_{3,6}^{14} e_{14} \\
[e_3, e_7] &= \alpha_{3,7}^{15} e_{15} & [e_4, e_5] &= \alpha_{4,5}^{14} e_{14} \\
[e_4, e_6] &= \alpha_{4,6}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4) &: \quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 &&= 0 \\
(e_1, e_2, e_5) &: \quad \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} &&= 0 \\
(e_1, e_3, e_4) &: \quad \alpha_{3,4}^{12} - \alpha_{3,5}^{13} &&= 0 \\
(e_1, e_2, e_6) &: \quad \alpha_{2,6}^{13} - \alpha_{2,7}^{14} - \alpha_{3,6}^{14} &&= 0 \\
(e_1, e_3, e_5) &: \quad \alpha_{3,5}^{13} - \alpha_{3,6}^{14} - \alpha_{4,5}^{14} &&= 0 \\
(e_1, e_2, e_7) &: \quad \alpha_{2,7}^{14} - \alpha_{2,8}^{15} - \alpha_{3,7}^{15} &&= 0 \\
(e_1, e_3, e_6) &: \quad \alpha_{3,6}^{14} - \alpha_{3,7}^{15} - \alpha_{4,6}^{15} &&= 0 \\
(e_1, e_4, e_5) &: \quad \alpha_{4,5}^{14} - \alpha_{4,6}^{15} &&= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{2,6}^{13} &\to x_1 \\
\alpha_{3,7}^{15} &\to x_2 \\
\alpha_{2,7}^{14} &\to x_3 \\
\alpha_{3,5}^{13} &\to x_4
\end{aligned}
$$

$$\alpha^{14}_{3,6} \to x_5$$

$$\alpha^{12}_{3,4} \to x_6$$

$$\alpha^{15}_{2,8} \to x_7$$

$$\alpha^{12}_{2,5} \to x_8$$

$$\alpha^{14}_{4,5} \to x_9$$

$$\alpha^{15}_{4,6} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_6 - x_8 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_4 + x_8 &&= 0\\
(e_1, e_3, e_4): &\quad -x_4 + x_6 &&= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_3 - x_5 &&= 0\\
(e_1, e_3, e_5): &\quad x_4 - x_5 - x_9 &&= 0\\
(e_1, e_2, e_7): &\quad -x_2 + x_3 - x_7 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_2 + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{10} + x_9 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 8 linear, 0 nonlinear)

$$x_1 - 2x_8 + 1 = 0$$

$$2x_{10} + x_2 + x_8 - 1 = 0$$

$$-x_{10} + x_3 - 3x_8 + 2 = 0$$

$$x_4 + x_8 - 1 = 0$$

$$x_{10} + x_5 + x_8 - 1 = 0$$

$$x_6 + x_8 - 1 = 0$$

$$-3x_{10} + x_7 - 4x_8 + 3 = 0$$

$$-x_{10} + x_9 = 0$$

$\mathfrak{m}_{1A}(8,15)$

m1A815 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14} \\
[e_1,e_{14}] &= e_{15} & [e_2,e_7] &= e_{15} \\
[e_3,e_6] &= -e_{15} & [e_4,e_5] &= e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(8,15)$

m3A815 (this line included for string searching purposes)
Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12} \\
[e_1,e_{12}] &= e_{13} & [e_1,e_{13}] &= e_{14} \\
[e_1,e_{14}] &= e_{15} & [e_2,e_5] &= e_{13} \\
[e_2,e_6] &= 2e_{14} & [e_2,e_7] &= \alpha_{2,7}^{15} e_{15} \\
[e_3,e_4] &= -e_{13} & [e_3,e_5] &= -e_{14} \\
[e_3,e_6] &= \alpha_{3,6}^{15} e_{15} & [e_4,e_5] &= \alpha_{4,5}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_6): &\quad -\alpha_{2,7}^{15} - \alpha_{3,6}^{15} + 2 &= 0 \\
(e_1,e_3,e_5): &\quad -\alpha_{3,6}^{15} - \alpha_{4,5}^{15} - 1 &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:

Change variables

$$\alpha_{4,5}^{15} \to x_1$$
$$\alpha_{3,6}^{15} \to x_2$$
$$\alpha_{2,7}^{15} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): \quad & -x_2 - x_3 + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -x_1 - x_2 - 1 && = 0
\end{aligned}$$

Groebner basis (3 variables, 2 linear, 0 nonlinear)

$$x_1 - x_3 + 3 = 0$$
$$x_2 + x_3 - 2 = 0$$

$\mathfrak{m}_{5A}(8, 15)$

m5A815 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{11} \\
[e_2, e_4] &= e_{12} & [e_2, e_5] &= \alpha_{2,5}^{13} e_{13} \\
[e_2, e_6] &= \alpha_{2,6}^{14} e_{14} & [e_2, e_7] &= \alpha_{2,7}^{15} e_{15} \\
[e_3, e_4] &= \alpha_{3,4}^{13} e_{13} & [e_3, e_5] &= \alpha_{3,5}^{14} e_{14} \\
[e_3, e_6] &= \alpha_{3,6}^{15} e_{15} & [e_4, e_5] &= \alpha_{4,5}^{15} e_{15}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{13} - \alpha_{2,6}^{14} - \alpha_{3,5}^{14} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{13} - \alpha_{3,5}^{14} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{14} - \alpha_{2,7}^{15} - \alpha_{3,6}^{15} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{14} - \alpha_{3,6}^{15} - \alpha_{4,5}^{15} &= 0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{15} \to x_1$$

$$\alpha_{3,6}^{15} \to x_2$$

$$\alpha_{2,7}^{15} \to x_3$$

$$\alpha_{3,4}^{13} \to x_4$$

$$\alpha_{2,5}^{13} \to x_5$$

$$\alpha_{2,6}^{14} \to x_6$$

$$\alpha_{3,5}^{14} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_4 - x_5 + 1 &= 0\\
(e_1, e_2, e_5): &\quad x_5 - x_6 - x_7 &= 0\\
(e_1, e_3, e_4): &\quad x_4 - x_7 &= 0\\
(e_1, e_2, e_6): &\quad -x_2 - x_3 + x_6 &= 0\\
(e_1, e_3, e_5): &\quad -x_1 - x_2 + x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 5 linear, 0 nonlinear)

$$x_1 - x_3 - 3x_7 + 1 = 0$$

$$x_2 + x_3 + 2x_7 - 1 = 0$$

$$x_4 - x_7 = 0$$

$$x_5 + x_7 - 1 = 0$$

$$x_6 + 2x_7 - 1 = 0$$

$\mathfrak{m}_{2A}(9, 15)$

m2A915 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_5] &= e_{14} \\
[e_2, e_6] &= 2e_{15} & [e_3, e_4] &= -e_{14} \\
[e_3, e_5] &= -e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{4A}(9, 15)$

m4A915 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{12} \\
[e_2, e_4] &= e_{13} & [e_2, e_5] &= \alpha_{2,5}^{14} e_{14} \\
[e_2, e_6] &= \alpha_{2,6}^{15} e_{15} & [e_3, e_4] &= \alpha_{3,4}^{14} e_{14} \\
[e_3, e_5] &= \alpha_{3,5}^{15} e_{15}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{14} - \alpha_{3,4}^{14} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{14} - \alpha_{2,6}^{15} - \alpha_{3,5}^{15} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{14} - \alpha_{3,5}^{15} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{3,5}^{15} \to x_1$$
$$\alpha_{2,6}^{15} \to x_2$$
$$\alpha_{2,5}^{14} \to x_3$$
$$\alpha_{3,4}^{14} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_4 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_1 - x_2 + x_3 &= 0\\
(e_1, e_3, e_4): &\quad -x_1 + x_4 &= 0
\end{aligned}$$

Groebner basis (4 variables, 3 linear, 0 nonlinear)

$$x_1 - x_4 = 0$$
$$x_2 + 2x_4 - 1 = 0$$
$$x_3 + x_4 - 1 = 0$$

$\mathfrak{m}_{1A}(10, 15)$

m1A1015 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14}\\
[e_1, e_{14}] &= e_{15} & [e_2, e_5] &= e_{15}\\
[e_3, e_4] &= -e_{15}
\end{aligned}$$

No non-trivial Jacobi tests

$\mathfrak{m}_{3A}(10, 15)$

m3A1015 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_1, e_{13}] = e_{14} \\
[e_1, e_{14}] = e_{15} & [e_2, e_3] = e_{13} \\
[e_2, e_4] = e_{14} & [e_2, e_5] = \alpha_{2,5}^{15} e_{15} \\
[e_3, e_4] = \alpha_{3,4}^{15} e_{15} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{2,5}^{15} - \alpha_{3,4}^{15} + 1 = 0$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{3,4}^{15} \to x_1$$
$$\alpha_{2,5}^{15} \to x_2$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - x_2 + 1 = 0$$

Groebner basis (2 variables, 1 linear, 0 nonlinear)

$$x_1 + x_2 - 1 = 0$$

$\mathfrak{m}_{2A}(11, 15)$

m2A1115 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{14} \\
[e_2, e_4] &= e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{1A}(12, 15)$

m1A1215 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_1, e_{13}] &= e_{14} \\
[e_1, e_{14}] &= e_{15} & [e_2, e_3] &= e_{15}
\end{aligned}
$$

No non-trivial Jacobi tests

$\mathfrak{m}_{2B}(2, 6)$

m2B26 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_2, e_3] &= e_5 \\
[e_2, e_5] &= e_6 & [e_3, e_4] &= -e_6
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_2, e_3] &= e_5 \\
[e_2, e_5] &= e_6 & [e_3, e_4] &= \alpha_{3,4}^{6} e_6
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_4): \quad -\alpha_{3,4}^{6} - 1 \qquad = 0$$

Solution 1:

$$\alpha_{3,4}^{6} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,4}^{6} \to x_1$$

Jacobi Tests

$$(e_1, e_2, e_4): \quad -x_1 - 1 \qquad = 0$$

Groebner basis (1 variables, 1 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

Solution 1:

$$x_1 = -1$$

$\mathfrak{m}_{2B}(2, 8)$

m2B28 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_5] &= e_7 \\
[e_2, e_7] &= e_8 & [e_3, e_4] &= -e_7 \\
[e_3, e_6] &= \alpha_{3,6}^{8} e_8 & [e_4, e_5] &= \alpha_{4,5}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6):& \quad -\alpha_{3,6}^8 - 1 &= 0\\
(e_1, e_3, e_5):& \quad -\alpha_{3,6}^8 - \alpha_{4,5}^8 &= 0\\
(e_2, e_3, e_4):& \quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(2, 8)$

m4B28 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_5\\
[e_2, e_4] &= e_6 & [e_2, e_5] &= 3e_7\\
[e_2, e_7] &= e_8 & [e_3, e_4] &= -2e_7\\
[e_3, e_6] &= -e_8 & [e_4, e_5] &= e_8
\end{aligned}$$

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_5\\
[e_2, e_4] &= e_6 & [e_2, e_5] &= \alpha_{2,5}^7 e_7\\
[e_2, e_7] &= e_8 & [e_3, e_4] &= \alpha_{3,4}^7 e_7\\
[e_3, e_6] &= \alpha_{3,6}^8 e_8 & [e_4, e_5] &= \alpha_{4,5}^8 e_8
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_4):& \quad -\alpha_{2,5}^7 - \alpha_{3,4}^7 + 1 &= 0\\
(e_1, e_2, e_6):& \quad -\alpha_{3,6}^8 - 1 &= 0\\
(e_1, e_3, e_5):& \quad -\alpha_{3,6}^8 - \alpha_{4,5}^8 &= 0\\
(e_2, e_3, e_4):& \quad \alpha_{3,4}^7 - \alpha_{3,6}^8 + \alpha_{4,5}^8 &= 0
\end{aligned}$$

Solution 1:

$$\alpha^8_{4,5} = 1$$
$$\alpha^7_{2,5} = 3$$
$$\alpha^7_{3,4} = -2$$
$$\alpha^8_{3,6} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^8_{4,5} \to x_1$$
$$\alpha^7_{2,5} \to x_2$$
$$\alpha^7_{3,4} \to x_3$$
$$\alpha^8_{3,6} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_3 + 1 &= 0\\
(e_1, e_2, e_6): &\quad -x_4 - 1 &= 0\\
(e_1, e_3, e_5): &\quad -x_1 - x_4 &= 0\\
(e_2, e_3, e_4): &\quad x_1 + x_3 - x_4 &= 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 - 3 = 0$$
$$x_3 + 2 = 0$$
$$x_4 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 3$$
$$x_3 = -2$$
$$x_4 = -1$$

$\mathfrak{m}_{3B}(3,8)$

m3B38 (this line included for string searching purposes)

Solution 1

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_2,e_3]=e_6$$
$$[e_2,e_4]=e_7 \qquad [e_2,e_7]=e_8$$
$$[e_3,e_6]=-e_8 \qquad [e_4,e_5]=e_8$$

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_2,e_3]=e_6$$
$$[e_2,e_4]=e_7 \qquad [e_2,e_7]=e_8$$
$$[e_3,e_6]=\alpha_{3,6}^8 e_8 \qquad [e_4,e_5]=\alpha_{4,5}^8 e_8$$

Non-trivial Jacobi Tests:

$$(e_1,e_2,e_6): \quad -\alpha_{3,6}^8-1 \qquad =0$$
$$(e_1,e_3,e_5): \quad -\alpha_{3,6}^8-\alpha_{4,5}^8 \qquad =0$$

Solution 1:

$$\alpha_{4,5}^8=1$$
$$\alpha_{3,6}^8=-1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^8 \to x_1$$
$$\alpha_{3,6}^8 \to x_2$$

Jacobi Tests

$$(e_1,e_2,e_6): \quad -x_2-1 \qquad =0$$
$$(e_1,e_3,e_5): \quad -x_1-x_2 \qquad =0$$

Groebner basis (2 variables, 2 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -1$$

$\mathfrak{m}_{2B}(4,8)$

m2B48 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_7 \\
[e_2, e_7] &= e_8 & [e_3, e_6] &= -e_8 \\
[e_4, e_5] &= e_8
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_2, e_3] &= e_7 \\
[e_2, e_7] &= e_8 & [e_3, e_6] &= \alpha_{3,6}^{8} e_8 \\
[e_4, e_5] &= \alpha_{4,5}^{8} e_8
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{3,6}^{8} - 1 &&= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{8} - \alpha_{4,5}^{8} &&= 0
\end{aligned}
$$

Solution 1:

$$\alpha_{4,5}^{8} = 1$$
$$\alpha_{3,6}^{8} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,5}^{8} \to x_1$$

$$\alpha_{3,6}^{8} \to x_2$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - 1 &= 0\\
(e_1, e_3, e_5): &\quad -x_1 - x_2 &= 0
\end{aligned}$$

Groebner basis (2 variables, 2 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 + 1 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = -1$$

$\mathfrak{m}_{2B}(2, 10)$

m2B210 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_7] &= e_9\\
[e_2, e_9] &= e_{10} & [e_3, e_6] &= -e_9\\
[e_3, e_8] &= \alpha_{3,8}^{10} e_{10} & [e_4, e_5] &= e_9\\
[e_4, e_7] &= \alpha_{4,7}^{10} e_{10} & [e_5, e_6] &= \alpha_{5,6}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &= 0\\
(e_2, e_3, e_6): &\quad \text{no solutions}\\
(e_2, e_4, e_5): &\quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(2, 10)$

m4B210 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_2, e_5] = e_7 \\
[e_2, e_6] = 2e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_9] = e_{10} & [e_3, e_4] = -e_7 \\
[e_3, e_5] = -e_8 & [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
[e_3, e_8] = \alpha_{3,8}^{10} e_{10} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_7] = \alpha_{4,7}^{10} e_{10} & [e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_6): & -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 & = 0 \\
(e_1, e_3, e_5): & -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 & = 0 \\
(e_2, e_3, e_4): & -\alpha_{2,7}^{9} & = 0 \\
(e_1, e_2, e_8): & -\alpha_{3,8}^{10} - 1 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} & = 0 \\
(e_2, e_3, e_6): & \alpha_{3,6}^{9} - 2\alpha_{3,8}^{10} & = 0 \\
(e_2, e_4, e_5): & \alpha_{4,5}^{9} - \alpha_{4,7}^{10} & = 0
\end{array}
$$

No solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{array}{c}
\alpha_{4,7}^{10} \to x_1 \\
\alpha_{5,6}^{10} \to x_2 \\
\alpha_{2,7}^{9} \to x_3 \\
\alpha_{3,6}^{9} \to x_4 \\
\alpha_{4,5}^{9} \to x_5 \\
\alpha_{3,8}^{10} \to x_6
\end{array}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_3 - x_4 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_4 - x_5 - 1 &&= 0\\
(e_2, e_3, e_4): &\quad -x_3 &&= 0\\
(e_1, e_2, e_8): &\quad -x_6 - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -x_1 - x_6 &&= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_2 &&= 0\\
(e_2, e_3, e_6): &\quad x_4 - 2x_6 &&= 0\\
(e_2, e_4, e_5): &\quad -x_1 + x_5 &&= 0
\end{aligned}
$$

Groebner basis (6 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6B}(2, 10)$

m6B210 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_5\\
[e_2, e_4] &= e_6 & [e_2, e_5] &= 0\\
[e_2, e_6] &= -e_8 & [e_2, e_7] &= -e_9\\
[e_2, e_9] &= e_{10} & [e_3, e_4] &= e_7\\
[e_3, e_5] &= e_8 & [e_3, e_6] &= 0\\
[e_3, e_8] &= -e_{10} & [e_4, e_5] &= e_9\\
[e_4, e_7] &= e_{10} & [e_5, e_6] &= -e_{10}
\end{aligned}
$$

Solution 2

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = 2e_7 \\
&[e_2, e_6] = 3e_8 && [e_2, e_7] = 7e_9 \\
&[e_2, e_9] = e_{10} && [e_3, e_4] = -e_7 \\
&[e_3, e_5] = -e_8 && [e_3, e_6] = -4e_9 \\
&[e_3, e_8] = -e_{10} && [e_4, e_5] = 3e_9 \\
&[e_4, e_7] = e_{10} && [e_5, e_6] = -e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_5 \\
&[e_2, e_4] = e_6 && [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
&[e_2, e_6] = \alpha_{2,6}^{8} e_8 && [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
&[e_2, e_9] = e_{10} && [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
&[e_3, e_5] = \alpha_{3,5}^{8} e_8 && [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
&[e_4, e_7] = \alpha_{4,7}^{10} e_{10} && [e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &&= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &&= 0\\
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{10} + \alpha_{3,6}^{9} - \alpha_{5,6}^{10} &&= 0\\
(e_2, e_4, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{10} + \alpha_{4,5}^{9} + \alpha_{5,6}^{10} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,7}^{10} &= 1\\
\alpha_{2,6}^{8} &= -1\\
\alpha_{5,6}^{10} &= -1\\
\alpha_{3,5}^{8} &= 1\\
\alpha_{2,5}^{7} &= 0\\
\alpha_{2,7}^{9} &= -1\\
\alpha_{3,6}^{9} &= 0\\
\alpha_{4,5}^{9} &= 1\\
\alpha_{3,4}^{7} &= 1\\
\alpha_{3,8}^{10} &= -1
\end{aligned}
$$

Solution 2:

$$\alpha^{10}_{4,7} = 1$$
$$\alpha^{8}_{2,6} = 3$$
$$\alpha^{10}_{5,6} = -1$$
$$\alpha^{8}_{3,5} = -1$$
$$\alpha^{7}_{2,5} = 2$$
$$\alpha^{9}_{2,7} = 7$$
$$\alpha^{9}_{3,6} = -4$$
$$\alpha^{9}_{4,5} = 3$$
$$\alpha^{7}_{3,4} = -1$$
$$\alpha^{10}_{3,8} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{10}_{4,7} \to x_1$$
$$\alpha^{8}_{2,6} \to x_2$$
$$\alpha^{10}_{5,6} \to x_3$$
$$\alpha^{8}_{3,5} \to x_4$$
$$\alpha^{7}_{2,5} \to x_5$$
$$\alpha^{9}_{2,7} \to x_6$$
$$\alpha^{9}_{3,6} \to x_7$$
$$\alpha^{9}_{4,5} \to x_8$$
$$\alpha^{7}_{3,4} \to x_9$$
$$\alpha^{10}_{3,8} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_5 - x_9 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_2 - x_4 + x_5 &&= 0\\
(e_1, e_3, e_4): &\quad -x_4 + x_9 &&= 0\\
(e_1, e_2, e_6): &\quad x_2 - x_6 - x_7 &&= 0\\
(e_1, e_3, e_5): &\quad x_4 - x_7 - x_8 &&= 0\\
(e_2, e_3, e_4): &\quad x_6 x_9 - x_7 + x_8 &&= 0\\
(e_1, e_2, e_8): &\quad -x_{10} - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -x_1 - x_{10} &&= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_3 &&= 0\\
(e_2, e_3, e_6): &\quad -x_{10} x_2 - x_3 + x_7 &&= 0\\
(e_2, e_4, e_5): &\quad -x_1 x_5 + x_3 + x_8 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 9 linear, 1 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 2x_9 - 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - x_9 = 0$$
$$x_5 + x_9 - 1 = 0$$
$$x_6 + 4x_9 - 3 = 0$$
$$x_7 - 2x_9 + 2 = 0$$
$$x_8 + x_9 - 2 = 0$$
$$x_9^2 - 1 = 0$$
$$x_{10} + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = -1$$
$$x_4 = 1$$
$$x_5 = 0$$
$$x_6 = -1$$
$$x_7 = 0$$

$$x_8 = 1$$
$$x_9 = 1$$
$$x_10 = -1$$

Solution 2:

$$x_1 = 1$$
$$x_2 = 3$$
$$x_3 = -1$$
$$x_4 = -1$$
$$x_5 = 2$$
$$x_6 = 7$$
$$x_7 = -4$$
$$x_8 = 3$$
$$x_9 = -1$$
$$x_10 = -1$$

$\mathfrak{m}_{3B}(3, 10)$

m3B310 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= 2e_9 & [e_2, e_9] &= e_{10} \\
[e_3, e_4] &= -e_8 & [e_3, e_5] &= -e_9 \\
[e_3, e_8] &= -e_{10} & [e_4, e_7] &= e_{10} \\
[e_5, e_6] &= -e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= 2e_9 & [e_2, e_9] &= e_{10} \\
[e_3, e_4] &= -e_8 & [e_3, e_5] &= -e_9 \\
[e_3, e_8] &= \alpha_{3,8}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{10} e_{10} \\
[e_5, e_6] &= \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{3,8}^{10} - 1 &&= 0
\end{aligned}$$

Solution 1:

$$\alpha_{5,6}^{10} = -1$$
$$\alpha_{4,7}^{10} = 1$$
$$\alpha_{3,8}^{10} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{5,6}^{10} \to x_1$$
$$\alpha_{4,7}^{10} \to x_2$$
$$\alpha_{3,8}^{10} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_3 &&= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_2 &&= 0\\
(e_2, e_3, e_5): &\quad -x_3 - 1 &&= 0
\end{aligned}$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = -1$$

$\mathfrak{m}_{5B}(3,10)$

m5B310 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1,e_2]=e_3 && [e_1,e_3]=e_4\\
&[e_1,e_4]=e_5 && [e_1,e_5]=e_6\\
&[e_1,e_6]=e_7 && [e_1,e_7]=e_8\\
&[e_1,e_8]=e_9 && [e_2,e_3]=e_6\\
&[e_2,e_4]=e_7 && [e_2,e_5]=\alpha_{2,5}^{8}e_8\\
&[e_2,e_6]=\alpha_{2,6}^{9}e_9 && [e_2,e_9]=e_{10}\\
&[e_3,e_4]=\alpha_{3,4}^{8}e_8 && [e_3,e_5]=\alpha_{3,5}^{9}e_9\\
&[e_3,e_8]=\alpha_{3,8}^{10}e_{10} && [e_4,e_7]=\alpha_{4,7}^{10}e_{10}\\
&[e_5,e_6]=\alpha_{5,6}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{8}-\alpha_{3,4}^{8}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{8}-\alpha_{2,6}^{9}-\alpha_{3,5}^{9} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{8}-\alpha_{3,5}^{9} &&=0\\
(e_1,e_2,e_8):&\quad -\alpha_{3,8}^{10}-1 &&=0\\
(e_1,e_3,e_7):&\quad -\alpha_{3,8}^{10}-\alpha_{4,7}^{10} &&=0\\
(e_1,e_4,e_6):&\quad -\alpha_{4,7}^{10}-\alpha_{5,6}^{10} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,5}^{8}\alpha_{3,8}^{10}+\alpha_{3,5}^{9}+\alpha_{5,6}^{10} &&=0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\begin{aligned}
\alpha_{4,7}^{10}&\to x_1\\
\alpha_{5,6}^{10}&\to x_2\\
\alpha_{3,4}^{8}&\to x_3\\
\alpha_{2,6}^{9}&\to x_4\\
\alpha_{2,5}^{8}&\to x_5\\
\alpha_{3,8}^{10}&\to x_6\\
\alpha_{3,5}^{9}&\to x_7
\end{aligned}
$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_3 - x_5 + 1 &= 0\\
(e_1, e_2, e_5): &\quad -x_4 + x_5 - x_7 &= 0\\
(e_1, e_3, e_4): &\quad x_3 - x_7 &= 0\\
(e_1, e_2, e_8): &\quad -x_6 - 1 &= 0\\
(e_1, e_3, e_7): &\quad -x_1 - x_6 &= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_2 &= 0\\
(e_2, e_3, e_5): &\quad x_2 - x_5x_6 + x_7 &= 0
\end{aligned}$$

Groebner basis (7 variables, 6 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 1 = 0$$
$$x_3 - x_7 = 0$$
$$x_4 + 2x_7 - 1 = 0$$
$$x_5 + x_7 - 1 = 0$$
$$x_6 + 1 = 0$$

$\mathfrak{m}_{2B}(4, 10)$

m2B410 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_5] &= e_9\\
[e_2, e_9] &= e_{10} & [e_3, e_4] &= -e_9\\
[e_3, e_8] &= \alpha_{3,8}^{10} e_{10} & [e_4, e_7] &= \alpha_{4,7}^{10} e_{10}\\
[e_5, e_6] &= \alpha_{5,6}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &= 0\\
(e_2, e_3, e_4): &\quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(4, 10)$

m4B410 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_7 \\
&[e_2, e_4] = e_8 && [e_2, e_5] = 3e_9 \\
&[e_2, e_9] = e_{10} && [e_3, e_4] = -2e_9 \\
&[e_3, e_8] = -e_{10} && [e_4, e_7] = e_{10} \\
&[e_5, e_6] = -e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_2, e_3] = e_7 \\
&[e_2, e_4] = e_8 && [e_2, e_5] = \alpha_{2,5}^{9} e_9 \\
&[e_2, e_9] = e_{10} && [e_3, e_4] = \alpha_{3,4}^{9} e_9 \\
&[e_3, e_8] = \alpha_{3,8}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{10} e_{10} \\
&[e_5, e_6] = \alpha_{5,6}^{10} e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{9} - \alpha_{3,4}^{9} + 1 && = 0 \\
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 && = 0 \\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} && = 0 \\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} && = 0 \\
(e_2, e_3, e_4): &\quad \alpha_{3,4}^{9} - \alpha_{3,8}^{10} + \alpha_{4,7}^{10} && = 0
\end{aligned}
$$

Solution 1:

$$\alpha^{10}_{4,7} = 1$$
$$\alpha^{10}_{5,6} = -1$$
$$\alpha^{9}_{2,5} = 3$$
$$\alpha^{9}_{3,4} = -2$$
$$\alpha^{10}_{3,8} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{10}_{4,7} \to x_1$$
$$\alpha^{10}_{5,6} \to x_2$$
$$\alpha^{9}_{2,5} \to x_3$$
$$\alpha^{9}_{3,4} \to x_4$$
$$\alpha^{10}_{3,8} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_3 - x_4 + 1 &&= 0\\
(e_1, e_2, e_8):&\quad -x_5 - 1 &&= 0\\
(e_1, e_3, e_7):&\quad -x_1 - x_5 &&= 0\\
(e_1, e_4, e_6):&\quad -x_1 - x_2 &&= 0\\
(e_2, e_3, e_4):&\quad x_1 + x_4 - x_5 &&= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 + 1 = 0$$
$$x_3 - 3 = 0$$
$$x_4 + 2 = 0$$
$$x_5 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -1$$
$$x_3 = 3$$
$$x_4 = -2$$
$$x_5 = -1$$

$\mathfrak{m}_{3B}(5,10)$

m3B510 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_8\\
[e_2,e_4] &= e_9 & [e_2,e_9] &= e_{10}\\
[e_3,e_8] &= -e_{10} & [e_4,e_7] &= e_{10}\\
[e_5,e_6] &= -e_{10}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_2,e_3] &= e_8\\
[e_2,e_4] &= e_9 & [e_2,e_9] &= e_{10}\\
[e_3,e_8] &= \alpha_{3,8}^{10}e_{10} & [e_4,e_7] &= \alpha_{4,7}^{10}e_{10}\\
[e_5,e_6] &= \alpha_{5,6}^{10}e_{10}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_8): &\quad -\alpha_{3,8}^{10}-1 &= 0\\
(e_1,e_3,e_7): &\quad -\alpha_{3,8}^{10}-\alpha_{4,7}^{10} &= 0\\
(e_1,e_4,e_6): &\quad -\alpha_{4,7}^{10}-\alpha_{5,6}^{10} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{5,6}^{10} &= -1\\
\alpha_{4,7}^{10} &= 1\\
\alpha_{3,8}^{10} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{5,6}^{10} \to x_1$$

$$\alpha_{4,7}^{10} \to x_2$$
$$\alpha_{3,8}^{10} \to x_3$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - 1 &= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_3 &= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_2 &= 0
\end{aligned}$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$x_1 + 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$

Solution 1:

$$x_1 = -1$$
$$x_2 = 1$$
$$x_3 = -1$$

$\mathfrak{m}_{2B}(6, 10)$

m2B610 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_9\\
[e_2, e_9] &= e_{10} & [e_3, e_8] &= -e_{10}\\
[e_4, e_7] &= e_{10} & [e_5, e_6] &= -e_{10}
\end{aligned}$$

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_2, e_3] &= e_9\\
[e_2, e_9] &= e_{10} & [e_3, e_8] &= \alpha_{3,8}^{10} e_{10}\\
[e_4, e_7] &= \alpha_{4,7}^{10} e_{10} & [e_5, e_6] &= \alpha_{5,6}^{10} e_{10}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{3,8}^{10} - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{10} - \alpha_{4,7}^{10} &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{10} - \alpha_{5,6}^{10} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{5,6}^{10} &= -1\\
\alpha_{4,7}^{10} &= 1\\
\alpha_{3,8}^{10} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{5,6}^{10} &\to x_1\\
\alpha_{4,7}^{10} &\to x_2\\
\alpha_{3,8}^{10} &\to x_3
\end{aligned}
$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -x_3 - 1 &&= 0\\
(e_1, e_3, e_7): &\quad -x_2 - x_3 &&= 0\\
(e_1, e_4, e_6): &\quad -x_1 - x_2 &&= 0
\end{aligned}
$$

Groebner basis (3 variables, 3 linear, 0 nonlinear)

$$
\begin{aligned}
x_1 + 1 &= 0\\
x_2 - 1 &= 0\\
x_3 + 1 &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
x_1 &= -1\\
x_2 &= 1\\
x_3 &= -1
\end{aligned}
$$

$\mathfrak{m}_{2B}(2,12)$

m2B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4 \\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6 \\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8 \\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10} \\
[e_1,e_{10}] &= e_{11} & [e_2,e_9] &= e_{11} \\
[e_2,e_{11}] &= e_{12} & [e_3,e_8] &= -e_{11} \\
[e_3,e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4,e_7] &= e_{11} \\
[e_4,e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5,e_6] &= -e_{11} \\
[e_5,e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1,e_3,e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1,e_4,e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1,e_5,e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2,e_3,e_8): &\quad \text{no solutions} \\
(e_2,e_4,e_7): &\quad \text{no solutions} \\
(e_2,e_5,e_6): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2,12)$

m4B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_7] = e_9 \\
[e_2, e_8] = 3e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{11}] = e_{12} & [e_3, e_6] = -e_9 \\
[e_3, e_7] = -2e_{10} & [e_3, e_8] = \alpha_{3,8}^{11} e_{11} \\
[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} & [e_4, e_5] = e_9 \\
[e_4, e_6] = e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_9] = \alpha_{4,9}^{12} e_{12} & [e_5, e_6] = \alpha_{5,6}^{11} e_{11} \\
[e_5, e_8] = \alpha_{5,8}^{12} e_{12} & [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1, e_2, e_8): & -\alpha_{2,9}^{11} - \alpha_{3,8}^{11} + 3 & = 0 \\
(e_1, e_3, e_7): & -\alpha_{3,8}^{11} - \alpha_{4,7}^{11} - 2 & = 0 \\
(e_1, e_4, e_6): & -\alpha_{4,7}^{11} - \alpha_{5,6}^{11} + 1 & = 0 \\
(e_2, e_3, e_6): & -\alpha_{2,9}^{11} & = 0 \\
(e_2, e_4, e_5): & \alpha_{2,9}^{11} & = 0 \\
(e_1, e_2, e_{10}): & -\alpha_{3,10}^{12} - 1 & = 0 \\
(e_1, e_3, e_9): & -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} & = 0 \\
(e_1, e_4, e_8): & -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} & = 0 \\
(e_1, e_5, e_7): & -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} & = 0 \\
(e_2, e_3, e_8): & -3\alpha_{3,10}^{12} + \alpha_{3,8}^{11} & = 0 \\
(e_2, e_4, e_7): & \alpha_{4,7}^{11} - \alpha_{4,9}^{12} & = 0 \\
(e_2, e_5, e_6): & \alpha_{5,6}^{11} & = 0 \\
(e_3, e_4, e_6): & \alpha_{3,10}^{12} + \alpha_{4,9}^{12} & = 0
\end{array}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{3,8}^{11} \to x_1$$

$$\alpha_{3,10}^{12} \to x_2$$

$$\alpha_{4,7}^{11} \to x_3$$

$$\alpha_{5,6}^{11} \to x_4$$

$$\alpha_{4,9}^{12} \to x_5$$

$$\alpha_{5,8}^{12} \to x_6$$

$$\alpha_{6,7}^{12} \to x_7$$

$$\alpha_{2,9}^{11} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_8 + 3 &= 0\\
(e_1, e_3, e_7): &\quad -x_1 - x_3 - 2 &= 0\\
(e_1, e_4, e_6): &\quad -x_3 - x_4 + 1 &= 0\\
(e_2, e_3, e_6): &\quad -x_8 &= 0\\
(e_2, e_4, e_5): &\quad x_8 &= 0\\
(e_1, e_2, e_{10}): &\quad -x_2 - 1 &= 0\\
(e_1, e_3, e_9): &\quad -x_2 - x_5 &= 0\\
(e_1, e_4, e_8): &\quad -x_5 - x_6 &= 0\\
(e_1, e_5, e_7): &\quad -x_6 - x_7 &= 0\\
(e_2, e_3, e_8): &\quad x_1 - 3x_2 &= 0\\
(e_2, e_4, e_7): &\quad x_3 - x_5 &= 0\\
(e_2, e_5, e_6): &\quad x_4 &= 0\\
(e_3, e_4, e_6): &\quad x_2 + x_5 &= 0
\end{aligned}$$

Groebner basis (8 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{6B}(2, 12)$

m6B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_5] = e_7 \\
&[e_2, e_6] = 2e_8 && [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
&[e_2, e_8] = \alpha_{2,8}^{10} e_{10} && [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
&[e_2, e_{11}] = e_{12} && [e_3, e_4] = -e_7 \\
&[e_3, e_5] = -e_8 && [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
&[e_3, e_7] = \alpha_{3,7}^{10} e_{10} && [e_3, e_8] = \alpha_{3,8}^{11} e_{11} \\
&[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} && [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
&[e_4, e_6] = \alpha_{4,6}^{10} e_{10} && [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
&[e_4, e_9] = \alpha_{4,9}^{12} e_{12} && [e_5, e_6] = \alpha_{5,6}^{11} e_{11} \\
&[e_5, e_8] = \alpha_{5,8}^{12} e_{12} && [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{9} - \alpha_{3,6}^{9} + 2 &= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{9} - \alpha_{4,5}^{9} - 1 &= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,7}^{9} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,8}^{10} - \alpha_{3,7}^{10} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,9}^{11}\alpha_{3,6}^{9} - 2\alpha_{3,8}^{11} &= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,9}^{11}\alpha_{4,5}^{9} - \alpha_{4,7}^{11} &= 0\\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0\\
(e_2, e_3, e_8): &\quad -\alpha_{2,8}^{10}\alpha_{3,10}^{12} + \alpha_{3,8}^{11} &= 0\\
(e_2, e_4, e_7): &\quad -\alpha_{2,7}^{9}\alpha_{4,9}^{12} + \alpha_{4,7}^{11} &= 0\\
(e_2, e_5, e_6): &\quad \alpha_{5,6}^{11} - 2\alpha_{5,8}^{12} + \alpha_{6,7}^{12} &= 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,10}^{12}\alpha_{4,6}^{10} - \alpha_{3,6}^{9}\alpha_{4,9}^{12} - \alpha_{6,7}^{12} &= 0
\end{aligned}$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{9} \to x_1$$
$$\alpha_{3,7}^{10} \to x_2$$
$$\alpha_{3,8}^{11} \to x_3$$
$$\alpha_{2,8}^{10} \to x_4$$
$$\alpha_{3,6}^{9} \to x_5$$
$$\alpha_{4,5}^{9} \to x_6$$
$$\alpha_{4,7}^{11} \to x_7$$

$$\alpha^{10}_{4,6} \to x_8$$

$$\alpha^{11}_{5,6} \to x_9$$

$$\alpha^{12}_{3,10} \to x_{10}$$

$$\alpha^{12}_{4,9} \to x_{11}$$

$$\alpha^{12}_{5,8} \to x_{12}$$

$$\alpha^{12}_{6,7} \to x_{13}$$

$$\alpha^{11}_{2,9} \to x_{14}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_5 + 2 &= 0\\
(e_1, e_3, e_5): &\quad -x_5 - x_6 - 1 &= 0\\
(e_2, e_3, e_4): &\quad -x_1 &= 0\\
(e_1, e_2, e_7): &\quad x_1 - x_2 - x_4 &= 0\\
(e_1, e_3, e_6): &\quad -x_2 + x_5 - x_8 &= 0\\
(e_1, e_4, e_5): &\quad x_6 - x_8 &= 0\\
(e_2, e_3, e_5): &\quad -x_2 - x_4 &= 0\\
(e_1, e_2, e_8): &\quad -x_{14} - x_3 + x_4 &= 0\\
(e_1, e_3, e_7): &\quad x_2 - x_3 - x_7 &= 0\\
(e_1, e_4, e_6): &\quad -x_7 + x_8 - x_9 &= 0\\
(e_2, e_3, e_6): &\quad x_{14}x_5 - 2x_3 &= 0\\
(e_2, e_4, e_5): &\quad x_{14}x_6 - x_7 &= 0\\
(e_1, e_2, e_{10}): &\quad -x_{10} - 1 &= 0\\
(e_1, e_3, e_9): &\quad -x_{10} - x_{11} &= 0\\
(e_1, e_4, e_8): &\quad -x_{11} - x_{12} &= 0\\
(e_1, e_5, e_7): &\quad -x_{12} - x_{13} &= 0\\
(e_2, e_3, e_8): &\quad -x_{10}x_4 + x_3 &= 0\\
(e_2, e_4, e_7): &\quad -x_1x_{11} + x_7 &= 0\\
(e_2, e_5, e_6): &\quad -2x_{12} + x_{13} + x_9 &= 0\\
(e_3, e_4, e_6): &\quad x_{10}x_8 - x_{11}x_5 - x_{13} &= 0
\end{aligned}$$

Groebner basis (14 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{8B}(2,12)$

m8B212 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_2,e_3]=e_5 \\
[e_2,e_4]=e_6 & [e_2,e_5]=\alpha_{2,5}^{7}e_7 \\
[e_2,e_6]=\alpha_{2,6}^{8}e_8 & [e_2,e_7]=\alpha_{2,7}^{9}e_9 \\
[e_2,e_8]=\alpha_{2,8}^{10}e_{10} & [e_2,e_9]=\alpha_{2,9}^{11}e_{11} \\
[e_2,e_{11}]=e_{12} & [e_3,e_4]=\alpha_{3,4}^{7}e_7 \\
[e_3,e_5]=\alpha_{3,5}^{8}e_8 & [e_3,e_6]=\alpha_{3,6}^{9}e_9 \\
[e_3,e_7]=\alpha_{3,7}^{10}e_{10} & [e_3,e_8]=\alpha_{3,8}^{11}e_{11} \\
[e_3,e_{10}]=\alpha_{3,10}^{12}e_{12} & [e_4,e_5]=\alpha_{4,5}^{9}e_9 \\
[e_4,e_6]=\alpha_{4,6}^{10}e_{10} & [e_4,e_7]=\alpha_{4,7}^{11}e_{11} \\
[e_4,e_9]=\alpha_{4,9}^{12}e_{12} & [e_5,e_6]=\alpha_{5,6}^{11}e_{11} \\
[e_5,e_8]=\alpha_{5,8}^{12}e_{12} & [e_6,e_7]=\alpha_{6,7}^{12}e_{12}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{7} - \alpha_{3,4}^{7} + 1 &= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{7} - \alpha_{2,6}^{8} - \alpha_{3,5}^{8} &= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{7} - \alpha_{3,5}^{8} &= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{8} - \alpha_{2,7}^{9} - \alpha_{3,6}^{9} &= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{8} - \alpha_{3,6}^{9} - \alpha_{4,5}^{9} &= 0\\
(e_2, e_3, e_4): &\quad \alpha_{2,7}^{9}\alpha_{3,4}^{7} - \alpha_{3,6}^{9} + \alpha_{4,5}^{9} &= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{9} - \alpha_{2,8}^{10} - \alpha_{3,7}^{10} &= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{9} - \alpha_{3,7}^{10} - \alpha_{4,6}^{10} &= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{9} - \alpha_{4,6}^{10} &= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{3,7}^{10} + \alpha_{2,8}^{10}\alpha_{3,5}^{8} &= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{10} - \alpha_{2,9}^{11} - \alpha_{3,8}^{11} &= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{10} - \alpha_{3,8}^{11} - \alpha_{4,7}^{11} &= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{10} - \alpha_{4,7}^{11} - \alpha_{5,6}^{11} &= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,6}^{8}\alpha_{3,8}^{11} + \alpha_{2,9}^{11}\alpha_{3,6}^{9} - \alpha_{5,6}^{11} &= 0\\
(e_2, e_4, e_5): &\quad -\alpha_{2,5}^{7}\alpha_{4,7}^{11} + \alpha_{2,9}^{11}\alpha_{4,5}^{9} + \alpha_{5,6}^{11} &= 0\\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0\\
(e_2, e_3, e_8): &\quad -\alpha_{2,8}^{10}\alpha_{3,10}^{12} + \alpha_{3,8}^{11} - \alpha_{5,8}^{12} &= 0\\
(e_2, e_4, e_7): &\quad -\alpha_{2,7}^{9}\alpha_{4,9}^{12} + \alpha_{4,7}^{11} - \alpha_{6,7}^{12} &= 0\\
(e_2, e_5, e_6): &\quad \alpha_{2,5}^{7}\alpha_{6,7}^{12} - \alpha_{2,6}^{8}\alpha_{5,8}^{12} + \alpha_{5,6}^{11} &= 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,10}^{12}\alpha_{4,6}^{10} + \alpha_{3,4}^{7}\alpha_{6,7}^{12} - \alpha_{3,6}^{9}\alpha_{4,9}^{12} &= 0
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{8} \to x_1$$

$$\alpha_{3,5}^{8} \to x_2$$

$$\alpha_{2,5}^{7} \to x_3$$

$$\alpha_{2,7}^{9} \to x_4$$

$$\alpha_{3,7}^{10} \to x_5$$

$$\alpha_{2,8}^{10} \to x_6$$

$$\alpha_{3,6}^{9} \to x_7$$

$$\alpha_{4,5}^{9} \to x_8$$

$$\alpha_{3,8}^{11} \to x_9$$

$$\alpha_{4,6}^{10} \to x_{10}$$

$$\alpha_{4,7}^{11} \to x_{11}$$

$$\alpha_{5,6}^{11} \to x_{12}$$

$$\alpha_{3,4}^{7} \to x_{13}$$

$$\alpha_{4,9}^{12} \to x_{14}$$

$$\alpha_{5,8}^{12} \to x_{15}$$

$$\alpha_{3,10}^{12} \to x_{16}$$

$$\alpha_{2,9}^{11} \to x_{17}$$

$$\alpha_{6,7}^{12} \to x_{18}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -x_{13}-x_3+1 &&=0\\
(e_1,e_2,e_5): &\quad -x_1-x_2+x_3 &&=0\\
(e_1,e_3,e_4): &\quad x_{13}-x_2 &&=0\\
(e_1,e_2,e_6): &\quad x_1-x_4-x_7 &&=0\\
(e_1,e_3,e_5): &\quad x_2-x_7-x_8 &&=0\\
(e_2,e_3,e_4): &\quad x_{13}x_4-x_7+x_8 &&=0\\
(e_1,e_2,e_7): &\quad x_4-x_5-x_6 &&=0\\
(e_1,e_3,e_6): &\quad -x_{10}-x_5+x_7 &&=0\\
(e_1,e_4,e_5): &\quad -x_{10}+x_8 &&=0\\
(e_2,e_3,e_5): &\quad x_2x_6-x_3x_5 &&=0\\
(e_1,e_2,e_8): &\quad -x_{17}+x_6-x_9 &&=0\\
(e_1,e_3,e_7): &\quad -x_{11}+x_5-x_9 &&=0\\
(e_1,e_4,e_6): &\quad x_{10}-x_{11}-x_{12} &&=0\\
(e_2,e_3,e_6): &\quad -x_1x_9-x_{12}+x_{17}x_7 &&=0\\
(e_2,e_4,e_5): &\quad -x_{11}x_3+x_{12}+x_{17}x_8 &&=0\\
(e_1,e_2,e_{10}): &\quad -x_{16}-1 &&=0\\
(e_1,e_3,e_9): &\quad -x_{14}-x_{16} &&=0\\
(e_1,e_4,e_8): &\quad -x_{14}-x_{15} &&=0\\
(e_1,e_5,e_7): &\quad -x_{15}-x_{18} &&=0\\
(e_2,e_3,e_8): &\quad -x_{15}-x_{16}x_6+x_9 &&=0\\
(e_2,e_4,e_7): &\quad x_{11}-x_{14}x_4-x_{18} &&=0\\
(e_2,e_5,e_6): &\quad -x_1x_{15}+x_{12}+x_{18}x_3 &&=0\\
(e_3,e_4,e_6): &\quad x_{10}x_{16}+x_{13}x_{18}-x_{14}x_7 &&=0
\end{aligned}
$$

Groebner basis (18 variables, 6 linear, 12 nonlinear)

$$x_1+\frac{3x_{17}^3}{80}-\frac{37x_{17}^2}{80}-\frac{59x_{17}}{80}+\frac{77}{80}=0$$

$$-\frac{3x_{17}^3}{160}+\frac{37x_{17}^2}{160}+\frac{59x_{17}}{160}+x_2-\frac{157}{160}=0$$

$$\frac{3x_{17}^3}{160}-\frac{37x_{17}^2}{160}-\frac{59x_{17}}{160}+x_3-\frac{3}{160}=0$$

$$\frac{x_{17}^3}{32}-\frac{37x_{17}^2}{96}-\frac{25x_{17}}{32}+x_4+\frac{109}{96}=0$$

$$\frac{x_{17}^3}{32}-\frac{37x_{17}^2}{96}-\frac{9x_{17}}{32}+x_5+\frac{61}{96}=0$$

$$-\frac{x_{17}}{2} + x_6 + \frac{1}{2} = 0$$
$$\frac{x_{17}^3}{160} - \frac{37x_{17}^2}{480} + \frac{7x_{17}}{160} + x_7 - \frac{83}{480} = 0$$
$$-\frac{x_{17}^3}{40} + \frac{37x_{17}^2}{120} + \frac{13x_{17}}{40} + x_8 - \frac{97}{120} = 0$$
$$\frac{x_{17}}{2} + x_9 + \frac{1}{2} = 0$$
$$x_{10} - \frac{x_{17}^3}{40} + \frac{37x_{17}^2}{120} + \frac{13x_{17}}{40} - \frac{97}{120} = 0$$
$$x_{11} + \frac{x_{17}^3}{32} - \frac{37x_{17}^2}{96} - \frac{25x_{17}}{32} + \frac{13}{96} = 0$$
$$x_{12} - \frac{9x_{17}^3}{160} + \frac{111x_{17}^2}{160} + \frac{177x_{17}}{160} - \frac{151}{160} = 0$$
$$x_{13} - \frac{3x_{17}^3}{160} + \frac{37x_{17}^2}{160} + \frac{59x_{17}}{160} - \frac{157}{160} = 0$$
$$x_{14} - 1 = 0$$
$$x_{15} + 1 = 0$$
$$x_{16} + 1 = 0$$
$$x_{17}^4 - \frac{28x_{17}^3}{3} - 50x_{17}^2 - \frac{140x_{17}}{3} + 57 = 0$$
$$x_{18} - 1 = 0$$

$\mathfrak{m}_{3B}(3, 12)$

m3B312 (this line included for string searching purposes)

Original brackets:

$$\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_2, e_7] = e_{10} \\
[e_2, e_8] = 3e_{11} & [e_2, e_{11}] = e_{12} \\
[e_3, e_6] = -e_{10} & [e_3, e_7] = -2e_{11} \\
[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} & [e_4, e_5] = e_{10} \\
[e_4, e_6] = e_{11} & [e_4, e_9] = \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] = \alpha_{5,8}^{12} e_{12} & [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{array}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha^{12}_{3,10} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha^{12}_{3,10} - \alpha^{12}_{4,9} &&= 0\\
(e_1, e_4, e_8): &\quad -\alpha^{12}_{4,9} - \alpha^{12}_{5,8} &&= 0\\
(e_1, e_5, e_7): &\quad -\alpha^{12}_{5,8} - \alpha^{12}_{6,7} &&= 0\\
(e_2, e_3, e_7): &\quad -\alpha^{12}_{3,10} - 2 &&= 0\\
(e_2, e_4, e_6): &\quad \text{no solutions}\\
(e_3, e_4, e_5): &\quad \alpha^{12}_{3,10} &&= 0
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{5B}(3, 12)$

m5B312 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_8\\
[e_2, e_6] &= 2e_9 & [e_2, e_7] &= \alpha^{10}_{2,7} e_{10}\\
[e_2, e_8] &= \alpha^{11}_{2,8} e_{11} & [e_2, e_{11}] &= e_{12}\\
[e_3, e_4] &= -e_8 & [e_3, e_5] &= -e_9\\
[e_3, e_6] &= \alpha^{10}_{3,6} e_{10} & [e_3, e_7] &= \alpha^{11}_{3,7} e_{11}\\
[e_3, e_{10}] &= \alpha^{12}_{3,10} e_{12} & [e_4, e_5] &= \alpha^{10}_{4,5} e_{10}\\
[e_4, e_6] &= \alpha^{11}_{4,6} e_{11} & [e_4, e_9] &= \alpha^{12}_{4,9} e_{12}\\
[e_5, e_8] &= \alpha^{12}_{5,8} e_{12} & [e_6, e_7] &= \alpha^{12}_{6,7} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &&= 0\\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0\\
(e_2, e_3, e_7): &\quad -\alpha_{2,7}^{10}\alpha_{3,10}^{12} + \alpha_{3,7}^{11} &&= 0\\
(e_2, e_4, e_6): &\quad \alpha_{4,6}^{11} - 2\alpha_{4,9}^{12} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{12}\alpha_{4,5}^{10} + \alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,7}^{10} \to x_1$$

$$\alpha_{3,10}^{12} \to x_2$$

$$\alpha_{3,7}^{11} \to x_3$$

$$\alpha_{4,9}^{12} \to x_4$$

$$\alpha_{3,6}^{10} \to x_5$$

$$\alpha_{5,8}^{12} \to x_6$$

$$\alpha_{2,8}^{11} \to x_7$$

$$\alpha_{4,5}^{10} \to x_8$$

$$\alpha_{6,7}^{12} \to x_9$$

$$\alpha_{4,6}^{11} \to x_{10}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_1 - x_5 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_5 - x_8 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad x_1 - x_3 - x_7 &&= 0\\
(e_1, e_3, e_6): &\quad -x_{10} - x_3 + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{10} + x_8 &&= 0\\
(e_2, e_3, e_4): &\quad -x_7 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_2 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_2 - x_4 &&= 0\\
(e_1, e_4, e_8): &\quad -x_4 - x_6 &&= 0\\
(e_1, e_5, e_7): &\quad -x_6 - x_9 &&= 0\\
(e_2, e_3, e_7): &\quad -x_1 x_2 + x_3 &&= 0\\
(e_2, e_4, e_6): &\quad x_{10} - 2x_4 &&= 0\\
(e_3, e_4, e_5): &\quad x_2 x_8 + x_4 - x_6 &&= 0
\end{aligned}
$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{7B}(3, 12)$

m7B312 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_3] = e_6 \\
&[e_2, e_4] = e_7 && [e_2, e_5] = \frac{8e_8}{5} \\
&[e_2, e_6] = \frac{11e_9}{5} && [e_2, e_7] = 4e_{10} \\
&[e_2, e_8] = 7e_{11} && [e_2, e_{11}] = e_{12} \\
&[e_3, e_4] = -\frac{3e_8}{5} && [e_3, e_5] = -\frac{3e_9}{5} \\
&[e_3, e_6] = -\frac{9e_{10}}{5} && [e_3, e_7] = -3e_{11} \\
&[e_3, e_{10}] = -e_{12} && [e_4, e_5] = \frac{6e_{10}}{5} \\
&[e_4, e_6] = \frac{6e_{11}}{5} && [e_4, e_9] = e_{12} \\
&[e_5, e_8] = -e_{12} && [e_6, e_7] = e_{12}
\end{aligned}
$$

Solution 2

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_3] = e_6 \\
&[e_2, e_4] = e_7 && [e_2, e_5] = e_8 \\
&[e_2, e_6] = e_9 && [e_2, e_7] = e_{10} \\
&[e_2, e_8] = e_{11} && [e_2, e_{11}] = e_{12} \\
&[e_3, e_4] = 0 && [e_3, e_5] = 0 \\
&[e_3, e_6] = 0 && [e_3, e_7] = 0 \\
&[e_3, e_{10}] = -e_{12} && [e_4, e_5] = 0 \\
&[e_4, e_6] = 0 && [e_4, e_9] = e_{12} \\
&[e_5, e_8] = -e_{12} && [e_6, e_7] = e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_2, e_3] = e_6 \\
&[e_2, e_4] = e_7 && [e_2, e_5] = \alpha_{2,5}^{8} e_8 \\
&[e_2, e_6] = \alpha_{2,6}^{9} e_9 && [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
&[e_2, e_8] = \alpha_{2,8}^{11} e_{11} && [e_2, e_{11}] = e_{12} \\
&[e_3, e_4] = \alpha_{3,4}^{8} e_8 && [e_3, e_5] = \alpha_{3,5}^{9} e_9 \\
&[e_3, e_6] = \alpha_{3,6}^{10} e_{10} && [e_3, e_7] = \alpha_{3,7}^{11} e_{11} \\
&[e_3, e_{10}] = \alpha_{3,10}^{12} e_{12} && [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
&[e_4, e_6] = \alpha_{4,6}^{11} e_{11} && [e_4, e_9] = \alpha_{4,9}^{12} e_{12} \\
&[e_5, e_8] = \alpha_{5,8}^{12} e_{12} && [e_6, e_7] = \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{8} - \alpha_{3,4}^{8} + 1 &= 0 \\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{8} - \alpha_{2,6}^{9} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{8} - \alpha_{3,5}^{9} &= 0 \\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{9} - \alpha_{2,7}^{10} - \alpha_{3,6}^{10} &= 0 \\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{9} - \alpha_{3,6}^{10} - \alpha_{4,5}^{10} &= 0 \\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &= 0 \\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &= 0 \\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &= 0 \\
(e_2, e_3, e_4): &\quad \alpha_{2,8}^{11}\alpha_{3,4}^{8} - \alpha_{3,7}^{11} + \alpha_{4,6}^{11} &= 0 \\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_7): &\quad -\alpha_{2,7}^{10}\alpha_{3,10}^{12} + \alpha_{3,7}^{11} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_4, e_6): &\quad -\alpha_{2,6}^{9}\alpha_{4,9}^{12} + \alpha_{4,6}^{11} + \alpha_{6,7}^{12} &= 0 \\
(e_3, e_4, e_5): &\quad \alpha_{3,10}^{12}\alpha_{4,5}^{10} + \alpha_{3,4}^{8}\alpha_{5,8}^{12} - \alpha_{3,5}^{9}\alpha_{4,9}^{12} &= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{8}_{3,4} = -3/5$$
$$\alpha^{10}_{2,7} = 4$$
$$\alpha^{9}_{2,6} = 11/5$$
$$\alpha^{8}_{2,5} = 8/5$$
$$\alpha^{10}_{4,5} = 6/5$$
$$\alpha^{11}_{3,7} = -3$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{10}_{3,6} = -9/5$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{11}_{2,8} = 7$$
$$\alpha^{9}_{3,5} = -3/5$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{11}_{4,6} = 6/5$$

Solution 2:

$$\alpha^{8}_{3,4} = 0$$
$$\alpha^{10}_{2,7} = 1$$
$$\alpha^{9}_{2,6} = 1$$
$$\alpha^{8}_{2,5} = 1$$
$$\alpha^{10}_{4,5} = 0$$
$$\alpha^{11}_{3,7} = 0$$
$$\alpha^{12}_{3,10} = -1$$
$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{10}_{3,6} = 0$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{11}_{2,8} = 1$$
$$\alpha^{9}_{3,5} = 0$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{11}_{4,6} = 0$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{8}_{3,4} \to x_1$$
$$\alpha^{10}_{2,7} \to x_2$$
$$\alpha^{9}_{2,6} \to x_3$$
$$\alpha^{8}_{2,5} \to x_4$$
$$\alpha^{10}_{4,5} \to x_5$$
$$\alpha^{11}_{3,7} \to x_6$$
$$\alpha^{12}_{3,10} \to x_7$$
$$\alpha^{12}_{4,9} \to x_8$$
$$\alpha^{10}_{3,6} \to x_9$$
$$\alpha^{12}_{5,8} \to x_{10}$$
$$\alpha^{11}_{2,8} \to x_{11}$$
$$\alpha^{9}_{3,5} \to x_{12}$$
$$\alpha^{12}_{6,7} \to x_{13}$$
$$\alpha^{11}_{4,6} \to x_{14}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -x_1 - x_4 + 1 &&= 0\\
(e_1, e_2, e_5):&\quad -x_{12} - x_3 + x_4 &&= 0\\
(e_1, e_3, e_4):&\quad x_1 - x_{12} &&= 0\\
(e_1, e_2, e_6):&\quad -x_2 + x_3 - x_9 &&= 0\\
(e_1, e_3, e_5):&\quad x_{12} - x_5 - x_9 &&= 0\\
(e_1, e_2, e_7):&\quad -x_{11} + x_2 - x_6 &&= 0\\
(e_1, e_3, e_6):&\quad -x_{14} - x_6 + x_9 &&= 0\\
(e_1, e_4, e_5):&\quad -x_{14} + x_5 &&= 0\\
(e_2, e_3, e_4):&\quad x_1 x_{11} + x_{14} - x_6 &&= 0\\
(e_1, e_2, e_{10}):&\quad -x_7 - 1 &&= 0\\
(e_1, e_3, e_9):&\quad -x_7 - x_8 &&= 0\\
(e_1, e_4, e_8):&\quad -x_{10} - x_8 &&= 0\\
(e_1, e_5, e_7):&\quad -x_{10} - x_{13} &&= 0\\
(e_2, e_3, e_7):&\quad -x_{13} - x_2 x_7 + x_6 &&= 0\\
(e_2, e_4, e_6):&\quad x_{13} + x_{14} - x_3 x_8 &&= 0\\
(e_3, e_4, e_5):&\quad x_1 x_{10} - x_{12} x_8 + x_5 x_7 &&= 0
\end{aligned}
$$

Groebner basis (14 variables, 13 linear, 1 nonlinear)

$$x_1 + \frac{x_{14}}{2} = 0$$

$$-\frac{5x_{14}}{2} + x_2 - 1 = 0$$

$$-x_{14} + x_3 - 1 = 0$$

$$-\frac{x_{14}}{2} + x_4 - 1 = 0$$

$$-x_{14} + x_5 = 0$$

$$\frac{5x_{14}}{2} + x_6 = 0$$

$$x_7 + 1 = 0$$

$$x_8 - 1 = 0$$

$$\frac{3x_{14}}{2} + x_9 = 0$$

$$x_{10} + 1 = 0$$

$$x_{11} - 5x_{14} - 1 = 0$$

$$x_{12} + \frac{x_{14}}{2} = 0$$

$$x_{13} - 1 = 0$$

$$x_{14}^2 - \frac{6x_{14}}{5} = 0$$

Solution 1:

$$x_1 = -3/5$$

$$x_2 = 4$$

$$x_3 = 11/5$$

$$x_4 = 8/5$$

$$x_5 = 6/5$$

$$x_6 = -3$$

$$x_7 = -1$$

$$x_8 = 1$$

$$x_9 = -9/5$$

$$x_10 = -1$$

$$x_11 = 7$$

$$x_12 = -3/5$$

$$x_13 = 1$$

$$x_14 = 6/5$$

Solution 2:

$$x_1 = 0$$
$$x_2 = 1$$
$$x_3 = 1$$
$$x_4 = 1$$
$$x_5 = 0$$
$$x_6 = 0$$
$$x_7 = -1$$
$$x_8 = 1$$
$$x_9 = 0$$
$$x_10 = -1$$
$$x_11 = 1$$
$$x_12 = 0$$
$$x_13 = 1$$
$$x_14 = 0$$

$\mathfrak{m}_{2B}(4, 12)$

m2B412 (this line included for string searching purposes)

Original brackets:

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_7] &= e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_6] &= -e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_8] &= \alpha_{5,8}^{12} e_{12} \\
[e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &&= 0\\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &&= 0\\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &&= 0\\
(e_2, e_3, e_6): &\quad \text{no solutions}\\
(e_2, e_4, e_5): &\quad \text{no solutions}
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{4B}(4, 12)$

m4B412 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_9\\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= 4e_{11}\\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= -e_9\\
[e_3, e_5] &= -e_{10} & [e_3, e_6] &= -2e_{11}\\
[e_3, e_{10}] &= -e_{12} & [e_4, e_5] &= e_{11}\\
[e_4, e_9] &= e_{12} & [e_5, e_8] &= -e_{12}\\
[e_6, e_7] &= e_{12}
\end{aligned}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_9 \\
[e_2, e_6] &= 2e_{10} & [e_2, e_7] &= \alpha_{2,7}^{11} e_{11} \\
[e_2, e_{11}] &= e_{12} & [e_3, e_4] &= -e_9 \\
[e_3, e_5] &= -e_{10} & [e_3, e_6] &= \alpha_{3,6}^{11} e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_5] &= \alpha_{4,5}^{11} e_{11} \\
[e_4, e_9] &= \alpha_{4,9}^{12} e_{12} & [e_5, e_8] &= \alpha_{5,8}^{12} e_{12} \\
[e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &= 0 \\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &= 0 \\
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_6): &\quad -2\alpha_{3,10}^{12} + \alpha_{3,6}^{11} &= 0 \\
(e_2, e_4, e_5): &\quad \alpha_{4,5}^{11} - \alpha_{4,9}^{12} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,5}^{11} &= 1 \\
\alpha_{2,7}^{11} &= 4 \\
\alpha_{3,10}^{12} &= -1 \\
\alpha_{6,7}^{12} &= 1 \\
\alpha_{4,9}^{12} &= 1 \\
\alpha_{5,8}^{12} &= -1 \\
\alpha_{3,6}^{11} &= -2
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{4,5} \to x_1$$
$$\alpha^{11}_{2,7} \to x_2$$
$$\alpha^{12}_{3,10} \to x_3$$
$$\alpha^{12}_{6,7} \to x_4$$
$$\alpha^{12}_{4,9} \to x_5$$
$$\alpha^{12}_{5,8} \to x_6$$
$$\alpha^{11}_{3,6} \to x_7$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_7 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_1 - x_7 - 1 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_3 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_3 - x_5 &&= 0\\
(e_1, e_4, e_8): &\quad -x_5 - x_6 &&= 0\\
(e_1, e_5, e_7): &\quad -x_4 - x_6 &&= 0\\
(e_2, e_3, e_6): &\quad -2x_3 + x_7 &&= 0\\
(e_2, e_4, e_5): &\quad x_1 - x_5 &&= 0
\end{aligned}$$

Groebner basis (7 variables, 7 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 - 4 = 0$$
$$x_3 + 1 = 0$$
$$x_4 - 1 = 0$$
$$x_5 - 1 = 0$$
$$x_6 + 1 = 0$$
$$x_7 + 2 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 4$$
$$x_3 = -1$$
$$x_4 = 1$$
$$x_5 = 1$$
$$x_6 = -1$$
$$x_7 = -2$$

$\mathfrak{m}_{6B}(4,12)$

m6B412 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_7\\
[e_2,e_4] &= e_8 & [e_2,e_5] &= \alpha_{2,5}^{9}e_9\\
[e_2,e_6] &= \alpha_{2,6}^{10}e_{10} & [e_2,e_7] &= \alpha_{2,7}^{11}e_{11}\\
[e_2,e_{11}] &= e_{12} & [e_3,e_4] &= \alpha_{3,4}^{9}e_9\\
[e_3,e_5] &= \alpha_{3,5}^{10}e_{10} & [e_3,e_6] &= \alpha_{3,6}^{11}e_{11}\\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_5] &= \alpha_{4,5}^{11}e_{11}\\
[e_4,e_9] &= \alpha_{4,9}^{12}e_{12} & [e_5,e_8] &= \alpha_{5,8}^{12}e_{12}\\
[e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{9}-\alpha_{3,4}^{9}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{9}-\alpha_{2,6}^{10}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{9}-\alpha_{3,5}^{10} &&=0\\
(e_1,e_2,e_6):&\quad \alpha_{2,6}^{10}-\alpha_{2,7}^{11}-\alpha_{3,6}^{11} &&=0\\
(e_1,e_3,e_5):&\quad \alpha_{3,5}^{10}-\alpha_{3,6}^{11}-\alpha_{4,5}^{11} &&=0\\
(e_1,e_2,e_{10}):&\quad -\alpha_{3,10}^{12}-1 &&=0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &&=0\\
(e_1,e_4,e_8):&\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &&=0\\
(e_1,e_5,e_7):&\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha_{2,6}^{10}\alpha_{3,10}^{12}+\alpha_{3,6}^{11}+\alpha_{6,7}^{12} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha_{2,5}^{9}\alpha_{4,9}^{12}+\alpha_{4,5}^{11}+\alpha_{5,8}^{12} &&=0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$
\alpha_{2,6}^{10}\to x_1
$$
$$
\alpha_{2,5}^{9}\to x_2
$$

$$\alpha^{9}_{3,4} \to x_3$$
$$\alpha^{11}_{4,5} \to x_4$$
$$\alpha^{11}_{2,7} \to x_5$$
$$\alpha^{12}_{3,10} \to x_6$$
$$\alpha^{12}_{6,7} \to x_7$$
$$\alpha^{12}_{4,9} \to x_8$$
$$\alpha^{10}_{3,5} \to x_9$$
$$\alpha^{12}_{5,8} \to x_{10}$$
$$\alpha^{11}_{3,6} \to x_{11}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4):&\quad -x_2 - x_3 + 1 &= 0\\
(e_1, e_2, e_5):&\quad -x_1 + x_2 - x_9 &= 0\\
(e_1, e_3, e_4):&\quad x_3 - x_9 &= 0\\
(e_1, e_2, e_6):&\quad x_1 - x_{11} - x_5 &= 0\\
(e_1, e_3, e_5):&\quad -x_{11} - x_4 + x_9 &= 0\\
(e_1, e_2, e_{10}):&\quad -x_6 - 1 &= 0\\
(e_1, e_3, e_9):&\quad -x_6 - x_8 &= 0\\
(e_1, e_4, e_8):&\quad -x_{10} - x_8 &= 0\\
(e_1, e_5, e_7):&\quad -x_{10} - x_7 &= 0\\
(e_2, e_3, e_6):&\quad -x_1x_6 + x_{11} + x_7 &= 0\\
(e_2, e_4, e_5):&\quad x_{10} - x_2x_8 + x_4 &= 0
\end{aligned}$$

Groebner basis (11 variables, 10 linear, 0 nonlinear)

$$x_1 + x_{11} + 1 = 0$$
$$\frac{x_{11}}{2} + x_2 = 0$$
$$-\frac{x_{11}}{2} + x_3 - 1 = 0$$
$$\frac{x_{11}}{2} + x_4 - 1 = 0$$
$$2x_{11} + x_5 + 1 = 0$$
$$x_6 + 1 = 0$$
$$x_7 - 1 = 0$$
$$x_8 - 1 = 0$$
$$-\frac{x_{11}}{2} + x_9 - 1 = 0$$
$$x_{10} + 1 = 0$$

$\mathfrak{m}_{3B}(5, 12)$

m3B512 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{10} \\
[e_2, e_6] &= 2e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= -e_{10} & [e_3, e_5] &= -e_{11} \\
[e_3, e_{10}] &= -e_{12} & [e_4, e_9] &= e_{12} \\
[e_5, e_8] &= -e_{12} & [e_6, e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_5] &= e_{10} \\
[e_2, e_6] &= 2e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_4] &= -e_{10} & [e_3, e_5] &= -e_{11} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}): &\quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9): &\quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8): &\quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7): &\quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0 \\
(e_2, e_3, e_5): &\quad -\alpha_{3,10}^{12} - 1 &= 0
\end{aligned}
$$

Solution 1:

$$\alpha_{4,9}^{12} = 1$$
$$\alpha_{6,7}^{12} = 1$$
$$\alpha_{5,8}^{12} = -1$$
$$\alpha_{3,10}^{12} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,9}^{12} \to x_1$$
$$\alpha_{6,7}^{12} \to x_2$$
$$\alpha_{5,8}^{12} \to x_3$$
$$\alpha_{3,10}^{12} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_4 - 1 &&= 0 \\
(e_1, e_3, e_9): &\quad -x_1 - x_4 &&= 0 \\
(e_1, e_4, e_8): &\quad -x_1 - x_3 &&= 0 \\
(e_1, e_5, e_7): &\quad -x_2 - x_3 &&= 0 \\
(e_2, e_3, e_5): &\quad -x_4 - 1 &&= 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = -1$$

$\mathfrak{m}_{5B}(5,12)$

m5B512 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_8\\
[e_2,e_4] &= e_9 & [e_2,e_5] &= \alpha_{2,5}^{10}e_{10}\\
[e_2,e_6] &= \alpha_{2,6}^{11}e_{11} & [e_2,e_{11}] &= e_{12}\\
[e_3,e_4] &= \alpha_{3,4}^{10}e_{10} & [e_3,e_5] &= \alpha_{3,5}^{11}e_{11}\\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_9] &= \alpha_{4,9}^{12}e_{12}\\
[e_5,e_8] &= \alpha_{5,8}^{12}e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha_{2,5}^{10}-\alpha_{3,4}^{10}+1 &= 0\\
(e_1,e_2,e_5):&\quad \alpha_{2,5}^{10}-\alpha_{2,6}^{11}-\alpha_{3,5}^{11} &= 0\\
(e_1,e_3,e_4):&\quad \alpha_{3,4}^{10}-\alpha_{3,5}^{11} &= 0\\
(e_1,e_2,e_{10}):&\quad -\alpha_{3,10}^{12}-1 &= 0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &= 0\\
(e_1,e_4,e_8):&\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &= 0\\
(e_1,e_5,e_7):&\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &= 0\\
(e_2,e_3,e_5):&\quad -\alpha_{2,5}^{10}\alpha_{3,10}^{12}+\alpha_{3,5}^{11}+\alpha_{5,8}^{12} &= 0
\end{aligned}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{3,5}^{11} \to x_1$$

$$\alpha_{2,5}^{10} \to x_2$$

$$\alpha_{3,10}^{12} \to x_3$$

$$\alpha_{4,9}^{12} \to x_4$$

$$\alpha_{3,4}^{10} \to x_5$$

$$\alpha_{5,8}^{12} \to x_6$$

$$\alpha_{6,7}^{12} \to x_7$$

$$\alpha_{2,6}^{11} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_2 - x_5 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 + x_2 - x_8 &&= 0\\
(e_1, e_3, e_4): &\quad -x_1 + x_5 &&= 0\\
(e_1, e_2, e_{10}): &\quad -x_3 - 1 &&= 0\\
(e_1, e_3, e_9): &\quad -x_3 - x_4 &&= 0\\
(e_1, e_4, e_8): &\quad -x_4 - x_6 &&= 0\\
(e_1, e_5, e_7): &\quad -x_6 - x_7 &&= 0\\
(e_2, e_3, e_5): &\quad x_1 - x_2 x_3 + x_6 &&= 0
\end{aligned}$$

Groebner basis (8 variables, 7 linear, 0 nonlinear)

$$x_1 + \frac{x_8}{2} - \frac{1}{2} = 0$$

$$x_2 - \frac{x_8}{2} - \frac{1}{2} = 0$$

$$x_3 + 1 = 0$$

$$x_4 - 1 = 0$$

$$x_5 + \frac{x_8}{2} - \frac{1}{2} = 0$$

$$x_6 + 1 = 0$$

$$x_7 - 1 = 0$$

$\mathfrak{m}_{2B}(6, 12)$

m2B612 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_5] &= e_{11}\\
[e_2,e_{11}] &= e_{12} & [e_3,e_4] &= -e_{11}\\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_9] &= \alpha_{4,9}^{12}e_{12}\\
[e_5,e_8] &= \alpha_{5,8}^{12}e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_{10}):&\quad -\alpha_{3,10}^{12}-1 &&= 0\\
(e_1,e_3,e_9):&\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &&= 0\\
(e_1,e_4,e_8):&\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &&= 0\\
(e_1,e_5,e_7):&\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &&= 0\\
(e_2,e_3,e_4):&\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(6,12)$

m4B612 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_9\\
[e_2,e_4] &= e_{10} & [e_2,e_5] &= 3e_{11}\\
[e_2,e_{11}] &= e_{12} & [e_3,e_4] &= -2e_{11}\\
[e_3,e_{10}] &= -e_{12} & [e_4,e_9] &= e_{12}\\
[e_5,e_8] &= -e_{12} & [e_6,e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_2,e_3] &= e_9\\
[e_2,e_4] &= e_{10} & [e_2,e_5] &= \alpha_{2,5}^{11}e_{11}\\
[e_2,e_{11}] &= e_{12} & [e_3,e_4] &= \alpha_{3,4}^{11}e_{11}\\
[e_3,e_{10}] &= \alpha_{3,10}^{12}e_{12} & [e_4,e_9] &= \alpha_{4,9}^{12}e_{12}\\
[e_5,e_8] &= \alpha_{5,8}^{12}e_{12} & [e_6,e_7] &= \alpha_{6,7}^{12}e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha_{2,5}^{11}-\alpha_{3,4}^{11}+1 &&= 0\\
(e_1,e_2,e_{10}): &\quad -\alpha_{3,10}^{12}-1 &&= 0\\
(e_1,e_3,e_9): &\quad -\alpha_{3,10}^{12}-\alpha_{4,9}^{12} &&= 0\\
(e_1,e_4,e_8): &\quad -\alpha_{4,9}^{12}-\alpha_{5,8}^{12} &&= 0\\
(e_1,e_5,e_7): &\quad -\alpha_{5,8}^{12}-\alpha_{6,7}^{12} &&= 0\\
(e_2,e_3,e_4): &\quad -\alpha_{3,10}^{12}+\alpha_{3,4}^{11}+\alpha_{4,9}^{12} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{3,10}^{12} &= -1\\
\alpha_{3,4}^{11} &= -2\\
\alpha_{4,9}^{12} &= 1\\
\alpha_{2,5}^{11} &= 3\\
\alpha_{5,8}^{12} &= -1\\
\alpha_{6,7}^{12} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{3,10}^{12} \to x_1$$

$$\alpha_{3,4}^{11} \to x_2$$

$$\alpha^{12}_{4,9} \to x_3$$

$$\alpha^{11}_{2,5} \to x_4$$

$$\alpha^{12}_{5,8} \to x_5$$

$$\alpha^{12}_{6,7} \to x_6$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): \quad & -x_2 - x_4 + 1 && = 0 \\
(e_1, e_2, e_{10}): \quad & -x_1 - 1 && = 0 \\
(e_1, e_3, e_9): \quad & -x_1 - x_3 && = 0 \\
(e_1, e_4, e_8): \quad & -x_3 - x_5 && = 0 \\
(e_1, e_5, e_7): \quad & -x_5 - x_6 && = 0 \\
(e_2, e_3, e_4): \quad & -x_1 + x_2 + x_3 && = 0
\end{aligned}$$

Groebner basis (6 variables, 6 linear, 0 nonlinear)

$$x_1 + 1 = 0$$

$$x_2 + 2 = 0$$

$$x_3 - 1 = 0$$

$$x_4 - 3 = 0$$

$$x_5 + 1 = 0$$

$$x_6 - 1 = 0$$

Solution 1:

$$x_1 = -1$$

$$x_2 = -2$$

$$x_3 = 1$$

$$x_4 = 3$$

$$x_5 = -1$$

$$x_6 = 1$$

$\mathfrak{m}_{3B}(7, 12)$

m3B712 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_{10}] &= -e_{12} & [e_4, e_9] &= e_{12} \\
[e_5, e_8] &= -e_{12} & [e_6, e_7] &= e_{12}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_{11}] &= e_{12} \\
[e_3, e_{10}] &= \alpha_{3,10}^{12} e_{12} & [e_4, e_9] &= \alpha_{4,9}^{12} e_{12} \\
[e_5, e_8] &= \alpha_{5,8}^{12} e_{12} & [e_6, e_7] &= \alpha_{6,7}^{12} e_{12}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}):& \quad -\alpha_{3,10}^{12} - 1 &= 0 \\
(e_1, e_3, e_9):& \quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} &= 0 \\
(e_1, e_4, e_8):& \quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} &= 0 \\
(e_1, e_5, e_7):& \quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} &= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{12}_{4,9} = 1$$
$$\alpha^{12}_{6,7} = 1$$
$$\alpha^{12}_{5,8} = -1$$
$$\alpha^{12}_{3,10} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{4,9} \to x_1$$
$$\alpha^{12}_{6,7} \to x_2$$
$$\alpha^{12}_{5,8} \to x_3$$
$$\alpha^{12}_{3,10} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): \quad & -x_4 - 1 && = 0 \\
(e_1, e_3, e_9): \quad & -x_1 - x_4 && = 0 \\
(e_1, e_4, e_8): \quad & -x_1 - x_3 && = 0 \\
(e_1, e_5, e_7): \quad & -x_2 - x_3 && = 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = -1$$

$\mathfrak{m}_{2B}(8, 12)$

m2B812 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_{11}$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_{10}] = -e_{12}$$
$$[e_4, e_9] = e_{12} \qquad [e_5, e_8] = -e_{12}$$
$$[e_6, e_7] = e_{12}$$

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_2, e_3] = e_{11}$$
$$[e_2, e_{11}] = e_{12} \qquad [e_3, e_{10}] = \alpha_{3,10}^{12} e_{12}$$
$$[e_4, e_9] = \alpha_{4,9}^{12} e_{12} \qquad [e_5, e_8] = \alpha_{5,8}^{12} e_{12}$$
$$[e_6, e_7] = \alpha_{6,7}^{12} e_{12}$$

Non-trivial Jacobi Tests:

$$(e_1, e_2, e_{10}): \quad -\alpha_{3,10}^{12} - 1 = 0$$
$$(e_1, e_3, e_9): \quad -\alpha_{3,10}^{12} - \alpha_{4,9}^{12} = 0$$
$$(e_1, e_4, e_8): \quad -\alpha_{4,9}^{12} - \alpha_{5,8}^{12} = 0$$
$$(e_1, e_5, e_7): \quad -\alpha_{5,8}^{12} - \alpha_{6,7}^{12} = 0$$

Solution 1:

$$\alpha_{4,9}^{12} = 1$$
$$\alpha_{6,7}^{12} = 1$$
$$\alpha_{5,8}^{12} = -1$$
$$\alpha_{3,10}^{12} = -1$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,9}^{12} \to x_1$$
$$\alpha_{6,7}^{12} \to x_2$$
$$\alpha_{5,8}^{12} \to x_3$$
$$\alpha_{3,10}^{12} \to x_4$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_4 - 1 &= 0\\
(e_1, e_3, e_9): &\quad -x_1 - x_4 &= 0\\
(e_1, e_4, e_8): &\quad -x_1 - x_3 &= 0\\
(e_1, e_5, e_7): &\quad -x_2 - x_3 &= 0
\end{aligned}$$

Groebner basis (4 variables, 4 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = -1$$

$\mathfrak{m}_{2B}(2, 14)$

m2B214 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_{11}] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_{10}] &= -e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_9] &= e_{13} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_8] &= -e_{13} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_7] &= e_{13} \\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_{10}): &\quad \text{no solutions} \\
(e_2, e_4, e_9): &\quad \text{no solutions} \\
(e_2, e_5, e_8): &\quad \text{no solutions} \\
(e_2, e_6, e_7): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(2, 14)$

m4B214 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_9] &= e_{11} \\
[e_2, e_{10}] &= 4e_{12} & [e_2, e_{11}] &= \alpha_{2,11}^{13} e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_8] &= -e_{11} \\
[e_3, e_9] &= -3e_{12} & [e_3, e_{10}] &= \alpha_{3,10}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_7] &= e_{11} \\
[e_4, e_8] &= 2e_{12} & [e_4, e_9] &= \alpha_{4,9}^{13} e_{13} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_6] &= -e_{11} \\
[e_5, e_7] &= -e_{12} & [e_5, e_8] &= \alpha_{5,8}^{13} e_{13} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_7] &= \alpha_{6,7}^{13} e_{13} \\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{10}):& \quad -\alpha_{2,11}^{13} - \alpha_{3,10}^{13} + 4 &= 0\\
(e_1, e_3, e_9):& \quad -\alpha_{3,10}^{13} - \alpha_{4,9}^{13} - 3 &= 0\\
(e_1, e_4, e_8):& \quad -\alpha_{4,9}^{13} - \alpha_{5,8}^{13} + 2 &= 0\\
(e_1, e_5, e_7):& \quad -\alpha_{5,8}^{13} - \alpha_{6,7}^{13} - 1 &= 0\\
(e_2, e_3, e_8):& \quad -\alpha_{2,11}^{13} &= 0\\
(e_2, e_4, e_7):& \quad \alpha_{2,11}^{13} &= 0\\
(e_2, e_5, e_6):& \quad -\alpha_{2,11}^{13} &= 0\\
(e_1, e_2, e_{12}):& \quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}):& \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}):& \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9):& \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8):& \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_{10}):& \quad \alpha_{3,10}^{13} - 4\alpha_{3,12}^{14} &= 0\\
(e_2, e_4, e_9):& \quad -\alpha_{4,11}^{14} + \alpha_{4,9}^{13} &= 0\\
(e_2, e_5, e_8):& \quad \alpha_{5,8}^{13} &= 0\\
(e_2, e_6, e_7):& \quad \alpha_{6,7}^{13} &= 0\\
(e_3, e_4, e_8):& \quad 2\alpha_{3,12}^{14} + \alpha_{4,11}^{14} &= 0\\
(e_3, e_5, e_7):& \quad -\alpha_{3,12}^{14} &= 0\\
(e_4, e_5, e_6):& \quad -\alpha_{4,11}^{14} &= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,11}^{13} \to x_1$$

$$\alpha_{5,8}^{13} \to x_2$$

$$\alpha_{4,11}^{14} \to x_3$$

$$\alpha_{6,9}^{14} \to x_4$$

$$\alpha_{3,12}^{14} \to x_5$$

$$\alpha_{3,10}^{13} \to x_6$$

$$\alpha_{7,8}^{14} \to x_7$$

$$\alpha_{6,7}^{13} \to x_8$$

$$\alpha_{5,10}^{14} \to x_9$$

$$\alpha_{4,9}^{13} \to x_{10}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{10}): &\quad -x_1 - x_6 + 4 &&= 0\\
(e_1, e_3, e_9): &\quad -x_{10} - x_6 - 3 &&= 0\\
(e_1, e_4, e_8): &\quad -x_{10} - x_2 + 2 &&= 0\\
(e_1, e_5, e_7): &\quad -x_2 - x_8 - 1 &&= 0\\
(e_2, e_3, e_8): &\quad -x_1 &&= 0\\
(e_2, e_4, e_7): &\quad x_1 &&= 0\\
(e_2, e_5, e_6): &\quad -x_1 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_5 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_3 - x_5 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_3 - x_9 &&= 0\\
(e_1, e_5, e_9): &\quad -x_4 - x_9 &&= 0\\
(e_1, e_6, e_8): &\quad -x_4 - x_7 &&= 0\\
(e_2, e_3, e_{10}): &\quad -4x_5 + x_6 &&= 0\\
(e_2, e_4, e_9): &\quad x_{10} - x_3 &&= 0\\
(e_2, e_5, e_8): &\quad x_2 &&= 0\\
(e_2, e_6, e_7): &\quad x_8 &&= 0\\
(e_3, e_4, e_8): &\quad x_3 + 2x_5 &&= 0\\
(e_3, e_5, e_7): &\quad -x_5 &&= 0\\
(e_4, e_5, e_6): &\quad -x_3 &&= 0
\end{aligned}$$

Groebner basis (10 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{10B}(2, 14)$

m10B214 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_3] = e_5 \\
[e_2, e_4] = e_6 & [e_2, e_5] = \alpha_{2,5}^{7} e_7 \\
[e_2, e_6] = \alpha_{2,6}^{8} e_8 & [e_2, e_7] = \alpha_{2,7}^{9} e_9 \\
[e_2, e_8] = \alpha_{2,8}^{10} e_{10} & [e_2, e_9] = \alpha_{2,9}^{11} e_{11} \\
[e_2, e_{10}] = \alpha_{2,10}^{12} e_{12} & [e_2, e_{11}] = \alpha_{2,11}^{13} e_{13} \\
[e_2, e_{13}] = e_{14} & [e_3, e_4] = \alpha_{3,4}^{7} e_7 \\
[e_3, e_5] = \alpha_{3,5}^{8} e_8 & [e_3, e_6] = \alpha_{3,6}^{9} e_9 \\
[e_3, e_7] = \alpha_{3,7}^{10} e_{10} & [e_3, e_8] = \alpha_{3,8}^{11} e_{11} \\
[e_3, e_9] = \alpha_{3,9}^{12} e_{12} & [e_3, e_{10}] = \alpha_{3,10}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_5] = \alpha_{4,5}^{9} e_9 \\
[e_4, e_6] = \alpha_{4,6}^{10} e_{10} & [e_4, e_7] = \alpha_{4,7}^{11} e_{11} \\
[e_4, e_8] = \alpha_{4,8}^{12} e_{12} & [e_4, e_9] = \alpha_{4,9}^{13} e_{13} \\
[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} & [e_5, e_6] = \alpha_{5,6}^{11} e_{11} \\
[e_5, e_7] = \alpha_{5,7}^{12} e_{12} & [e_5, e_8] = \alpha_{5,8}^{13} e_{13} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_7] = \alpha_{6,7}^{13} e_{13} \\
[e_6, e_9] = \alpha_{6,9}^{14} e_{14} & [e_7, e_8] = \alpha_{7,8}^{14} e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -\alpha^{7}_{2,5}-\alpha^{7}_{3,4}+1 &&=0\\
(e_1,e_2,e_5):&\quad \alpha^{7}_{2,5}-\alpha^{8}_{2,6}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_3,e_4):&\quad \alpha^{7}_{3,4}-\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_6):&\quad \alpha^{8}_{2,6}-\alpha^{9}_{2,7}-\alpha^{9}_{3,6} &&=0\\
(e_1,e_3,e_5):&\quad \alpha^{8}_{3,5}-\alpha^{9}_{3,6}-\alpha^{9}_{4,5} &&=0\\
(e_2,e_3,e_4):&\quad \alpha^{9}_{2,7}\alpha^{7}_{3,4}-\alpha^{9}_{3,6}+\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_7):&\quad \alpha^{9}_{2,7}-\alpha^{10}_{2,8}-\alpha^{10}_{3,7} &&=0\\
(e_1,e_3,e_6):&\quad \alpha^{9}_{3,6}-\alpha^{10}_{3,7}-\alpha^{10}_{4,6} &&=0\\
(e_1,e_4,e_5):&\quad \alpha^{9}_{4,5}-\alpha^{10}_{4,6} &&=0\\
(e_2,e_3,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{10}_{3,7}+\alpha^{10}_{2,8}\alpha^{8}_{3,5} &&=0\\
(e_1,e_2,e_8):&\quad \alpha^{10}_{2,8}-\alpha^{11}_{2,9}-\alpha^{11}_{3,8} &&=0\\
(e_1,e_3,e_7):&\quad \alpha^{10}_{3,7}-\alpha^{11}_{3,8}-\alpha^{11}_{4,7} &&=0\\
(e_1,e_4,e_6):&\quad \alpha^{10}_{4,6}-\alpha^{11}_{4,7}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_3,e_6):&\quad -\alpha^{8}_{2,6}\alpha^{11}_{3,8}+\alpha^{11}_{2,9}\alpha^{9}_{3,6}-\alpha^{11}_{5,6} &&=0\\
(e_2,e_4,e_5):&\quad -\alpha^{7}_{2,5}\alpha^{11}_{4,7}+\alpha^{11}_{2,9}\alpha^{9}_{4,5}+\alpha^{11}_{5,6} &&=0\\
(e_1,e_2,e_9):&\quad -\alpha^{12}_{2,10}+\alpha^{11}_{2,9}-\alpha^{12}_{3,9} &&=0\\
(e_1,e_3,e_8):&\quad \alpha^{11}_{3,8}-\alpha^{12}_{3,9}-\alpha^{12}_{4,8} &&=0\\
(e_1,e_4,e_7):&\quad \alpha^{11}_{4,7}-\alpha^{12}_{4,8}-\alpha^{12}_{5,7} &&=0\\
(e_1,e_5,e_6):&\quad \alpha^{11}_{5,6}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_3,e_7):&\quad \alpha^{12}_{2,10}\alpha^{10}_{3,7}-\alpha^{9}_{2,7}\alpha^{12}_{3,9}-\alpha^{12}_{5,7} &&=0\\
(e_2,e_4,e_6):&\quad \alpha^{12}_{2,10}\alpha^{10}_{4,6}-\alpha^{8}_{2,6}\alpha^{12}_{4,8} &&=0\\
(e_3,e_4,e_5):&\quad \alpha^{7}_{3,4}\alpha^{12}_{5,7}-\alpha^{8}_{3,5}\alpha^{12}_{4,8}+\alpha^{12}_{3,9}\alpha^{9}_{4,5} &&=0\\
(e_1,e_2,e_{10}):&\quad \alpha^{12}_{2,10}-\alpha^{13}_{2,11}-\alpha^{13}_{3,10} &&=0\\
(e_1,e_3,e_9):&\quad -\alpha^{13}_{3,10}+\alpha^{12}_{3,9}-\alpha^{13}_{4,9} &&=0\\
(e_1,e_4,e_8):&\quad \alpha^{12}_{4,8}-\alpha^{13}_{4,9}-\alpha^{13}_{5,8} &&=0\\
(e_1,e_5,e_7):&\quad \alpha^{12}_{5,7}-\alpha^{13}_{5,8}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_3,e_8):&\quad \alpha^{13}_{2,11}\alpha^{11}_{3,8}-\alpha^{10}_{2,8}\alpha^{13}_{3,10}-\alpha^{13}_{5,8} &&=0\\
(e_2,e_4,e_7):&\quad \alpha^{13}_{2,11}\alpha^{11}_{4,7}-\alpha^{9}_{2,7}\alpha^{13}_{4,9}-\alpha^{13}_{6,7} &&=0\\
(e_2,e_5,e_6):&\quad \alpha^{13}_{2,11}\alpha^{11}_{5,6}+\alpha^{7}_{2,5}\alpha^{13}_{6,7}-\alpha^{8}_{2,6}\alpha^{13}_{5,8} &&=0\\
(e_3,e_4,e_6):&\quad \alpha^{13}_{3,10}\alpha^{10}_{4,6}+\alpha^{7}_{3,4}\alpha^{13}_{6,7}-\alpha^{9}_{3,6}\alpha^{13}_{4,9} &&=0\\
(e_1,e_2,e_{12}):&\quad -\alpha^{14}_{3,12}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -\alpha^{14}_{3,12}-\alpha^{14}_{4,11} &&=0\\
(e_1,e_4,e_{10}):&\quad -\alpha^{14}_{4,11}-\alpha^{14}_{5,10} &&=0\\
(e_1,e_5,e_9):&\quad -\alpha^{14}_{5,10}-\alpha^{14}_{6,9} &&=0\\
(e_1,e_6,e_8):&\quad -\alpha^{14}_{6,9}-\alpha^{14}_{7,8} &&=0\\
(e_2,e_3,e_{10}):&\quad -\alpha^{12}_{2,10}\alpha^{14}_{3,12}+\alpha^{13}_{3,10}-\alpha^{14}_{5,10} &&=0\\
(e_2,e_4,e_9):&\quad -\alpha^{11}_{2,9}\alpha^{14}_{4,11}+\alpha^{13}_{4,9}-\alpha^{14}_{6,9} &&=0\\
(e_2,e_5,e_8):&\quad -\alpha^{7}_{2,5}\alpha^{14}_{7,8}-\alpha^{10}_{2,8}\alpha^{14}_{5,10}+\alpha^{13}_{5,8} &&=0\\
(e_2,e_6,e_7):&\quad \alpha^{8}_{2,6}\alpha^{14}_{7,8}-\alpha^{9}_{2,7}\alpha^{14}_{6,9}+\alpha^{13}_{6,7} &&=0\\
(e_3,e_4,e_8):&\quad \alpha^{14}_{3,12}\alpha^{12}_{4,8}-\alpha^{7}_{3,4}\alpha^{14}_{7,8}-\alpha^{11}_{3,8}\alpha^{14}_{4,11} &&=0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{11}_{3,8} \to x_1$$

$$\alpha^{11}_{4,7} \to x_2$$

$$\alpha^{14}_{6,9} \to x_3$$

$$\alpha^{14}_{3,12} \to x_4$$

$$\alpha^{14}_{7,8} \to x_5$$

$$\alpha^{13}_{6,7} \to x_6$$

$$\alpha^{12}_{5,7} \to x_7$$

$$\alpha^{8}_{2,6} \to x_8$$

$$\alpha^{12}_{3,9} \to x_9$$

$$\alpha^{12}_{4,8} \to x_{10}$$

$$\alpha^{7}_{2,5} \to x_{11}$$

$$\alpha^{9}_{2,7} \to x_{12}$$

$$\alpha^{10}_{3,7} \to x_{13}$$

$$\alpha^{10}_{2,8} \to x_{14}$$

$$\alpha^{9}_{4,5} \to x_{15}$$

$$\alpha^{13}_{3,10} \to x_{16}$$

$$\alpha^{14}_{5,10} \to x_{17}$$

$$\alpha^{13}_{5,8} \to x_{18}$$

$$\alpha^{14}_{4,11} \to x_{19}$$

$$\alpha^{8}_{3,5} \to x_{20}$$

$$\alpha^{11}_{5,6} \to x_{21}$$

$$\alpha^{10}_{4,6} \to x_{22}$$

$$\alpha^{11}_{2,9} \to x_{23}$$

$$\alpha^{13}_{2,11} \to x_{24}$$

$$\alpha^{13}_{4,9} \to x_{25}$$

$$\alpha^{9}_{3,6} \to x_{26}$$

$$\alpha_{3,4}^{7} \to x_{27}$$

$$\alpha_{2,10}^{12} \to x_{28}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_{11}-x_{27}+1 &&=0\\
(e_1,e_2,e_5):&\quad x_{11}-x_{20}-x_8 &&=0\\
(e_1,e_3,e_4):&\quad -x_{20}+x_{27} &&=0\\
(e_1,e_2,e_6):&\quad -x_{12}-x_{26}+x_8 &&=0\\
(e_1,e_3,e_5):&\quad -x_{15}+x_{20}-x_{26} &&=0\\
(e_2,e_3,e_4):&\quad x_{12}x_{27}+x_{15}-x_{26} &&=0\\
(e_1,e_2,e_7):&\quad x_{12}-x_{13}-x_{14} &&=0\\
(e_1,e_3,e_6):&\quad -x_{13}-x_{22}+x_{26} &&=0\\
(e_1,e_4,e_5):&\quad x_{15}-x_{22} &&=0\\
(e_2,e_3,e_5):&\quad -x_{11}x_{13}+x_{14}x_{20} &&=0\\
(e_1,e_2,e_8):&\quad -x_1+x_{14}-x_{23} &&=0\\
(e_1,e_3,e_7):&\quad -x_1+x_{13}-x_2 &&=0\\
(e_1,e_4,e_6):&\quad -x_2-x_{21}+x_{22} &&=0\\
(e_2,e_3,e_6):&\quad -x_1x_8-x_{21}+x_{23}x_{26} &&=0\\
(e_2,e_4,e_5):&\quad -x_{11}x_2+x_{15}x_{23}+x_{21} &&=0\\
(e_1,e_2,e_9):&\quad x_{23}-x_{28}-x_9 &&=0\\
(e_1,e_3,e_8):&\quad x_1-x_{10}-x_9 &&=0\\
(e_1,e_4,e_7):&\quad -x_{10}+x_2-x_7 &&=0\\
(e_1,e_5,e_6):&\quad x_{21}-x_7 &&=0\\
(e_2,e_3,e_7):&\quad -x_{12}x_9+x_{13}x_{28}-x_7 &&=0\\
(e_2,e_4,e_6):&\quad -x_{10}x_8+x_{22}x_{28} &&=0\\
(e_3,e_4,e_5):&\quad -x_{10}x_{20}+x_{15}x_9+x_{27}x_7 &&=0\\
(e_1,e_2,e_{10}):&\quad -x_{16}-x_{24}+x_{28} &&=0\\
(e_1,e_3,e_9):&\quad -x_{16}-x_{25}+x_9 &&=0\\
(e_1,e_4,e_8):&\quad x_{10}-x_{18}-x_{25} &&=0\\
(e_1,e_5,e_7):&\quad -x_{18}-x_6+x_7 &&=0\\
(e_2,e_3,e_8):&\quad x_1x_{24}-x_{14}x_{16}-x_{18} &&=0\\
(e_2,e_4,e_7):&\quad -x_{12}x_{25}+x_2x_{24}-x_6 &&=0\\
(e_2,e_5,e_6):&\quad x_{11}x_6-x_{18}x_8+x_{21}x_{24} &&=0\\
(e_3,e_4,e_6):&\quad x_{16}x_{22}-x_{25}x_{26}+x_{27}x_6 &&=0\\
(e_1,e_2,e_{12}):&\quad -x_4-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -x_{19}-x_4 &&=0\\
(e_1,e_4,e_{10}):&\quad -x_{17}-x_{19} &&=0\\
(e_1,e_5,e_9):&\quad -x_{17}-x_3 &&=0\\
(e_1,e_6,e_8):&\quad -x_3-x_5 &&=0\\
(e_2,e_3,e_{10}):&\quad x_{16}-x_{17}-x_{28}x_4 &&=0\\
(e_2,e_4,e_9):&\quad -x_{19}x_{23}+x_{25}-x_3 &&=0\\
(e_2,e_5,e_8):&\quad -x_{11}x_5-x_{14}x_{17}+x_{18} &&=0\\
(e_2,e_6,e_7):&\quad -x_{12}x_3+x_5x_8+x_6 &&=0\\
(e_3,e_4,e_8):&\quad -x_1x_{19}+x_{10}x_4-x_{27}x_5 &&=0\\
(e_3,e_5,e_7):&\quad -x_{13}x_{17}+x_{20}x_5+x_4x_7 &&=0\\
(e_4,e_5,e_6):&\quad x_{15}x_3-x_{17}x_{22}+x_{19}x_{21} &&=0
\end{aligned}
$$

Groebner basis (28 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{3B}(3, 14)$

m3B314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_9] &= e_{12} \\
[e_2, e_{10}] &= 4e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_8] &= -e_{12} & [e_3, e_9] &= -3e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_7] &= e_{12} \\
[e_4, e_8] &= 2e_{13} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_6] &= -e_{12} & [e_5, e_7] &= -e_{13} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_9): &\quad -\alpha_{3,12}^{14} - 3 &&= 0 \\
(e_2, e_4, e_8): &\quad \text{no solutions} \\
(e_2, e_5, e_7): &\quad \text{no solutions} \\
(e_3, e_4, e_7): &\quad \alpha_{3,12}^{14} &&= 0 \\
(e_3, e_5, e_6): &\quad -\alpha_{3,12}^{14} &&= 0
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{5B}(3,14)$

m5B314 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1,e_2] &= e_3 & [e_1,e_3] &= e_4\\
[e_1,e_4] &= e_5 & [e_1,e_5] &= e_6\\
[e_1,e_6] &= e_7 & [e_1,e_7] &= e_8\\
[e_1,e_8] &= e_9 & [e_1,e_9] &= e_{10}\\
[e_1,e_{10}] &= e_{11} & [e_1,e_{11}] &= e_{12}\\
[e_1,e_{12}] &= e_{13} & [e_2,e_7] &= e_{10}\\
[e_2,e_8] &= 3e_{11} & [e_2,e_9] &= \alpha_{2,9}^{12}e_{12}\\
[e_2,e_{10}] &= \alpha_{2,10}^{13}e_{13} & [e_2,e_{13}] &= e_{14}\\
[e_3,e_6] &= -e_{10} & [e_3,e_7] &= -2e_{11}\\
[e_3,e_8] &= \alpha_{3,8}^{12}e_{12} & [e_3,e_9] &= \alpha_{3,9}^{13}e_{13}\\
[e_3,e_{12}] &= \alpha_{3,12}^{14}e_{14} & [e_4,e_5] &= e_{10}\\
[e_4,e_6] &= e_{11} & [e_4,e_7] &= \alpha_{4,7}^{12}e_{12}\\
[e_4,e_8] &= \alpha_{4,8}^{13}e_{13} & [e_4,e_{11}] &= \alpha_{4,11}^{14}e_{14}\\
[e_5,e_6] &= \alpha_{5,6}^{12}e_{12} & [e_5,e_7] &= \alpha_{5,7}^{13}e_{13}\\
[e_5,e_{10}] &= \alpha_{5,10}^{14}e_{14} & [e_6,e_9] &= \alpha_{6,9}^{14}e_{14}\\
[e_7,e_8] &= \alpha_{7,8}^{14}e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{12} - \alpha_{4,7}^{12} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{12} - \alpha_{5,6}^{12} + 1 &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6): &\quad -\alpha_{2,10}^{13} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13} &&= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_9): &\quad -\alpha_{2,9}^{12}\alpha_{3,12}^{14} + \alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_8): &\quad -3\alpha_{4,11}^{14} + \alpha_{4,8}^{13} &&= 0\\
(e_2, e_5, e_7): &\quad -\alpha_{5,10}^{14} + \alpha_{5,7}^{13} &&= 0\\
(e_3, e_4, e_7): &\quad \alpha_{3,12}^{14}\alpha_{4,7}^{12} + 2\alpha_{4,11}^{14} &&= 0\\
(e_3, e_5, e_6): &\quad \alpha_{3,12}^{14}\alpha_{5,6}^{12} + \alpha_{5,10}^{14} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{12} \to x_1$$

$$\alpha_{5,7}^{13} \to x_2$$

$$\alpha_{4,11}^{14} \to x_3$$

$$\alpha_{3,8}^{12} \to x_4$$

$$\alpha_{4,8}^{13} \to x_5$$

$$\alpha_{6,9}^{14} \to x_6$$

$$\alpha_{3,12}^{14} \to x_7$$

$$\alpha_{2,10}^{13} \to x_8$$

$$\alpha_{4,7}^{12} \to x_9$$
$$\alpha_{5,6}^{12} \to x_{10}$$
$$\alpha_{7,8}^{14} \to x_{11}$$
$$\alpha_{3,9}^{13} \to x_{12}$$
$$\alpha_{5,10}^{14} \to x_{13}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_1 - x_4 + 3 &= 0\\
(e_1, e_3, e_7): &\quad -x_4 - x_9 - 2 &= 0\\
(e_1, e_4, e_6): &\quad -x_{10} - x_9 + 1 &= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_{12} - x_8 &= 0\\
(e_1, e_3, e_8): &\quad -x_{12} + x_4 - x_5 &= 0\\
(e_1, e_4, e_7): &\quad -x_2 - x_5 + x_9 &= 0\\
(e_1, e_5, e_6): &\quad x_{10} - x_2 &= 0\\
(e_2, e_3, e_6): &\quad -x_8 &= 0\\
(e_2, e_4, e_5): &\quad x_8 &= 0\\
(e_1, e_2, e_{12}): &\quad -x_7 - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -x_3 - x_7 &= 0\\
(e_1, e_4, e_{10}): &\quad -x_{13} - x_3 &= 0\\
(e_1, e_5, e_9): &\quad -x_{13} - x_6 &= 0\\
(e_1, e_6, e_8): &\quad -x_{11} - x_6 &= 0\\
(e_2, e_3, e_9): &\quad -x_1 x_7 + x_{12} &= 0\\
(e_2, e_4, e_8): &\quad -3x_3 + x_5 &= 0\\
(e_2, e_5, e_7): &\quad -x_{13} + x_2 &= 0\\
(e_3, e_4, e_7): &\quad 2x_3 + x_7 x_9 &= 0\\
(e_3, e_5, e_6): &\quad x_{10} x_7 + x_{13} &= 0
\end{aligned}$$

Groebner basis (13 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{7B}(3, 14)$

m7B314 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_5] = e_8 \\
[e_2, e_6] = 2e_9 & [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
[e_2, e_8] = \alpha_{2,8}^{11} e_{11} & [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} & [e_2, e_{13}] = e_{14} \\
[e_3, e_4] = -e_8 & [e_3, e_5] = -e_9 \\
[e_3, e_6] = \alpha_{3,6}^{10} e_{10} & [e_3, e_7] = \alpha_{3,7}^{11} e_{11} \\
[e_3, e_8] = \alpha_{3,8}^{12} e_{12} & [e_3, e_9] = \alpha_{3,9}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
[e_4, e_6] = \alpha_{4,6}^{11} e_{11} & [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
[e_4, e_8] = \alpha_{4,8}^{13} e_{13} & [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
[e_5, e_6] = \alpha_{5,6}^{12} e_{12} & [e_5, e_7] = \alpha_{5,7}^{13} e_{13} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] = \alpha_{7,8}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -\alpha_{2,7}^{10} - \alpha_{3,6}^{10} + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -\alpha_{3,6}^{10} - \alpha_{4,5}^{10} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{10} - \alpha_{2,8}^{11} - \alpha_{3,7}^{11} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{10} - \alpha_{3,7}^{11} - \alpha_{4,6}^{11} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{10} - \alpha_{4,6}^{11} &&= 0\\
(e_2, e_3, e_4): &\quad -\alpha_{2,8}^{11} &&= 0\\
(e_1, e_2, e_8): &\quad \alpha_{2,8}^{11} - \alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_3, e_7): &\quad \alpha_{3,7}^{11} - \alpha_{3,8}^{12} - \alpha_{4,7}^{12} &&= 0\\
(e_1, e_4, e_6): &\quad \alpha_{4,6}^{11} - \alpha_{4,7}^{12} - \alpha_{5,6}^{12} &&= 0\\
(e_2, e_3, e_5): &\quad -\alpha_{2,9}^{12} - \alpha_{3,8}^{12} &&= 0\\
(e_1, e_2, e_9): &\quad -\alpha_{2,10}^{13} + \alpha_{2,9}^{12} - \alpha_{3,9}^{13} &&= 0\\
(e_1, e_3, e_8): &\quad \alpha_{3,8}^{12} - \alpha_{3,9}^{13} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_4, e_7): &\quad \alpha_{4,7}^{12} - \alpha_{4,8}^{13} - \alpha_{5,7}^{13} &&= 0\\
(e_1, e_5, e_6): &\quad \alpha_{5,6}^{12} - \alpha_{5,7}^{13} &&= 0\\
(e_2, e_3, e_6): &\quad \alpha_{2,10}^{13}\alpha_{3,6}^{10} - 2\alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_5): &\quad \alpha_{2,10}^{13}\alpha_{4,5}^{10} - \alpha_{4,8}^{13} &&= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_9): &\quad -\alpha_{2,9}^{12}\alpha_{3,12}^{14} + \alpha_{3,9}^{13} &&= 0\\
(e_2, e_4, e_8): &\quad -\alpha_{2,8}^{11}\alpha_{4,11}^{14} + \alpha_{4,8}^{13} &&= 0\\
(e_2, e_5, e_7): &\quad -\alpha_{2,7}^{10}\alpha_{5,10}^{14} + \alpha_{5,7}^{13} + \alpha_{7,8}^{14} &&= 0\\
(e_3, e_4, e_7): &\quad \alpha_{3,12}^{14}\alpha_{4,7}^{12} - \alpha_{3,7}^{11}\alpha_{4,11}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_3, e_5, e_6): &\quad \alpha_{3,12}^{14}\alpha_{5,6}^{12} - \alpha_{3,6}^{10}\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0
\end{aligned}
$$

No solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,9}^{12} \to x_1$$

$$\alpha_{5,7}^{13} \to x_2$$

$$\alpha^{14}_{4,11} \to x_3$$

$$\alpha^{10}_{2,7} \to x_4$$

$$\alpha^{12}_{3,8} \to x_5$$

$$\alpha^{13}_{4,8} \to x_6$$

$$\alpha^{14}_{6,9} \to x_7$$

$$\alpha^{11}_{3,7} \to x_8$$

$$\alpha^{13}_{2,10} \to x_9$$

$$\alpha^{14}_{3,12} \to x_{10}$$

$$\alpha^{10}_{3,6} \to x_{11}$$

$$\alpha^{12}_{4,7} \to x_{12}$$

$$\alpha^{11}_{2,8} \to x_{13}$$

$$\alpha^{12}_{5,6} \to x_{14}$$

$$\alpha^{14}_{7,8} \to x_{15}$$

$$\alpha^{10}_{4,5} \to x_{16}$$

$$\alpha^{13}_{3,9} \to x_{17}$$

$$\alpha^{11}_{4,6} \to x_{18}$$

$$\alpha^{14}_{5,10} \to x_{19}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{11} - x_4 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{11} - x_{16} - 1 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{13} + x_4 - x_8 &&= 0\\
(e_1, e_3, e_6): &\quad x_{11} - x_{18} - x_8 &&= 0\\
(e_1, e_4, e_5): &\quad x_{16} - x_{18} &&= 0\\
(e_2, e_3, e_4): &\quad -x_{13} &&= 0\\
(e_1, e_2, e_8): &\quad -x_1 + x_{13} - x_5 &&= 0\\
(e_1, e_3, e_7): &\quad -x_{12} - x_5 + x_8 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{12} - x_{14} + x_{18} &&= 0\\
(e_2, e_3, e_5): &\quad -x_1 - x_5 &&= 0\\
(e_1, e_2, e_9): &\quad x_1 - x_{17} - x_9 &&= 0\\
(e_1, e_3, e_8): &\quad -x_{17} + x_5 - x_6 &&= 0\\
(e_1, e_4, e_7): &\quad x_{12} - x_2 - x_6 &&= 0\\
(e_1, e_5, e_6): &\quad x_{14} - x_2 &&= 0\\
(e_2, e_3, e_6): &\quad x_{11}x_9 - 2x_{17} &&= 0\\
(e_2, e_4, e_5): &\quad x_{16}x_9 - x_6 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_{10} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_{10} - x_3 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_{19} - x_3 &&= 0\\
(e_1, e_5, e_9): &\quad -x_{19} - x_7 &&= 0\\
(e_1, e_6, e_8): &\quad -x_{15} - x_7 &&= 0\\
(e_2, e_3, e_9): &\quad -x_1x_{10} + x_{17} &&= 0\\
(e_2, e_4, e_8): &\quad -x_{13}x_3 + x_6 &&= 0\\
(e_2, e_5, e_7): &\quad x_{15} - x_{19}x_4 + x_2 &&= 0\\
(e_3, e_4, e_7): &\quad x_{10}x_{12} - x_{15} - x_3x_8 &&= 0\\
(e_3, e_5, e_6): &\quad x_{10}x_{14} - x_{11}x_{19} - x_7 &&= 0
\end{aligned}
$$

Groebner basis (19 variables, 1 linear, 0 nonlinear)

$$1 = 0$$

$\mathfrak{m}_{9B}(3, 14)$

m9B314 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= -2e_8 \\
[e_2, e_6] &= -5e_9 & [e_2, e_7] &= -5e_{10} \\
[e_2, e_8] &= -2e_{11} & [e_2, e_9] &= e_{12} \\
[e_2, e_{10}] &= e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_4] &= 3e_8 & [e_3, e_5] &= 3e_9 \\
[e_3, e_6] &= 0 & [e_3, e_7] &= -3e_{11} \\
[e_3, e_8] &= -3e_{12} & [e_3, e_9] &= 0 \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_5] &= 3e_{10} \\
[e_4, e_6] &= 3e_{11} & [e_4, e_7] &= 0 \\
[e_4, e_8] &= -3e_{13} & [e_4, e_{11}] &= e_{14} \\
[e_5, e_6] &= 3e_{12} & [e_5, e_7] &= 3e_{13} \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}
$$

Solution 2

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{10e_8}{7}$$
$$[e_2, e_6] = \frac{13e_9}{7} \qquad [e_2, e_7] = \frac{19e_{10}}{7}$$
$$[e_2, e_8] = 4e_{11} \qquad [e_2, e_9] = 7e_{12}$$
$$[e_2, e_{10}] = 13e_{13} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_4] = -\frac{3e_8}{7} \qquad [e_3, e_5] = -\frac{3e_9}{7}$$
$$[e_3, e_6] = -\frac{6e_{10}}{7} \qquad [e_3, e_7] = -\frac{9e_{11}}{7}$$
$$[e_3, e_8] = -3e_{12} \qquad [e_3, e_9] = -6e_{13}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_5] = \frac{3e_{10}}{7}$$
$$[e_4, e_6] = \frac{3e_{11}}{7} \qquad [e_4, e_7] = \frac{12e_{12}}{7}$$
$$[e_4, e_8] = 3e_{13} \qquad [e_4, e_{11}] = e_{14}$$
$$[e_5, e_6] = -\frac{9e_{12}}{7} \qquad [e_5, e_7] = -\frac{9e_{13}}{7}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Solution 3

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_6 \\
[e_2, e_4] &= e_7 & [e_2, e_5] &= e_8 \\
[e_2, e_6] &= e_9 & [e_2, e_7] &= e_{10} \\
[e_2, e_8] &= e_{11} & [e_2, e_9] &= e_{12} \\
[e_2, e_{10}] &= e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_4] &= 0 & [e_3, e_5] &= 0 \\
[e_3, e_6] &= 0 & [e_3, e_7] &= 0 \\
[e_3, e_8] &= 0 & [e_3, e_9] &= 0 \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_5] &= 0 \\
[e_4, e_6] &= 0 & [e_4, e_7] &= 0 \\
[e_4, e_8] &= 0 & [e_4, e_{11}] &= e_{14} \\
[e_5, e_6] &= 0 & [e_5, e_7] &= 0 \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}
$$

Solution 4

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_6$$
$$[e_2, e_4] = e_7 \qquad [e_2, e_5] = \frac{e_8}{4}$$
$$[e_2, e_6] = -\frac{e_9}{2} \qquad [e_2, e_7] = -\frac{23e_{10}}{28}$$
$$[e_2, e_8] = -\frac{5e_{11}}{7} \qquad [e_2, e_9] = -\frac{5e_{12}}{4}$$
$$[e_2, e_{10}] = -\frac{7e_{13}}{2} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_4] = \frac{3e_8}{4} \qquad [e_3, e_5] = \frac{3e_9}{4}$$
$$[e_3, e_6] = \frac{9e_{10}}{28} \qquad [e_3, e_7] = -\frac{3e_{11}}{28}$$
$$[e_3, e_8] = \frac{15e_{12}}{28} \qquad [e_3, e_9] = \frac{9e_{13}}{4}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_5] = \frac{3e_{10}}{7}$$
$$[e_4, e_6] = \frac{3e_{11}}{7} \qquad [e_4, e_7] = -\frac{9e_{12}}{14}$$
$$[e_4, e_8] = -\frac{12e_{13}}{7} \qquad [e_4, e_{11}] = e_{14}$$
$$[e_5, e_6] = \frac{15e_{12}}{14} \qquad [e_5, e_7] = \frac{15e_{13}}{14}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Original brackets:

$$
\begin{aligned}
&[e_1, e_2] = e_3 && [e_1, e_3] = e_4 \\
&[e_1, e_4] = e_5 && [e_1, e_5] = e_6 \\
&[e_1, e_6] = e_7 && [e_1, e_7] = e_8 \\
&[e_1, e_8] = e_9 && [e_1, e_9] = e_{10} \\
&[e_1, e_{10}] = e_{11} && [e_1, e_{11}] = e_{12} \\
&[e_1, e_{12}] = e_{13} && [e_2, e_3] = e_6 \\
&[e_2, e_4] = e_7 && [e_2, e_5] = \alpha_{2,5}^{8} e_8 \\
&[e_2, e_6] = \alpha_{2,6}^{9} e_9 && [e_2, e_7] = \alpha_{2,7}^{10} e_{10} \\
&[e_2, e_8] = \alpha_{2,8}^{11} e_{11} && [e_2, e_9] = \alpha_{2,9}^{12} e_{12} \\
&[e_2, e_{10}] = \alpha_{2,10}^{13} e_{13} && [e_2, e_{13}] = e_{14} \\
&[e_3, e_4] = \alpha_{3,4}^{8} e_8 && [e_3, e_5] = \alpha_{3,5}^{9} e_9 \\
&[e_3, e_6] = \alpha_{3,6}^{10} e_{10} && [e_3, e_7] = \alpha_{3,7}^{11} e_{11} \\
&[e_3, e_8] = \alpha_{3,8}^{12} e_{12} && [e_3, e_9] = \alpha_{3,9}^{13} e_{13} \\
&[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} && [e_4, e_5] = \alpha_{4,5}^{10} e_{10} \\
&[e_4, e_6] = \alpha_{4,6}^{11} e_{11} && [e_4, e_7] = \alpha_{4,7}^{12} e_{12} \\
&[e_4, e_8] = \alpha_{4,8}^{13} e_{13} && [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
&[e_5, e_6] = \alpha_{5,6}^{12} e_{12} && [e_5, e_7] = \alpha_{5,7}^{13} e_{13} \\
&[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} && [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
&[e_7, e_8] = \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rll}
(e_1,e_2,e_4): & -\alpha_{2,5}^{8}-\alpha_{3,4}^{8}+1 & =0\\
(e_1,e_2,e_5): & \alpha_{2,5}^{8}-\alpha_{2,6}^{9}-\alpha_{3,5}^{9} & =0\\
(e_1,e_3,e_4): & \alpha_{3,4}^{8}-\alpha_{3,5}^{9} & =0\\
(e_1,e_2,e_6): & \alpha_{2,6}^{9}-\alpha_{2,7}^{10}-\alpha_{3,6}^{10} & =0\\
(e_1,e_3,e_5): & \alpha_{3,5}^{9}-\alpha_{3,6}^{10}-\alpha_{4,5}^{10} & =0\\
(e_1,e_2,e_7): & \alpha_{2,7}^{10}-\alpha_{2,8}^{11}-\alpha_{3,7}^{11} & =0\\
(e_1,e_3,e_6): & \alpha_{3,6}^{10}-\alpha_{3,7}^{11}-\alpha_{4,6}^{11} & =0\\
(e_1,e_4,e_5): & \alpha_{4,5}^{10}-\alpha_{4,6}^{11} & =0\\
(e_2,e_3,e_4): & \alpha_{2,8}^{11}\alpha_{3,4}^{8}-\alpha_{3,7}^{11}+\alpha_{4,6}^{11} & =0\\
(e_1,e_2,e_8): & \alpha_{2,8}^{11}-\alpha_{2,9}^{12}-\alpha_{3,8}^{12} & =0\\
(e_1,e_3,e_7): & \alpha_{3,7}^{11}-\alpha_{3,8}^{12}-\alpha_{4,7}^{12} & =0\\
(e_1,e_4,e_6): & \alpha_{4,6}^{11}-\alpha_{4,7}^{12}-\alpha_{5,6}^{12} & =0\\
(e_2,e_3,e_5): & -\alpha_{2,5}^{8}\alpha_{3,8}^{12}+\alpha_{2,9}^{12}\alpha_{3,5}^{9}+\alpha_{5,6}^{12} & =0\\
(e_1,e_2,e_9): & -\alpha_{2,10}^{13}+\alpha_{2,9}^{12}-\alpha_{3,9}^{13} & =0\\
(e_1,e_3,e_8): & \alpha_{3,8}^{12}-\alpha_{3,9}^{13}-\alpha_{4,8}^{13} & =0\\
(e_1,e_4,e_7): & \alpha_{4,7}^{12}-\alpha_{4,8}^{13}-\alpha_{5,7}^{13} & =0\\
(e_1,e_5,e_6): & \alpha_{5,6}^{12}-\alpha_{5,7}^{13} & =0\\
(e_2,e_3,e_6): & \alpha_{2,10}^{13}\alpha_{3,6}^{10}-\alpha_{2,6}^{9}\alpha_{3,9}^{13} & =0\\
(e_2,e_4,e_5): & \alpha_{2,10}^{13}\alpha_{4,5}^{10}-\alpha_{2,5}^{8}\alpha_{4,8}^{13}+\alpha_{5,7}^{13} & =0\\
(e_1,e_2,e_{12}): & -\alpha_{3,12}^{14}-1 & =0\\
(e_1,e_3,e_{11}): & -\alpha_{3,12}^{14}-\alpha_{4,11}^{14} & =0\\
(e_1,e_4,e_{10}): & -\alpha_{4,11}^{14}-\alpha_{5,10}^{14} & =0\\
(e_1,e_5,e_9): & -\alpha_{5,10}^{14}-\alpha_{6,9}^{14} & =0\\
(e_1,e_6,e_8): & -\alpha_{6,9}^{14}-\alpha_{7,8}^{14} & =0\\
(e_2,e_3,e_9): & -\alpha_{2,9}^{12}\alpha_{3,12}^{14}+\alpha_{3,9}^{13}-\alpha_{6,9}^{14} & =0\\
(e_2,e_4,e_8): & -\alpha_{2,8}^{11}\alpha_{4,11}^{14}+\alpha_{4,8}^{13}-\alpha_{7,8}^{14} & =0\\
(e_2,e_5,e_7): & \alpha_{2,5}^{8}\alpha_{7,8}^{14}-\alpha_{2,7}^{10}\alpha_{5,10}^{14}+\alpha_{5,7}^{13} & =0\\
(e_3,e_4,e_7): & \alpha_{3,12}^{14}\alpha_{4,7}^{12}+\alpha_{3,4}^{8}\alpha_{7,8}^{14}-\alpha_{3,7}^{11}\alpha_{4,11}^{14} & =0\\
(e_3,e_5,e_6): & \alpha_{3,12}^{14}\alpha_{5,6}^{12}+\alpha_{3,5}^{9}\alpha_{6,9}^{14}-\alpha_{3,6}^{10}\alpha_{5,10}^{14} & =0
\end{array}
$$

Solution 1:

$$\alpha_{3,8}^{12} = -3$$
$$\alpha_{6,9}^{14} = 1$$
$$\alpha_{3,12}^{14} = -1$$
$$\alpha_{2,10}^{13} = 1$$
$$\alpha_{3,6}^{10} = 0$$
$$\alpha_{7,8}^{14} = -1$$
$$\alpha_{3,9}^{13} = 0$$
$$\alpha_{2,9}^{12} = 1$$
$$\alpha_{3,4}^{8} = 3$$
$$\alpha_{4,8}^{13} = -3$$
$$\alpha_{3,7}^{11} = -3$$
$$\alpha_{4,7}^{12} = 0$$
$$\alpha_{2,8}^{11} = -2$$
$$\alpha_{5,6}^{12} = 3$$
$$\alpha_{4,5}^{10} = 3$$
$$\alpha_{3,5}^{9} = 3$$
$$\alpha_{5,10}^{14} = -1$$
$$\alpha_{4,11}^{14} = 1$$
$$\alpha_{2,7}^{10} = -5$$
$$\alpha_{2,5}^{8} = -2$$
$$\alpha_{5,7}^{13} = 3$$
$$\alpha_{2,6}^{9} = -5$$
$$\alpha_{4,6}^{11} = 3$$

Solution 2:

$$\alpha_{3,8}^{12} = -3$$
$$\alpha_{6,9}^{14} = 1$$
$$\alpha_{3,12}^{14} = -1$$
$$\alpha_{2,10}^{13} = 13$$
$$\alpha_{3,6}^{10} = -6/7$$
$$\alpha_{7,8}^{14} = -1$$
$$\alpha_{3,9}^{13} = -6$$
$$\alpha_{2,9}^{12} = 7$$
$$\alpha_{3,4}^{8} = -3/7$$
$$\alpha_{4,8}^{13} = 3$$
$$\alpha_{3,7}^{11} = -9/7$$
$$\alpha_{4,7}^{12} = 12/7$$
$$\alpha_{2,8}^{11} = 4$$
$$\alpha_{5,6}^{12} = -9/7$$
$$\alpha_{4,5}^{10} = 3/7$$
$$\alpha_{3,5}^{9} = -3/7$$
$$\alpha_{5,10}^{14} = -1$$
$$\alpha_{4,11}^{14} = 1$$
$$\alpha_{2,7}^{10} = 19/7$$
$$\alpha_{2,5}^{8} = 10/7$$
$$\alpha_{5,7}^{13} = -9/7$$
$$\alpha_{2,6}^{9} = 13/7$$
$$\alpha_{4,6}^{11} = 3/7$$

Solution 3:

$$\alpha_{3,8}^{12} = 0$$
$$\alpha_{6,9}^{14} = 1$$
$$\alpha_{3,12}^{14} = -1$$
$$\alpha_{2,10}^{13} = 1$$
$$\alpha_{3,6}^{10} = 0$$
$$\alpha_{7,8}^{14} = -1$$
$$\alpha_{3,9}^{13} = 0$$
$$\alpha_{2,9}^{12} = 1$$
$$\alpha_{3,4}^{8} = 0$$
$$\alpha_{4,8}^{13} = 0$$
$$\alpha_{3,7}^{11} = 0$$
$$\alpha_{4,7}^{12} = 0$$
$$\alpha_{2,8}^{11} = 1$$
$$\alpha_{5,6}^{12} = 0$$
$$\alpha_{4,5}^{10} = 0$$
$$\alpha_{3,5}^{9} = 0$$
$$\alpha_{5,10}^{14} = -1$$
$$\alpha_{4,11}^{14} = 1$$
$$\alpha_{2,7}^{10} = 1$$
$$\alpha_{2,5}^{8} = 1$$
$$\alpha_{5,7}^{13} = 0$$
$$\alpha_{2,6}^{9} = 1$$
$$\alpha_{4,6}^{11} = 0$$

Solution 4:

$$\alpha^{12}_{3,8} = 15/28$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{13}_{2,10} = -7/2$$
$$\alpha^{10}_{3,6} = 9/28$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{13}_{3,9} = 9/4$$
$$\alpha^{12}_{2,9} = -5/4$$
$$\alpha^{8}_{3,4} = 3/4$$
$$\alpha^{13}_{4,8} = -12/7$$
$$\alpha^{11}_{3,7} = -3/28$$
$$\alpha^{12}_{4,7} = -9/14$$
$$\alpha^{11}_{2,8} = -5/7$$
$$\alpha^{12}_{5,6} = 15/14$$
$$\alpha^{10}_{4,5} = 3/7$$
$$\alpha^{9}_{3,5} = 3/4$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{10}_{2,7} = -23/28$$
$$\alpha^{8}_{2,5} = 1/4$$
$$\alpha^{13}_{5,7} = 15/14$$
$$\alpha^{9}_{2,6} = -1/2$$
$$\alpha^{11}_{4,6} = 3/7$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{12}_{3,8} \to x_1$$
$$\alpha^{14}_{6,9} \to x_2$$
$$\alpha^{14}_{3,12} \to x_3$$
$$\alpha^{13}_{2,10} \to x_4$$
$$\alpha^{10}_{3,6} \to x_5$$

$$\alpha_{7,8}^{14} \to x_6$$

$$\alpha_{3,9}^{13} \to x_7$$

$$\alpha_{2,9}^{12} \to x_8$$

$$\alpha_{3,4}^{8} \to x_9$$

$$\alpha_{4,8}^{13} \to x_{10}$$

$$\alpha_{3,7}^{11} \to x_{11}$$

$$\alpha_{4,7}^{12} \to x_{12}$$

$$\alpha_{2,8}^{11} \to x_{13}$$

$$\alpha_{5,6}^{12} \to x_{14}$$

$$\alpha_{4,5}^{10} \to x_{15}$$

$$\alpha_{3,5}^{9} \to x_{16}$$

$$\alpha_{5,10}^{14} \to x_{17}$$

$$\alpha_{4,11}^{14} \to x_{18}$$

$$\alpha_{2,7}^{10} \to x_{19}$$

$$\alpha_{2,5}^{8} \to x_{20}$$

$$\alpha_{5,7}^{13} \to x_{21}$$

$$\alpha_{2,6}^{9} \to x_{22}$$

$$\alpha_{4,6}^{11} \to x_{23}$$

Jacobi Tests

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{20} - x_9 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{16} + x_{20} - x_{22} &&= 0\\
(e_1, e_3, e_4): &\quad -x_{16} + x_9 &&= 0\\
(e_1, e_2, e_6): &\quad -x_{19} + x_{22} - x_5 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{15} + x_{16} - x_5 &&= 0\\
(e_1, e_2, e_7): &\quad -x_{11} - x_{13} + x_{19} &&= 0\\
(e_1, e_3, e_6): &\quad -x_{11} - x_{23} + x_5 &&= 0\\
(e_1, e_4, e_5): &\quad x_{15} - x_{23} &&= 0\\
(e_2, e_3, e_4): &\quad -x_{11} + x_{13}x_9 + x_{23} &&= 0\\
(e_1, e_2, e_8): &\quad -x_1 + x_{13} - x_8 &&= 0\\
(e_1, e_3, e_7): &\quad -x_1 + x_{11} - x_{12} &&= 0\\
(e_1, e_4, e_6): &\quad -x_{12} - x_{14} + x_{23} &&= 0\\
(e_2, e_3, e_5): &\quad -x_1x_{20} + x_{14} + x_{16}x_8 &&= 0\\
(e_1, e_2, e_9): &\quad -x_4 - x_7 + x_8 &&= 0\\
(e_1, e_3, e_8): &\quad x_1 - x_{10} - x_7 &&= 0\\
(e_1, e_4, e_7): &\quad -x_{10} + x_{12} - x_{21} &&= 0\\
(e_1, e_5, e_6): &\quad x_{14} - x_{21} &&= 0\\
(e_2, e_3, e_6): &\quad -x_{22}x_7 + x_4x_5 &&= 0\\
(e_2, e_4, e_5): &\quad -x_{10}x_{20} + x_{15}x_4 + x_{21} &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_3 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_{18} - x_3 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_{17} - x_{18} &&= 0\\
(e_1, e_5, e_9): &\quad -x_{17} - x_2 &&= 0\\
(e_1, e_6, e_8): &\quad -x_2 - x_6 &&= 0\\
(e_2, e_3, e_9): &\quad -x_2 - x_3x_8 + x_7 &&= 0\\
(e_2, e_4, e_8): &\quad x_{10} - x_{13}x_{18} - x_6 &&= 0\\
(e_2, e_5, e_7): &\quad -x_{17}x_{19} + x_{20}x_6 + x_{21} &&= 0\\
(e_3, e_4, e_7): &\quad -x_{11}x_{18} + x_{12}x_3 + x_6x_9 &&= 0\\
(e_3, e_5, e_6): &\quad x_{14}x_3 + x_{16}x_2 - x_{17}x_5 &&= 0
\end{aligned}
$$

Groebner basis (23 variables, 21 linear, 3 nonlinear)

$$x_1 + \frac{3x_{22}}{2} + 4x_{23} - \frac{3}{2} = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$

$$-7x_{22} - 14x_{23} + x_4 + 6 = 0$$
$$\frac{x_{22}}{2} + x_{23} + x_5 - \frac{1}{2} = 0$$
$$x_6 + 1 = 0$$
$$\frac{7x_{22}}{2} + 7x_{23} + x_7 - \frac{7}{2} = 0$$
$$-\frac{7x_{22}}{2} - 7x_{23} + x_8 + \frac{5}{2} = 0$$
$$\frac{x_{22}}{2} + x_9 - \frac{1}{2} = 0$$
$$x_{10} - 2x_{22} - 3x_{23} + 2 = 0$$
$$x_{11} + \frac{x_{22}}{2} + 2x_{23} - \frac{1}{2} = 0$$
$$x_{12} - x_{22} - 2x_{23} + 1 = 0$$
$$x_{13} - 2x_{22} - 3x_{23} + 1 = 0$$
$$x_{14} + x_{22} + x_{23} - 1 = 0$$
$$x_{15} - x_{23} = 0$$
$$x_{16} + \frac{x_{22}}{2} - \frac{1}{2} = 0$$
$$x_{17} + 1 = 0$$
$$x_{18} - 1 = 0$$
$$x_{19} - \frac{3x_{22}}{2} - x_{23} + \frac{1}{2} = 0$$
$$x_{20} - \frac{x_{22}}{2} - \frac{1}{2} = 0$$
$$x_{21} + x_{22} + x_{23} - 1 = 0$$
$$x_{22}^2 - \frac{19x_{22}}{14} - 3x_{23}^2 - \frac{12x_{23}}{7} + \frac{5}{14} = 0$$
$$x_{22}x_{23} - \frac{3x_{22}}{7} + 2x_{23}^2 - \frac{13x_{23}}{7} + \frac{3}{7} = 0$$
$$x_{23}^3 - \frac{24x_{23}^2}{7} + \frac{9x_{23}}{7} = 0$$

Solution 1:

$$x_1 = -3$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = 1$$
$$x_5 = 0$$

$$x_6 = -1$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = 3$$
$$x_10 = -3$$
$$x_11 = -3$$
$$x_12 = 0$$
$$x_13 = -2$$
$$x_14 = 3$$
$$x_15 = 3$$
$$x_16 = 3$$
$$x_17 = -1$$
$$x_18 = 1$$
$$x_19 = -5$$
$$x_20 = -2$$
$$x_21 = 3$$
$$x_22 = -5$$
$$x_23 = 3$$

Solution 2:

$$x_1 = -3$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = 13$$
$$x_5 = -6/7$$
$$x_6 = -1$$
$$x_7 = -6$$
$$x_8 = 7$$
$$x_9 = -3/7$$
$$x_10 = 3$$
$$x_11 = -9/7$$
$$x_12 = 12/7$$
$$x_13 = 4$$

$$x_14 = -9/7$$
$$x_15 = 3/7$$
$$x_16 = -3/7$$
$$x_17 = -1$$
$$x_18 = 1$$
$$x_19 = 19/7$$
$$x_20 = 10/7$$
$$x_21 = -9/7$$
$$x_22 = 13/7$$
$$x_23 = 3/7$$

Solution 3:

$$x_1 = 0$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = 1$$
$$x_5 = 0$$
$$x_6 = -1$$
$$x_7 = 0$$
$$x_8 = 1$$
$$x_9 = 0$$
$$x_10 = 0$$
$$x_11 = 0$$
$$x_12 = 0$$
$$x_13 = 1$$
$$x_14 = 0$$
$$x_15 = 0$$
$$x_16 = 0$$
$$x_17 = -1$$
$$x_18 = 1$$
$$x_19 = 1$$
$$x_20 = 1$$
$$x_21 = 0$$

$$x_22 = 1$$
$$x_23 = 0$$

Solution 4:

$$x_1 = 15/28$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = -7/2$$
$$x_5 = 9/28$$
$$x_6 = -1$$
$$x_7 = 9/4$$
$$x_8 = -5/4$$
$$x_9 = 3/4$$
$$x_10 = -12/7$$
$$x_11 = -3/28$$
$$x_12 = -9/14$$
$$x_13 = -5/7$$
$$x_14 = 15/14$$
$$x_15 = 3/7$$
$$x_16 = 3/4$$
$$x_17 = -1$$
$$x_18 = 1$$
$$x_19 = -23/28$$
$$x_20 = 1/4$$
$$x_21 = 15/14$$
$$x_22 = -1/2$$
$$x_23 = 3/7$$

$\mathfrak{m}_{2B}(4, 14)$

m2B414 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_9] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_8] &= -e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_7] &= e_{13} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_6] &= -e_{13} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_8): &\quad \text{no solutions} \\
(e_2, e_4, e_7): &\quad \text{no solutions} \\
(e_2, e_5, e_6): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(4, 14)$

m4B414 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{11} \\
[e_2, e_8] &= 3e_{12} & [e_2, e_9] &= 6e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_6] &= -e_{11} \\
[e_3, e_7] &= -2e_{12} & [e_3, e_8] &= -3e_{13} \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_5] &= e_{11} \\
[e_4, e_6] &= e_{12} & [e_4, e_7] &= e_{13} \\
[e_4, e_{11}] &= e_{14} & [e_5, e_6] &= 0 \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{11} \\
[e_2, e_8] &= 3e_{12} & [e_2, e_9] &= \alpha_{2,9}^{13} e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_6] &= -e_{11} \\
[e_3, e_7] &= -2e_{12} & [e_3, e_8] &= \alpha_{3,8}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= e_{11} \\
[e_4, e_6] &= e_{12} & [e_4, e_7] &= \alpha_{4,7}^{13} e_{13} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_6] &= \alpha_{5,6}^{13} e_{13} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_8): &\quad -\alpha_{2,9}^{13} - \alpha_{3,8}^{13} + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -\alpha_{3,8}^{13} - \alpha_{4,7}^{13} - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -\alpha_{4,7}^{13} - \alpha_{5,6}^{13} + 1 &&= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_8): &\quad -3\alpha_{3,12}^{14} + \alpha_{3,8}^{13} &&= 0\\
(e_2, e_4, e_7): &\quad -\alpha_{4,11}^{14} + \alpha_{4,7}^{13} &&= 0\\
(e_2, e_5, e_6): &\quad \alpha_{5,6}^{13} &&= 0\\
(e_3, e_4, e_6): &\quad \alpha_{3,12}^{14} + \alpha_{4,11}^{14} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,11}^{14} &= 1\\
\alpha_{6,9}^{14} &= 1\\
\alpha_{3,12}^{14} &= -1\\
\alpha_{5,6}^{13} &= 0\\
\alpha_{2,9}^{13} &= 6\\
\alpha_{7,8}^{14} &= -1\\
\alpha_{5,10}^{14} &= -1\\
\alpha_{4,7}^{13} &= 1\\
\alpha_{3,8}^{13} &= -3
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$\alpha_{4,11}^{14} \to x_1$$

$$\alpha_{6,9}^{14} \to x_2$$

$$\alpha_{3,12}^{14} \to x_3$$

$$\alpha_{5,6}^{13} \to x_4$$

$$\alpha_{2,9}^{13} \to x_5$$

$$\alpha_{7,8}^{14} \to x_6$$

$$\alpha_{5,10}^{14} \to x_7$$

$$\alpha_{4,7}^{13} \to x_8$$

$$\alpha_{3,8}^{13} \to x_9$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_8): &\quad -x_5 - x_9 + 3 &&= 0\\
(e_1, e_3, e_7): &\quad -x_8 - x_9 - 2 &&= 0\\
(e_1, e_4, e_6): &\quad -x_4 - x_8 + 1 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_3 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_3 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_1 - x_7 &&= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_7 &&= 0\\
(e_1, e_6, e_8): &\quad -x_2 - x_6 &&= 0\\
(e_2, e_3, e_8): &\quad -3x_3 + x_9 &&= 0\\
(e_2, e_4, e_7): &\quad -x_1 + x_8 &&= 0\\
(e_2, e_5, e_6): &\quad x_4 &&= 0\\
(e_3, e_4, e_6): &\quad x_1 + x_3 &&= 0
\end{aligned}$$

Groebner basis (9 variables, 9 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 - 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 = 0$$

$$x_5 - 6 = 0$$

$$x_6 + 1 = 0$$

$$x_7 + 1 = 0$$

$$x_8 - 1 = 0$$

$$x_9 + 3 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = 1$$

$$x_3 = -1$$
$$x_4 = 0$$
$$x_5 = 6$$
$$x_6 = -1$$
$$x_7 = -1$$
$$x_8 = 1$$
$$x_9 = -3$$

$\mathfrak{m}_{6B}(4, 14)$

m6B414 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_5] = e_9$$
$$[e_2, e_6] = 2e_{10} \qquad [e_2, e_7] = \frac{5e_{11}}{3}$$
$$[e_2, e_8] = 0 \qquad [e_2, e_9] = 0$$
$$[e_2, e_{13}] = e_{14} \qquad [e_3, e_4] = -e_9$$
$$[e_3, e_5] = -e_{10} \qquad [e_3, e_6] = \frac{e_{11}}{3}$$
$$[e_3, e_7] = \frac{5e_{12}}{3} \qquad [e_3, e_8] = 0$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_5] = -\frac{4e_{11}}{3}$$
$$[e_4, e_6] = -\frac{4e_{12}}{3} \qquad [e_4, e_7] = \frac{5e_{13}}{3}$$
$$[e_4, e_{11}] = e_{14} \qquad [e_5, e_6] = -3e_{13}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_5] = e_9$$
$$[e_2, e_6] = 2e_{10} \qquad [e_2, e_7] = \alpha_{2,7}^{11} e_{11}$$
$$[e_2, e_8] = \alpha_{2,8}^{12} e_{12} \qquad [e_2, e_9] = \alpha_{2,9}^{13} e_{13}$$
$$[e_2, e_{13}] = e_{14} \qquad [e_3, e_4] = -e_9$$
$$[e_3, e_5] = -e_{10} \qquad [e_3, e_6] = \alpha_{3,6}^{11} e_{11}$$
$$[e_3, e_7] = \alpha_{3,7}^{12} e_{12} \qquad [e_3, e_8] = \alpha_{3,8}^{13} e_{13}$$
$$[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} \qquad [e_4, e_5] = \alpha_{4,5}^{11} e_{11}$$
$$[e_4, e_6] = \alpha_{4,6}^{12} e_{12} \qquad [e_4, e_7] = \alpha_{4,7}^{13} e_{13}$$
$$[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \qquad [e_5, e_6] = \alpha_{5,6}^{13} e_{13}$$
$$[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} \qquad [e_6, e_9] = \alpha_{6,9}^{14} e_{14}$$
$$[e_7, e_8] = \alpha_{7,8}^{14} e_{14}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6):&\quad -\alpha_{2,7}^{11} - \alpha_{3,6}^{11} + 2 &= 0\\
(e_1, e_3, e_5):&\quad -\alpha_{3,6}^{11} - \alpha_{4,5}^{11} - 1 &= 0\\
(e_1, e_2, e_7):&\quad \alpha_{2,7}^{11} - \alpha_{2,8}^{12} - \alpha_{3,7}^{12} &= 0\\
(e_1, e_3, e_6):&\quad \alpha_{3,6}^{11} - \alpha_{3,7}^{12} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_4, e_5):&\quad \alpha_{4,5}^{11} - \alpha_{4,6}^{12} &= 0\\
(e_1, e_2, e_8):&\quad \alpha_{2,8}^{12} - \alpha_{2,9}^{13} - \alpha_{3,8}^{13} &= 0\\
(e_1, e_3, e_7):&\quad \alpha_{3,7}^{12} - \alpha_{3,8}^{13} - \alpha_{4,7}^{13} &= 0\\
(e_1, e_4, e_6):&\quad \alpha_{4,6}^{12} - \alpha_{4,7}^{13} - \alpha_{5,6}^{13} &= 0\\
(e_2, e_3, e_4):&\quad -\alpha_{2,9}^{13} &= 0\\
(e_1, e_2, e_{12}):&\quad -\alpha_{3,12}^{14} - 1 &= 0\\
(e_1, e_3, e_{11}):&\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0\\
(e_1, e_4, e_{10}):&\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0\\
(e_1, e_5, e_9):&\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0\\
(e_1, e_6, e_8):&\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0\\
(e_2, e_3, e_8):&\quad -\alpha_{2,8}^{12}\alpha_{3,12}^{14} + \alpha_{3,8}^{13} &= 0\\
(e_2, e_4, e_7):&\quad -\alpha_{2,7}^{11}\alpha_{4,11}^{14} + \alpha_{4,7}^{13} &= 0\\
(e_2, e_5, e_6):&\quad -2\alpha_{5,10}^{14} + \alpha_{5,6}^{13} + \alpha_{6,9}^{14} &= 0\\
(e_3, e_4, e_6):&\quad \alpha_{3,12}^{14}\alpha_{4,6}^{12} - \alpha_{3,6}^{11}\alpha_{4,11}^{14} - \alpha_{6,9}^{14} &= 0
\end{aligned}
$$

Solution 1:

$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{11}_{4,5} = -4/3$$
$$\alpha^{11}_{2,7} = 5/3$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{12}_{2,8} = 0$$
$$\alpha^{13}_{5,6} = -3$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{12}_{3,7} = 5/3$$
$$\alpha^{13}_{3,8} = 0$$
$$\alpha^{13}_{2,9} = 0$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{11}_{3,6} = 1/3$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{13}_{4,7} = 5/3$$
$$\alpha^{12}_{4,6} = -4/3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{14}_{4,11} \to x_1$$
$$\alpha^{11}_{4,5} \to x_2$$
$$\alpha^{11}_{2,7} \to x_3$$
$$\alpha^{14}_{6,9} \to x_4$$
$$\alpha^{12}_{2,8} \to x_5$$
$$\alpha^{13}_{5,6} \to x_6$$
$$\alpha^{14}_{3,12} \to x_7$$
$$\alpha^{12}_{3,7} \to x_8$$
$$\alpha^{13}_{3,8} \to x_9$$
$$\alpha^{13}_{2,9} \to x_{10}$$
$$\alpha^{14}_{7,8} \to x_{11}$$
$$\alpha^{11}_{3,6} \to x_{12}$$

$$\alpha^{14}_{5,10} \to x_{13}$$

$$\alpha^{13}_{4,7} \to x_{14}$$

$$\alpha^{12}_{4,6} \to x_{15}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_{12} - x_3 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_{12} - x_2 - 1 &&= 0\\
(e_1, e_2, e_7): &\quad x_3 - x_5 - x_8 &&= 0\\
(e_1, e_3, e_6): &\quad x_{12} - x_{15} - x_8 &&= 0\\
(e_1, e_4, e_5): &\quad -x_{15} + x_2 &&= 0\\
(e_1, e_2, e_8): &\quad -x_{10} + x_5 - x_9 &&= 0\\
(e_1, e_3, e_7): &\quad -x_{14} + x_8 - x_9 &&= 0\\
(e_1, e_4, e_6): &\quad -x_{14} + x_{15} - x_6 &&= 0\\
(e_2, e_3, e_4): &\quad -x_{10} &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_7 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_7 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_1 - x_{13} &&= 0\\
(e_1, e_5, e_9): &\quad -x_{13} - x_4 &&= 0\\
(e_1, e_6, e_8): &\quad -x_{11} - x_4 &&= 0\\
(e_2, e_3, e_8): &\quad -x_5 x_7 + x_9 &&= 0\\
(e_2, e_4, e_7): &\quad -x_1 x_3 + x_{14} &&= 0\\
(e_2, e_5, e_6): &\quad -2x_{13} + x_4 + x_6 &&= 0\\
(e_3, e_4, e_6): &\quad -x_1 x_{12} + x_{15} x_7 - x_4 &&= 0
\end{aligned}$$

Groebner basis (15 variables, 15 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 + \frac{4}{3} = 0$$

$$x_3 - \frac{5}{3} = 0$$

$$x_4 - 1 = 0$$

$$x_5 = 0$$

$$x_6 + 3 = 0$$

$$x_7 + 1 = 0$$

$$x_8 - \frac{5}{3} = 0$$
$$x_9 = 0$$
$$x_{10} = 0$$
$$x_{11} + 1 = 0$$
$$x_{12} - \frac{1}{3} = 0$$
$$x_{13} + 1 = 0$$
$$x_{14} - \frac{5}{3} = 0$$
$$x_{15} + \frac{4}{3} = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = -4/3$$
$$x_3 = 5/3$$
$$x_4 = 1$$
$$x_5 = 0$$
$$x_6 = -3$$
$$x_7 = -1$$
$$x_8 = 5/3$$
$$x_9 = 0$$
$$x_1 0 = 0$$
$$x_1 1 = -1$$
$$x_1 2 = 1/3$$
$$x_1 3 = -1$$
$$x_1 4 = 5/3$$
$$x_1 5 = -4/3$$

$\mathfrak{m}_{8B}(4,14)$

m8B414 (this line included for string searching purposes)

Original brackets:

$$[e_1,e_2]=e_3 \qquad [e_1,e_3]=e_4$$
$$[e_1,e_4]=e_5 \qquad [e_1,e_5]=e_6$$
$$[e_1,e_6]=e_7 \qquad [e_1,e_7]=e_8$$
$$[e_1,e_8]=e_9 \qquad [e_1,e_9]=e_{10}$$
$$[e_1,e_{10}]=e_{11} \qquad [e_1,e_{11}]=e_{12}$$
$$[e_1,e_{12}]=e_{13} \qquad [e_2,e_3]=e_7$$
$$[e_2,e_4]=e_8 \qquad [e_2,e_5]=\alpha_{2,5}^{9}e_9$$
$$[e_2,e_6]=\alpha_{2,6}^{10}e_{10} \qquad [e_2,e_7]=\alpha_{2,7}^{11}e_{11}$$
$$[e_2,e_8]=\alpha_{2,8}^{12}e_{12} \qquad [e_2,e_9]=\alpha_{2,9}^{13}e_{13}$$
$$[e_2,e_{13}]=e_{14} \qquad [e_3,e_4]=\alpha_{3,4}^{9}e_9$$
$$[e_3,e_5]=\alpha_{3,5}^{10}e_{10} \qquad [e_3,e_6]=\alpha_{3,6}^{11}e_{11}$$
$$[e_3,e_7]=\alpha_{3,7}^{12}e_{12} \qquad [e_3,e_8]=\alpha_{3,8}^{13}e_{13}$$
$$[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} \qquad [e_4,e_5]=\alpha_{4,5}^{11}e_{11}$$
$$[e_4,e_6]=\alpha_{4,6}^{12}e_{12} \qquad [e_4,e_7]=\alpha_{4,7}^{13}e_{13}$$
$$[e_4,e_{11}]=\alpha_{4,11}^{14}e_{14} \qquad [e_5,e_6]=\alpha_{5,6}^{13}e_{13}$$
$$[e_5,e_{10}]=\alpha_{5,10}^{14}e_{14} \qquad [e_6,e_9]=\alpha_{6,9}^{14}e_{14}$$
$$[e_7,e_8]=\alpha_{7,8}^{14}e_{14}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1,e_2,e_4): &\quad -\alpha^{9}_{2,5}-\alpha^{9}_{3,4}+1 &&=0\\
(e_1,e_2,e_5): &\quad \alpha^{9}_{2,5}-\alpha^{10}_{2,6}-\alpha^{10}_{3,5} &&=0\\
(e_1,e_3,e_4): &\quad \alpha^{9}_{3,4}-\alpha^{10}_{3,5} &&=0\\
(e_1,e_2,e_6): &\quad \alpha^{10}_{2,6}-\alpha^{11}_{2,7}-\alpha^{11}_{3,6} &&=0\\
(e_1,e_3,e_5): &\quad \alpha^{10}_{3,5}-\alpha^{11}_{3,6}-\alpha^{11}_{4,5} &&=0\\
(e_1,e_2,e_7): &\quad \alpha^{11}_{2,7}-\alpha^{12}_{2,8}-\alpha^{12}_{3,7} &&=0\\
(e_1,e_3,e_6): &\quad \alpha^{11}_{3,6}-\alpha^{12}_{3,7}-\alpha^{12}_{4,6} &&=0\\
(e_1,e_4,e_5): &\quad \alpha^{11}_{4,5}-\alpha^{12}_{4,6} &&=0\\
(e_1,e_2,e_8): &\quad \alpha^{12}_{2,8}-\alpha^{13}_{2,9}-\alpha^{13}_{3,8} &&=0\\
(e_1,e_3,e_7): &\quad \alpha^{12}_{3,7}-\alpha^{13}_{3,8}-\alpha^{13}_{4,7} &&=0\\
(e_1,e_4,e_6): &\quad \alpha^{12}_{4,6}-\alpha^{13}_{4,7}-\alpha^{13}_{5,6} &&=0\\
(e_2,e_3,e_4): &\quad \alpha^{13}_{2,9}\alpha^{9}_{3,4}-\alpha^{13}_{3,8}+\alpha^{13}_{4,7} &&=0\\
(e_1,e_2,e_{12}): &\quad -\alpha^{14}_{3,12}-1 &&=0\\
(e_1,e_3,e_{11}): &\quad -\alpha^{14}_{3,12}-\alpha^{14}_{4,11} &&=0\\
(e_1,e_4,e_{10}): &\quad -\alpha^{14}_{4,11}-\alpha^{14}_{5,10} &&=0\\
(e_1,e_5,e_9): &\quad -\alpha^{14}_{5,10}-\alpha^{14}_{6,9} &&=0\\
(e_1,e_6,e_8): &\quad -\alpha^{14}_{6,9}-\alpha^{14}_{7,8} &&=0\\
(e_2,e_3,e_8): &\quad -\alpha^{12}_{2,8}\alpha^{14}_{3,12}+\alpha^{13}_{3,8}-\alpha^{14}_{7,8} &&=0\\
(e_2,e_4,e_7): &\quad -\alpha^{11}_{2,7}\alpha^{14}_{4,11}+\alpha^{13}_{4,7}+\alpha^{14}_{7,8} &&=0\\
(e_2,e_5,e_6): &\quad \alpha^{9}_{2,5}\alpha^{14}_{6,9}-\alpha^{10}_{2,6}\alpha^{14}_{5,10}+\alpha^{13}_{5,6} &&=0\\
(e_3,e_4,e_6): &\quad \alpha^{14}_{3,12}\alpha^{12}_{4,6}+\alpha^{9}_{3,4}\alpha^{14}_{6,9}-\alpha^{11}_{3,6}\alpha^{14}_{4,11} &&=0
\end{aligned}$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha^{10}_{2,6}\to x_1$$

$$\alpha^{14}_{4,11}\to x_2$$

$$\alpha^{9}_{2,5}\to x_3$$

$$\alpha^{9}_{3,4}\to x_4$$

$$\alpha^{11}_{4,5}\to x_5$$

$$\alpha^{11}_{2,7}\to x_6$$

$$\alpha_{6,9}^{14} \to x_7$$

$$\alpha_{2,8}^{12} \to x_8$$

$$\alpha_{5,6}^{13} \to x_9$$

$$\alpha_{3,12}^{14} \to x_{10}$$

$$\alpha_{3,7}^{12} \to x_{11}$$

$$\alpha_{3,5}^{10} \to x_{12}$$

$$\alpha_{3,8}^{13} \to x_{13}$$

$$\alpha_{2,9}^{13} \to x_{14}$$

$$\alpha_{7,8}^{14} \to x_{15}$$

$$\alpha_{3,6}^{11} \to x_{16}$$

$$\alpha_{5,10}^{14} \to x_{17}$$

$$\alpha_{4,7}^{13} \to x_{18}$$

$$\alpha_{4,6}^{12} \to x_{19}$$

Jacobi Tests

$$
\begin{aligned}
(e_1,e_2,e_4):&\quad -x_3-x_4+1 &&=0\\
(e_1,e_2,e_5):&\quad -x_1-x_{12}+x_3 &&=0\\
(e_1,e_3,e_4):&\quad -x_{12}+x_4 &&=0\\
(e_1,e_2,e_6):&\quad x_1-x_{16}-x_6 &&=0\\
(e_1,e_3,e_5):&\quad x_{12}-x_{16}-x_5 &&=0\\
(e_1,e_2,e_7):&\quad -x_{11}+x_6-x_8 &&=0\\
(e_1,e_3,e_6):&\quad -x_{11}+x_{16}-x_{19} &&=0\\
(e_1,e_4,e_5):&\quad -x_{19}+x_5 &&=0\\
(e_1,e_2,e_8):&\quad -x_{13}-x_{14}+x_8 &&=0\\
(e_1,e_3,e_7):&\quad x_{11}-x_{13}-x_{18} &&=0\\
(e_1,e_4,e_6):&\quad -x_{18}+x_{19}-x_9 &&=0\\
(e_2,e_3,e_4):&\quad -x_{13}+x_{14}x_4+x_{18} &&=0\\
(e_1,e_2,e_{12}):&\quad -x_{10}-1 &&=0\\
(e_1,e_3,e_{11}):&\quad -x_{10}-x_2 &&=0\\
(e_1,e_4,e_{10}):&\quad -x_{17}-x_2 &&=0\\
(e_1,e_5,e_9):&\quad -x_{17}-x_7 &&=0\\
(e_1,e_6,e_8):&\quad -x_{15}-x_7 &&=0\\
(e_2,e_3,e_8):&\quad -x_{10}x_8+x_{13}-x_{15} &&=0\\
(e_2,e_4,e_7):&\quad x_{15}+x_{18}-x_2x_6 &&=0\\
(e_2,e_5,e_6):&\quad -x_1x_{17}+x_3x_7+x_9 &&=0\\
(e_3,e_4,e_6):&\quad x_{10}x_{19}-x_{16}x_2+x_4x_7 &&=0
\end{aligned}
$$

Groebner basis (19 variables, 17 linear, 1 nonlinear)

$$x_1-\frac{2x_{18}}{3}+\frac{2x_{19}}{3}+\frac{1}{3}=0$$
$$x_2-1=0$$
$$-\frac{x_{18}}{3}+\frac{x_{19}}{3}+x_3-\frac{1}{3}=0$$
$$\frac{x_{18}}{3}-\frac{x_{19}}{3}+x_4-\frac{2}{3}=0$$
$$-x_{19}+x_5=0$$
$$-x_{18}+x_6+1=0$$
$$x_7-1=0$$
$$-\frac{4x_{18}}{3}-\frac{5x_{19}}{3}+x_8+\frac{5}{3}=0$$

$$x_{18} - x_{19} + x_9 = 0$$
$$x_{10} + 1 = 0$$
$$x_{11} + \frac{x_{18}}{3} + \frac{5x_{19}}{3} - \frac{2}{3} = 0$$
$$x_{12} + \frac{x_{18}}{3} - \frac{x_{19}}{3} - \frac{2}{3} = 0$$
$$x_{13} + \frac{4x_{18}}{3} + \frac{5x_{19}}{3} - \frac{2}{3} = 0$$
$$x_{14} - \frac{8x_{18}}{3} - \frac{10x_{19}}{3} + \frac{7}{3} = 0$$
$$x_{15} + 1 = 0$$
$$x_{16} + \frac{x_{18}}{3} + \frac{2x_{19}}{3} - \frac{2}{3} = 0$$
$$x_{17} + 1 = 0$$
$$x_{18}^2 + \frac{x_{18}x_{19}}{4} - \frac{11x_{18}}{2} - \frac{5x_{19}^2}{4} - \frac{7x_{19}}{2} + \frac{5}{2} = 0$$

$\mathfrak{m}_{3B}(5, 14)$

m3B514 (this line included for string searching purposes)

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_7] = e_{12}$$
$$[e_2, e_8] = 3e_{13} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_6] = -e_{12} \qquad [e_3, e_7] = -2e_{13}$$
$$[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} \qquad [e_4, e_5] = e_{12}$$
$$[e_4, e_6] = e_{13} \qquad [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14}$$
$$[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} \qquad [e_6, e_9] = \alpha_{6,9}^{14} e_{14}$$
$$[e_7, e_8] = \alpha_{7,8}^{14} e_{14}$$

Non-trivial Jacobi Tests:

$$\begin{aligned}
(e_1, e_2, e_{12}):&\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}):&\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}):&\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9):&\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8):&\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_7):&\quad -\alpha_{3,12}^{14} - 2 &&= 0\\
(e_2, e_4, e_6):&\quad \text{no solutions} &&\\
(e_3, e_4, e_5):&\quad \alpha_{3,12}^{14} &&= 0
\end{aligned}$$

There are no solutions.

$\mathfrak{m}_{5B}(5, 14)$

m5B514 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4\\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6\\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8\\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10}\\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12}\\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{10}\\
[e_2, e_6] &= 2e_{11} & [e_2, e_7] &= 5e_{12}\\
[e_2, e_8] &= 10e_{13} & [e_2, e_{13}] &= e_{14}\\
[e_3, e_4] &= -e_{10} & [e_3, e_5] &= -e_{11}\\
[e_3, e_6] &= -3e_{12} & [e_3, e_7] &= -5e_{13}\\
[e_3, e_{12}] &= -e_{14} & [e_4, e_5] &= 2e_{12}\\
[e_4, e_6] &= 2e_{13} & [e_4, e_{11}] &= e_{14}\\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14}\\
[e_7, e_8] &= -e_{14} & &
\end{aligned}$$

Original brackets:

$$
\begin{array}{ll}
[e_1, e_2] = e_3 & [e_1, e_3] = e_4 \\
[e_1, e_4] = e_5 & [e_1, e_5] = e_6 \\
[e_1, e_6] = e_7 & [e_1, e_7] = e_8 \\
[e_1, e_8] = e_9 & [e_1, e_9] = e_{10} \\
[e_1, e_{10}] = e_{11} & [e_1, e_{11}] = e_{12} \\
[e_1, e_{12}] = e_{13} & [e_2, e_5] = e_{10} \\
[e_2, e_6] = 2e_{11} & [e_2, e_7] = \alpha_{2,7}^{12} e_{12} \\
[e_2, e_8] = \alpha_{2,8}^{13} e_{13} & [e_2, e_{13}] = e_{14} \\
[e_3, e_4] = -e_{10} & [e_3, e_5] = -e_{11} \\
[e_3, e_6] = \alpha_{3,6}^{12} e_{12} & [e_3, e_7] = \alpha_{3,7}^{13} e_{13} \\
[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} & [e_4, e_5] = \alpha_{4,5}^{12} e_{12} \\
[e_4, e_6] = \alpha_{4,6}^{13} e_{13} & [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} & [e_6, e_9] = \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] = \alpha_{7,8}^{14} e_{14} &
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{rll}
(e_1, e_2, e_6): & -\alpha_{2,7}^{12} - \alpha_{3,6}^{12} + 2 & = 0 \\
(e_1, e_3, e_5): & -\alpha_{3,6}^{12} - \alpha_{4,5}^{12} - 1 & = 0 \\
(e_1, e_2, e_7): & \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} & = 0 \\
(e_1, e_3, e_6): & \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} & = 0 \\
(e_1, e_4, e_5): & \alpha_{4,5}^{12} - \alpha_{4,6}^{13} & = 0 \\
(e_1, e_2, e_{12}): & -\alpha_{3,12}^{14} - 1 & = 0 \\
(e_1, e_3, e_{11}): & -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} & = 0 \\
(e_1, e_4, e_{10}): & -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} & = 0 \\
(e_1, e_5, e_9): & -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} & = 0 \\
(e_1, e_6, e_8): & -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} & = 0 \\
(e_2, e_3, e_7): & -\alpha_{2,7}^{12}\alpha_{3,12}^{14} + \alpha_{3,7}^{13} & = 0 \\
(e_2, e_4, e_6): & -2\alpha_{4,11}^{14} + \alpha_{4,6}^{13} & = 0 \\
(e_3, e_4, e_5): & \alpha_{3,12}^{14}\alpha_{4,5}^{12} + \alpha_{4,11}^{14} - \alpha_{5,10}^{14} & = 0
\end{array}
$$

Solution 1:

$$\alpha^{13}_{4,6} = 2$$
$$\alpha^{12}_{4,5} = 2$$
$$\alpha^{14}_{4,11} = 1$$
$$\alpha^{13}_{2,8} = 10$$
$$\alpha^{13}_{3,7} = -5$$
$$\alpha^{14}_{3,12} = -1$$
$$\alpha^{12}_{2,7} = 5$$
$$\alpha^{14}_{6,9} = 1$$
$$\alpha^{14}_{7,8} = -1$$
$$\alpha^{14}_{5,10} = -1$$
$$\alpha^{12}_{3,6} = -3$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{13}_{4,6} \to x_1$$
$$\alpha^{12}_{4,5} \to x_2$$
$$\alpha^{14}_{4,11} \to x_3$$
$$\alpha^{13}_{2,8} \to x_4$$
$$\alpha^{13}_{3,7} \to x_5$$
$$\alpha^{14}_{3,12} \to x_6$$
$$\alpha^{12}_{2,7} \to x_7$$
$$\alpha^{14}_{6,9} \to x_8$$
$$\alpha^{14}_{7,8} \to x_9$$
$$\alpha^{14}_{5,10} \to x_{10}$$
$$\alpha^{12}_{3,6} \to x_{11}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): \quad & -x_{11} - x_7 + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -x_{11} - x_2 - 1 && = 0 \\
(e_1, e_2, e_7): \quad & -x_4 - x_5 + x_7 && = 0 \\
(e_1, e_3, e_6): \quad & -x_1 + x_{11} - x_5 && = 0 \\
(e_1, e_4, e_5): \quad & -x_1 + x_2 && = 0 \\
(e_1, e_2, e_{12}): \quad & -x_6 - 1 && = 0 \\
(e_1, e_3, e_{11}): \quad & -x_3 - x_6 && = 0 \\
(e_1, e_4, e_{10}): \quad & -x_{10} - x_3 && = 0 \\
(e_1, e_5, e_9): \quad & -x_{10} - x_8 && = 0 \\
(e_1, e_6, e_8): \quad & -x_8 - x_9 && = 0 \\
(e_2, e_3, e_7): \quad & x_5 - x_6 x_7 && = 0 \\
(e_2, e_4, e_6): \quad & x_1 - 2x_3 && = 0 \\
(e_3, e_4, e_5): \quad & -x_{10} + x_2 x_6 + x_3 && = 0
\end{aligned}$$

Groebner basis (11 variables, 11 linear, 0 nonlinear)

$$x_1 - 2 = 0$$
$$x_2 - 2 = 0$$
$$x_3 - 1 = 0$$
$$x_4 - 10 = 0$$
$$x_5 + 5 = 0$$
$$x_6 + 1 = 0$$
$$x_7 - 5 = 0$$
$$x_8 - 1 = 0$$
$$x_9 + 1 = 0$$
$$x_{10} + 1 = 0$$
$$x_{11} + 3 = 0$$

Solution 1:

$$x_1 = 2$$
$$x_2 = 2$$
$$x_3 = 1$$
$$x_4 = 10$$

$$x_5 = -5$$
$$x_6 = -1$$
$$x_7 = 5$$
$$x_8 = 1$$
$$x_9 = -1$$
$$x_10 = -1$$
$$x_11 = -3$$

$\mathfrak{m}_{7B}(5, 14)$

m7B514 (this line included for string searching purposes)
Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_3] = e_8$$
$$[e_2, e_4] = e_9 \qquad [e_2, e_5] = \alpha_{2,5}^{10} e_{10}$$
$$[e_2, e_6] = \alpha_{2,6}^{11} e_{11} \qquad [e_2, e_7] = \alpha_{2,7}^{12} e_{12}$$
$$[e_2, e_8] = \alpha_{2,8}^{13} e_{13} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_4] = \alpha_{3,4}^{10} e_{10} \qquad [e_3, e_5] = \alpha_{3,5}^{11} e_{11}$$
$$[e_3, e_6] = \alpha_{3,6}^{12} e_{12} \qquad [e_3, e_7] = \alpha_{3,7}^{13} e_{13}$$
$$[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} \qquad [e_4, e_5] = \alpha_{4,5}^{12} e_{12}$$
$$[e_4, e_6] = \alpha_{4,6}^{13} e_{13} \qquad [e_4, e_{11}] = \alpha_{4,11}^{14} e_{14}$$
$$[e_5, e_{10}] = \alpha_{5,10}^{14} e_{14} \qquad [e_6, e_9] = \alpha_{6,9}^{14} e_{14}$$
$$[e_7, e_8] = \alpha_{7,8}^{14} e_{14}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4): &\quad -\alpha_{2,5}^{10} - \alpha_{3,4}^{10} + 1 &&= 0\\
(e_1, e_2, e_5): &\quad \alpha_{2,5}^{10} - \alpha_{2,6}^{11} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_3, e_4): &\quad \alpha_{3,4}^{10} - \alpha_{3,5}^{11} &&= 0\\
(e_1, e_2, e_6): &\quad \alpha_{2,6}^{11} - \alpha_{2,7}^{12} - \alpha_{3,6}^{12} &&= 0\\
(e_1, e_3, e_5): &\quad \alpha_{3,5}^{11} - \alpha_{3,6}^{12} - \alpha_{4,5}^{12} &&= 0\\
(e_1, e_2, e_7): &\quad \alpha_{2,7}^{12} - \alpha_{2,8}^{13} - \alpha_{3,7}^{13} &&= 0\\
(e_1, e_3, e_6): &\quad \alpha_{3,6}^{12} - \alpha_{3,7}^{13} - \alpha_{4,6}^{13} &&= 0\\
(e_1, e_4, e_5): &\quad \alpha_{4,5}^{12} - \alpha_{4,6}^{13} &&= 0\\
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0\\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0\\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0\\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0\\
(e_2, e_3, e_7): &\quad -\alpha_{2,7}^{12}\alpha_{3,12}^{14} + \alpha_{3,7}^{13} + \alpha_{7,8}^{14} &&= 0\\
(e_2, e_4, e_6): &\quad -\alpha_{2,6}^{11}\alpha_{4,11}^{14} + \alpha_{4,6}^{13} + \alpha_{6,9}^{14} &&= 0\\
(e_3, e_4, e_5): &\quad \alpha_{3,12}^{14}\alpha_{4,5}^{12} + \alpha_{3,4}^{10}\alpha_{5,10}^{14} - \alpha_{3,5}^{11}\alpha_{4,11}^{14} &&= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{4,6}^{13} \to x_1$$

$$\alpha_{4,5}^{12} \to x_2$$

$$\alpha_{4,11}^{14} \to x_3$$

$$\alpha_{3,5}^{11} \to x_4$$

$$\alpha_{2,8}^{13} \to x_5$$

$$\alpha_{2,5}^{10} \to x_6$$

$$\alpha_{3,7}^{13} \to x_7$$

$$\alpha_{3,6}^{12} \to x_8$$

$$\alpha_{2,7}^{12} \to x_9$$

$$\alpha_{3,12}^{14} \to x_{10}$$

$$\alpha_{6,9}^{14} \to x_{11}$$

$$\alpha_{3,4}^{10} \to x_{12}$$

$$\alpha_{7,8}^{14} \to x_{13}$$

$$\alpha_{5,10}^{14} \to x_{14}$$

$$\alpha_{2,6}^{11} \to x_{15}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{12} - x_6 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_{15} - x_4 + x_6 &&= 0\\
(e_1, e_3, e_4): &\quad x_{12} - x_4 &&= 0\\
(e_1, e_2, e_6): &\quad x_{15} - x_8 - x_9 &&= 0\\
(e_1, e_3, e_5): &\quad -x_2 + x_4 - x_8 &&= 0\\
(e_1, e_2, e_7): &\quad -x_5 - x_7 + x_9 &&= 0\\
(e_1, e_3, e_6): &\quad -x_1 - x_7 + x_8 &&= 0\\
(e_1, e_4, e_5): &\quad -x_1 + x_2 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_{10} - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_{10} - x_3 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_{14} - x_3 &&= 0\\
(e_1, e_5, e_9): &\quad -x_{11} - x_{14} &&= 0\\
(e_1, e_6, e_8): &\quad -x_{11} - x_{13} &&= 0\\
(e_2, e_3, e_7): &\quad -x_{10}x_9 + x_{13} + x_7 &&= 0\\
(e_2, e_4, e_6): &\quad x_1 + x_{11} - x_{15}x_3 &&= 0\\
(e_3, e_4, e_5): &\quad x_{10}x_2 + x_{12}x_{14} - x_3x_4 &&= 0
\end{aligned}$$

Groebner basis (15 variables, 14 linear, 0 nonlinear)

$$x_1 - x_{15} + 1 = 0$$

$$-x_{15} + x_2 + 1 = 0$$

$$x_3 - 1 = 0$$

$$\frac{x_{15}}{2} + x_4 - \frac{1}{2} = 0$$

$$-5x_{15} + x_5 + 4 = 0$$

$$-\frac{x_{15}}{2} + x_6 - \frac{1}{2} = 0$$

$$\frac{5x_{15}}{2} + x_7 - \frac{5}{2} = 0$$

$$\frac{3x_{15}}{2} + x_8 - \frac{3}{2} = 0$$
$$-\frac{5x_{15}}{2} + x_9 + \frac{3}{2} = 0$$
$$x_{10} + 1 = 0$$
$$x_{11} - 1 = 0$$
$$x_{12} + \frac{x_{15}}{2} - \frac{1}{2} = 0$$
$$x_{13} + 1 = 0$$
$$x_{14} + 1 = 0$$

$\mathfrak{m}_{2B}(6, 14)$

m2B614 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_7] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_6] &= -e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_5] &= e_{13} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} \\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0 \\
(e_2, e_3, e_6): &\quad \text{no solutions} & \\
(e_2, e_4, e_5): &\quad \text{no solutions} &
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(6, 14)$

m4B614 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_5] = e_{11}$$
$$[e_2, e_6] = 2e_{12} \qquad [e_2, e_7] = 4e_{13}$$
$$[e_2, e_{13}] = e_{14} \qquad [e_3, e_4] = -e_{11}$$
$$[e_3, e_5] = -e_{12} \qquad [e_3, e_6] = -2e_{13}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_5] = e_{13}$$
$$[e_4, e_{11}] = e_{14} \qquad [e_5, e_{10}] = -e_{14}$$
$$[e_6, e_9] = e_{14} \qquad [e_7, e_8] = -e_{14}$$

Original brackets:

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_5] = e_{11}$$
$$[e_2, e_6] = 2e_{12} \qquad [e_2, e_7] = \alpha_{2,7}^{13} e_{13}$$
$$[e_2, e_{13}] = e_{14} \qquad [e_3, e_4] = -e_{11}$$
$$[e_3, e_5] = -e_{12} \qquad [e_3, e_6] = \alpha_{3,6}^{13} e_{13}$$
$$[e_3, e_{12}] = \alpha_{3,12}^{14} e_{14} \qquad [e_4, e_5] = \alpha_{4,5}^{13} e_{13}$$
$$[e_4, e_{11}] = \alpha_{4,11}^{14} e_{14} \qquad [e_5, e_{10}] = \alpha_{5,10}^{14} e_{14}$$
$$[e_6, e_9] = \alpha_{6,9}^{14} e_{14} \qquad [e_7, e_8] = \alpha_{7,8}^{14} e_{14}$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_6): \quad & -\alpha_{2,7}^{13} - \alpha_{3,6}^{13} + 2 && = 0 \\
(e_1, e_3, e_5): \quad & -\alpha_{3,6}^{13} - \alpha_{4,5}^{13} - 1 && = 0 \\
(e_1, e_2, e_{12}): \quad & -\alpha_{3,12}^{14} - 1 && = 0 \\
(e_1, e_3, e_{11}): \quad & -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} && = 0 \\
(e_1, e_4, e_{10}): \quad & -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} && = 0 \\
(e_1, e_5, e_9): \quad & -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} && = 0 \\
(e_1, e_6, e_8): \quad & -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} && = 0 \\
(e_2, e_3, e_6): \quad & -2\alpha_{3,12}^{14} + \alpha_{3,6}^{13} && = 0 \\
(e_2, e_4, e_5): \quad & -\alpha_{4,11}^{14} + \alpha_{4,5}^{13} && = 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,11}^{14} &= 1 \\
\alpha_{3,6}^{13} &= -2 \\
\alpha_{2,7}^{13} &= 4 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{3,12}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1 \\
\alpha_{7,8}^{14} &= -1 \\
\alpha_{4,5}^{13} &= 1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,11}^{14} &\to x_1 \\
\alpha_{3,6}^{13} &\to x_2 \\
\alpha_{2,7}^{13} &\to x_3 \\
\alpha_{6,9}^{14} &\to x_4 \\
\alpha_{3,12}^{14} &\to x_5 \\
\alpha_{5,10}^{14} &\to x_6 \\
\alpha_{7,8}^{14} &\to x_7
\end{aligned}
$$

$$\alpha_{4,5}^{13} \to x_8$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_6): &\quad -x_2 - x_3 + 2 &&= 0\\
(e_1, e_3, e_5): &\quad -x_2 - x_8 - 1 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_5 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_5 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_1 - x_6 &&= 0\\
(e_1, e_5, e_9): &\quad -x_4 - x_6 &&= 0\\
(e_1, e_6, e_8): &\quad -x_4 - x_7 &&= 0\\
(e_2, e_3, e_6): &\quad x_2 - 2x_5 &&= 0\\
(e_2, e_4, e_5): &\quad -x_1 + x_8 &&= 0
\end{aligned}$$

Groebner basis (8 variables, 8 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 + 2 = 0$$

$$x_3 - 4 = 0$$

$$x_4 - 1 = 0$$

$$x_5 + 1 = 0$$

$$x_6 + 1 = 0$$

$$x_7 + 1 = 0$$

$$x_8 - 1 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = -2$$

$$x_3 = 4$$

$$x_4 = 1$$

$$x_5 = -1$$

$$x_6 = -1$$

$$x_7 = -1$$

$$x_8 = 1$$

$\mathfrak{m}_{6B}(6,14)$

m6B614 (this line included for string searching purposes)

Original brackets:

$$
\begin{array}{ll}
[e_1,e_2]=e_3 & [e_1,e_3]=e_4 \\
[e_1,e_4]=e_5 & [e_1,e_5]=e_6 \\
[e_1,e_6]=e_7 & [e_1,e_7]=e_8 \\
[e_1,e_8]=e_9 & [e_1,e_9]=e_{10} \\
[e_1,e_{10}]=e_{11} & [e_1,e_{11}]=e_{12} \\
[e_1,e_{12}]=e_{13} & [e_2,e_3]=e_9 \\
[e_2,e_4]=e_{10} & [e_2,e_5]=\alpha_{2,5}^{11}e_{11} \\
[e_2,e_6]=\alpha_{2,6}^{12}e_{12} & [e_2,e_7]=\alpha_{2,7}^{13}e_{13} \\
[e_2,e_{13}]=e_{14} & [e_3,e_4]=\alpha_{3,4}^{11}e_{11} \\
[e_3,e_5]=\alpha_{3,5}^{12}e_{12} & [e_3,e_6]=\alpha_{3,6}^{13}e_{13} \\
[e_3,e_{12}]=\alpha_{3,12}^{14}e_{14} & [e_4,e_5]=\alpha_{4,5}^{13}e_{13} \\
[e_4,e_{11}]=\alpha_{4,11}^{14}e_{14} & [e_5,e_{10}]=\alpha_{5,10}^{14}e_{14} \\
[e_6,e_9]=\alpha_{6,9}^{14}e_{14} & [e_7,e_8]=\alpha_{7,8}^{14}e_{14}
\end{array}
$$

Non-trivial Jacobi Tests:

$$
\begin{array}{lll}
(e_1,e_2,e_4): & -\alpha_{2,5}^{11}-\alpha_{3,4}^{11}+1 & =0 \\
(e_1,e_2,e_5): & \alpha_{2,5}^{11}-\alpha_{2,6}^{12}-\alpha_{3,5}^{12} & =0 \\
(e_1,e_3,e_4): & \alpha_{3,4}^{11}-\alpha_{3,5}^{12} & =0 \\
(e_1,e_2,e_6): & \alpha_{2,6}^{12}-\alpha_{2,7}^{13}-\alpha_{3,6}^{13} & =0 \\
(e_1,e_3,e_5): & \alpha_{3,5}^{12}-\alpha_{3,6}^{13}-\alpha_{4,5}^{13} & =0 \\
(e_1,e_2,e_{12}): & -\alpha_{3,12}^{14}-1 & =0 \\
(e_1,e_3,e_{11}): & -\alpha_{3,12}^{14}-\alpha_{4,11}^{14} & =0 \\
(e_1,e_4,e_{10}): & -\alpha_{4,11}^{14}-\alpha_{5,10}^{14} & =0 \\
(e_1,e_5,e_9): & -\alpha_{5,10}^{14}-\alpha_{6,9}^{14} & =0 \\
(e_1,e_6,e_8): & -\alpha_{6,9}^{14}-\alpha_{7,8}^{14} & =0 \\
(e_2,e_3,e_6): & -\alpha_{2,6}^{12}\alpha_{3,12}^{14}+\alpha_{3,6}^{13}+\alpha_{6,9}^{14} & =0 \\
(e_2,e_4,e_5): & -\alpha_{2,5}^{11}\alpha_{4,11}^{14}+\alpha_{4,5}^{13}+\alpha_{5,10}^{14} & =0
\end{array}
$$

Infinite number of solutions.

How the solution(s) were or were not found:

Change variables

$$\alpha_{2,6}^{12} \to x_1$$
$$\alpha_{4,5}^{13} \to x_2$$
$$\alpha_{3,6}^{13} \to x_3$$
$$\alpha_{4,11}^{14} \to x_4$$
$$\alpha_{2,7}^{13} \to x_5$$
$$\alpha_{6,9}^{14} \to x_6$$
$$\alpha_{3,4}^{11} \to x_7$$
$$\alpha_{3,12}^{14} \to x_8$$
$$\alpha_{5,10}^{14} \to x_9$$
$$\alpha_{2,5}^{11} \to x_{10}$$
$$\alpha_{7,8}^{14} \to x_{11}$$
$$\alpha_{3,5}^{12} \to x_{12}$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_{10} - x_7 + 1 &&= 0\\
(e_1, e_2, e_5): &\quad -x_1 + x_{10} - x_{12} &&= 0\\
(e_1, e_3, e_4): &\quad -x_{12} + x_7 &&= 0\\
(e_1, e_2, e_6): &\quad x_1 - x_3 - x_5 &&= 0\\
(e_1, e_3, e_5): &\quad x_{12} - x_2 - x_3 &&= 0\\
(e_1, e_2, e_{12}): &\quad -x_8 - 1 &&= 0\\
(e_1, e_3, e_{11}): &\quad -x_4 - x_8 &&= 0\\
(e_1, e_4, e_{10}): &\quad -x_4 - x_9 &&= 0\\
(e_1, e_5, e_9): &\quad -x_6 - x_9 &&= 0\\
(e_1, e_6, e_8): &\quad -x_{11} - x_6 &&= 0\\
(e_2, e_3, e_6): &\quad -x_1 x_8 + x_3 + x_6 &&= 0\\
(e_2, e_4, e_5): &\quad -x_{10} x_4 + x_2 + x_9 &&= 0
\end{aligned}$$

Groebner basis (12 variables, 11 linear, 0 nonlinear)

$$x_1 + 2x_{12} - 1 = 0$$
$$x_{12} + x_2 - 2 = 0$$

$$-2x_{12} + x_3 + 2 = 0$$
$$x_4 - 1 = 0$$
$$4x_{12} + x_5 - 3 = 0$$
$$x_6 - 1 = 0$$
$$-x_{12} + x_7 = 0$$
$$x_8 + 1 = 0$$
$$x_9 + 1 = 0$$
$$x_{10} + x_{12} - 1 = 0$$
$$x_{11} + 1 = 0$$

$\mathfrak{m}_{3B}(7, 14)$

m3B714 (this line included for string searching purposes)

Solution 1

$$[e_1, e_2] = e_3 \qquad [e_1, e_3] = e_4$$
$$[e_1, e_4] = e_5 \qquad [e_1, e_5] = e_6$$
$$[e_1, e_6] = e_7 \qquad [e_1, e_7] = e_8$$
$$[e_1, e_8] = e_9 \qquad [e_1, e_9] = e_{10}$$
$$[e_1, e_{10}] = e_{11} \qquad [e_1, e_{11}] = e_{12}$$
$$[e_1, e_{12}] = e_{13} \qquad [e_2, e_5] = e_{12}$$
$$[e_2, e_6] = 2e_{13} \qquad [e_2, e_{13}] = e_{14}$$
$$[e_3, e_4] = -e_{12} \qquad [e_3, e_5] = -e_{13}$$
$$[e_3, e_{12}] = -e_{14} \qquad [e_4, e_{11}] = e_{14}$$
$$[e_5, e_{10}] = -e_{14} \qquad [e_6, e_9] = e_{14}$$
$$[e_7, e_8] = -e_{14}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{12} \\
[e_2, e_6] &= 2e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_4] &= -e_{12} & [e_3, e_5] &= -e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}):& \quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}):& \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}):& \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9):& \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8):& \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0 \\
(e_2, e_3, e_5):& \quad -\alpha_{3,12}^{14} - 1 &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,11}^{14} &= 1 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{3,12}^{14} &= -1 \\
\alpha_{7,8}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,11}^{14} &\to x_1 \\
\alpha_{6,9}^{14} &\to x_2
\end{aligned}
$$

$$\alpha_{3,12}^{14} \to x_3$$

$$\alpha_{7,8}^{14} \to x_4$$

$$\alpha_{5,10}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}): &\quad -x_3 - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_3 &= 0\\
(e_1, e_4, e_{10}): &\quad -x_1 - x_5 &= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_5 &= 0\\
(e_1, e_6, e_8): &\quad -x_2 - x_4 &= 0\\
(e_2, e_3, e_5): &\quad -x_3 - 1 &= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 - 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 + 1 = 0$$

$$x_5 + 1 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = 1$$

$$x_3 = -1$$

$$x_4 = -1$$

$$x_5 = -1$$

$\mathfrak{m}_{5B}(7, 14)$

m5B714 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{10} \\
[e_2, e_4] &= e_{11} & [e_2, e_5] &= \alpha_{2,5}^{12} e_{12} \\
[e_2, e_6] &= \alpha_{2,6}^{13} e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_4] &= \alpha_{3,4}^{12} e_{12} & [e_3, e_5] &= \alpha_{3,5}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):&\quad -\alpha_{2,5}^{12} - \alpha_{3,4}^{12} + 1 &= 0 \\
(e_1, e_2, e_5):&\quad \alpha_{2,5}^{12} - \alpha_{2,6}^{13} - \alpha_{3,5}^{13} &= 0 \\
(e_1, e_3, e_4):&\quad \alpha_{3,4}^{12} - \alpha_{3,5}^{13} &= 0 \\
(e_1, e_2, e_{12}):&\quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}):&\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}):&\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9):&\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8):&\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0 \\
(e_2, e_3, e_5):&\quad -\alpha_{2,5}^{12}\alpha_{3,12}^{14} + \alpha_{3,5}^{13} + \alpha_{5,10}^{14} &= 0
\end{aligned}
$$

Infinite number of solutions.
How the solution(s) were or were not found:
Change variables

$$\alpha_{2,6}^{13} \to x_1$$

$$\alpha_{4,11}^{14} \to x_2$$

$$\alpha_{3,5}^{13} \to x_3$$

$$\alpha_{6,9}^{14} \to x_4$$

$$\alpha_{3,12}^{14} \to x_5$$

$$\alpha_{3,4}^{12} \to x_6$$

$$\alpha_{5,10}^{14} \to x_7$$

$$\alpha_{7,8}^{14} \to x_8$$

$$\alpha_{2,5}^{12} \to x_9$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_4): &\quad -x_6 - x_9 + 1 && = 0 \\
(e_1, e_2, e_5): &\quad -x_1 - x_3 + x_9 && = 0 \\
(e_1, e_3, e_4): &\quad -x_3 + x_6 && = 0 \\
(e_1, e_2, e_{12}): &\quad -x_5 - 1 && = 0 \\
(e_1, e_3, e_{11}): &\quad -x_2 - x_5 && = 0 \\
(e_1, e_4, e_{10}): &\quad -x_2 - x_7 && = 0 \\
(e_1, e_5, e_9): &\quad -x_4 - x_7 && = 0 \\
(e_1, e_6, e_8): &\quad -x_4 - x_8 && = 0 \\
(e_2, e_3, e_5): &\quad x_3 - x_5 x_9 + x_7 && = 0
\end{aligned}$$

Groebner basis (9 variables, 8 linear, 0 nonlinear)

$$x_1 - 2x_9 + 1 = 0$$

$$x_2 - 1 = 0$$

$$x_3 + x_9 - 1 = 0$$

$$x_4 - 1 = 0$$

$$x_5 + 1 = 0$$

$$x_6 + x_9 - 1 = 0$$

$$x_7 + 1 = 0$$

$$x_8 + 1 = 0$$

$\mathfrak{m}_{2B}(8, 14)$

m2B814 (this line included for string searching purposes)

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_5] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_4] &= -e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0 \\
(e_2, e_3, e_4): &\quad \text{no solutions}
\end{aligned}
$$

There are no solutions.

$\mathfrak{m}_{4B}(8, 14)$

m4B814 (this line included for string searching purposes)

Solution 1

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{11} \\
[e_2, e_4] &= e_{12} & [e_2, e_5] &= 3e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_4] &= -2e_{13} \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_{11}] &= e_{14} \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}
$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{11} \\
[e_2, e_4] &= e_{12} & [e_2, e_5] &= \alpha_{2,5}^{13} e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_4] &= \alpha_{3,4}^{13} e_{13} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_4):& \quad -\alpha_{2,5}^{13} - \alpha_{3,4}^{13} + 1 &= 0 \\
(e_1, e_2, e_{12}):& \quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}):& \quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}):& \quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9):& \quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8):& \quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0 \\
(e_2, e_3, e_4):& \quad -\alpha_{3,12}^{14} + \alpha_{3,4}^{13} + \alpha_{4,11}^{14} &= 0
\end{aligned}
$$

Solution 1:

$$\begin{aligned}\alpha^{14}_{4,11} &= 1\\ \alpha^{14}_{6,9} &= 1\\ \alpha^{14}_{3,12} &= -1\\ \alpha^{13}_{3,4} &= -2\\ \alpha^{13}_{2,5} &= 3\\ \alpha^{14}_{7,8} &= -1\\ \alpha^{14}_{5,10} &= -1\end{aligned}$$

How the solution(s) were or were not found:
Change variables

$$\alpha^{14}_{4,11} \to x_1$$

$$\alpha^{14}_{6,9} \to x_2$$

$$\alpha^{14}_{3,12} \to x_3$$

$$\alpha^{13}_{3,4} \to x_4$$

$$\alpha^{13}_{2,5} \to x_5$$

$$\alpha^{14}_{7,8} \to x_6$$

$$\alpha^{14}_{5,10} \to x_7$$

Jacobi Tests

$$\begin{aligned}(e_1, e_2, e_4): &\quad -x_4 - x_5 + 1 &&= 0\\ (e_1, e_2, e_{12}): &\quad -x_3 - 1 &&= 0\\ (e_1, e_3, e_{11}): &\quad -x_1 - x_3 &&= 0\\ (e_1, e_4, e_{10}): &\quad -x_1 - x_7 &&= 0\\ (e_1, e_5, e_9): &\quad -x_2 - x_7 &&= 0\\ (e_1, e_6, e_8): &\quad -x_2 - x_6 &&= 0\\ (e_2, e_3, e_4): &\quad x_1 - x_3 + x_4 &&= 0\end{aligned}$$

Groebner basis (7 variables, 7 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 - 1 = 0$$

$$x_3 + 1 = 0$$
$$x_4 + 2 = 0$$
$$x_5 - 3 = 0$$
$$x_6 + 1 = 0$$
$$x_7 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = -2$$
$$x_5 = 3$$
$$x_6 = -1$$
$$x_7 = -1$$

$\mathfrak{m}_{3B}(9, 14)$

m3B914 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{12} \\
[e_2, e_4] &= e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_{12}] &= -e_{14} & [e_4, e_{11}] &= e_{14} \\
[e_5, e_{10}] &= -e_{14} & [e_6, e_9] &= e_{14} \\
[e_7, e_8] &= -e_{14}
\end{aligned}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{12} \\
[e_2, e_4] &= e_{13} & [e_2, e_{13}] &= e_{14} \\
[e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} & [e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} \\
[e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} & [e_6, e_9] &= \alpha_{6,9}^{14} e_{14} \\
[e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &&= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &&= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &&= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &&= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &&= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,11}^{14} &= 1 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{3,12}^{14} &= -1 \\
\alpha_{7,8}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,11}^{14} &\to x_1 \\
\alpha_{6,9}^{14} &\to x_2 \\
\alpha_{3,12}^{14} &\to x_3 \\
\alpha_{7,8}^{14} &\to x_4
\end{aligned}
$$

$$\alpha_{5,10}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}): \quad & -x_3 - 1 && = 0 \\
(e_1, e_3, e_{11}): \quad & -x_1 - x_3 && = 0 \\
(e_1, e_4, e_{10}): \quad & -x_1 - x_5 && = 0 \\
(e_1, e_5, e_9): \quad & -x_2 - x_5 && = 0 \\
(e_1, e_6, e_8): \quad & -x_2 - x_4 && = 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 - 1 = 0$$
$$x_2 - 1 = 0$$
$$x_3 + 1 = 0$$
$$x_4 + 1 = 0$$
$$x_5 + 1 = 0$$

Solution 1:

$$x_1 = 1$$
$$x_2 = 1$$
$$x_3 = -1$$
$$x_4 = -1$$
$$x_5 = -1$$

$\mathfrak{m}_{2B}(10, 14)$

m2B1014 (this line included for string searching purposes)

Solution 1

$$\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_{12}] &= -e_{14} \\
[e_4, e_{11}] &= e_{14} & [e_5, e_{10}] &= -e_{14} \\
[e_6, e_9] &= e_{14} & [e_7, e_8] &= -e_{14}
\end{aligned}$$

Original brackets:

$$
\begin{aligned}
[e_1, e_2] &= e_3 & [e_1, e_3] &= e_4 \\
[e_1, e_4] &= e_5 & [e_1, e_5] &= e_6 \\
[e_1, e_6] &= e_7 & [e_1, e_7] &= e_8 \\
[e_1, e_8] &= e_9 & [e_1, e_9] &= e_{10} \\
[e_1, e_{10}] &= e_{11} & [e_1, e_{11}] &= e_{12} \\
[e_1, e_{12}] &= e_{13} & [e_2, e_3] &= e_{13} \\
[e_2, e_{13}] &= e_{14} & [e_3, e_{12}] &= \alpha_{3,12}^{14} e_{14} \\
[e_4, e_{11}] &= \alpha_{4,11}^{14} e_{14} & [e_5, e_{10}] &= \alpha_{5,10}^{14} e_{14} \\
[e_6, e_9] &= \alpha_{6,9}^{14} e_{14} & [e_7, e_8] &= \alpha_{7,8}^{14} e_{14}
\end{aligned}
$$

Non-trivial Jacobi Tests:

$$
\begin{aligned}
(e_1, e_2, e_{12}): &\quad -\alpha_{3,12}^{14} - 1 &= 0 \\
(e_1, e_3, e_{11}): &\quad -\alpha_{3,12}^{14} - \alpha_{4,11}^{14} &= 0 \\
(e_1, e_4, e_{10}): &\quad -\alpha_{4,11}^{14} - \alpha_{5,10}^{14} &= 0 \\
(e_1, e_5, e_9): &\quad -\alpha_{5,10}^{14} - \alpha_{6,9}^{14} &= 0 \\
(e_1, e_6, e_8): &\quad -\alpha_{6,9}^{14} - \alpha_{7,8}^{14} &= 0
\end{aligned}
$$

Solution 1:

$$
\begin{aligned}
\alpha_{4,11}^{14} &= 1 \\
\alpha_{6,9}^{14} &= 1 \\
\alpha_{3,12}^{14} &= -1 \\
\alpha_{7,8}^{14} &= -1 \\
\alpha_{5,10}^{14} &= -1
\end{aligned}
$$

How the solution(s) were or were not found:
Change variables

$$
\begin{aligned}
\alpha_{4,11}^{14} &\to x_1 \\
\alpha_{6,9}^{14} &\to x_2 \\
\alpha_{3,12}^{14} &\to x_3 \\
\alpha_{7,8}^{14} &\to x_4
\end{aligned}
$$

$$\alpha_{5,10}^{14} \to x_5$$

Jacobi Tests

$$\begin{aligned}
(e_1, e_2, e_{12}): &\quad -x_3 - 1 &= 0\\
(e_1, e_3, e_{11}): &\quad -x_1 - x_3 &= 0\\
(e_1, e_4, e_{10}): &\quad -x_1 - x_5 &= 0\\
(e_1, e_5, e_9): &\quad -x_2 - x_5 &= 0\\
(e_1, e_6, e_8): &\quad -x_2 - x_4 &= 0
\end{aligned}$$

Groebner basis (5 variables, 5 linear, 0 nonlinear)

$$x_1 - 1 = 0$$

$$x_2 - 1 = 0$$

$$x_3 + 1 = 0$$

$$x_4 + 1 = 0$$

$$x_5 + 1 = 0$$

Solution 1:

$$x_1 = 1$$

$$x_2 = 1$$

$$x_3 = -1$$

$$x_4 = -1$$

$$x_5 = -1$$